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Abstract
Discrete models of labor supply derived from stochastic utility representations have gained popularity because they are more practical than approaches based on continuous working hours. The main reason is that these models make it easy to deal with nonlinear and nonconvex economic budget constraints as well as general utility specifications. In this paper we argue that concerns of practicality is not the only feature of interest of the random utility modeling framework. Random utility models also offer a powerful methodology for addressing new and hitherto neglected aspect of labor market behavior. There are at least two important aspects of the labor market that are less addressed in traditional approaches to labor supply modeling, namely that individuals have preferences over jobs, and that they face restrictions on the choice of jobs and hours of work. This paper discusses how the notion of “job choice”, can be accommodated within a discrete choice model based on stochastic utility. The discussion focuses on model interpretations, theoretical and practical advantages. In order to demonstrate the practical use of the modeling framework, we show labor supply effects of policy changes for a model of couples’ labor supply.

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1. Introduction

Recently, discrete models of labor supply based on stochastic utility specifications have gained widespread popularity, mainly because they are much more practical than the conventional continuous approach based on marginal calculus, see Van Soest (1995) and the survey by Creedy and Kalb (2005). For example, with the discrete choice approach, it is easy to deal with nonlinear and non-convex economic budget constraints, and to apply rather general functional form of the utility representations. The conventional discrete choice approach to labor supply differs from the corresponding continuous approach in that the set of feasible hours of work is approximated by a suitable and finite discrete set. With particular distributional assumptions about the stochastic disturbances in the utility function one can derive tractable expressions for the distribution of hours of work, such as the conditional multinomial logit models or conditional nested multinomial logit models.

From a theoretical perspective, however, the conventional discrete choice approach represents no essential departure from the standard one. This is because the only new assumption postulated is that the set of feasible hours of work is finite and that the random components of the utility function have particular distributional properties.

The main message that we would like to convey in this paper is that the theory of discrete choice, in addition to concerns of practicality, offers new opportunities to address hitherto neglected aspects of labor market behavior. There are at least two important aspects of the labor market that are not addressed in traditional approaches to labor supply modeling, namely that individuals have preferences over jobs, and that they face restrictions on the choice of jobs and hours of work. Standard labor supply models and the conventional discrete choice models suffer from their inability to deal with choice restrictions which is a feature of the labor market due to institutional regulations. For example, the labor market is typically regulated in such a way that most jobs offer only full time or a fixed part time amount of hours of work. This paper discusses how the notion of “job choice” and restrictions on hours of work can be accounted for within the framework of discrete choice and random utility representations. The motivation for advancing beyond the traditional approach is that disposable income and leisure are just two out of several job-related choice variables that matter to the individuals when they make their labor supply decisions, and that, as mentioned above, the presence of important

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1 Practicality is often the main entry into the discrete choice part of labor supply modeling, see Blundell and MaCurdy (1999), Blundell, MaCurdy and Meghir (2007) and Meghir and Phillips (2010). Discrete choice approaches receive relatively little attention compared to other frameworks of obtaining information about labor supply effects and is mainly referred to as a method to “make estimation problems manageable” (Blundell, MaCurdy and Meghir, 2006, p. 74) and useful in order to include programme participation in models, along the lines of Keane and Moffitt (1998) and Brewer et al. (2006).
choice restrictions. This concern has been emphasized in other parts of the labor market literature, see for instance Sattinger (1993), Van Ophem, Hartog and Vijverberg (1993), but the approach we follow in this paper differs from theirs. Here, the modeling approach departs from the notion of latent jobs, characterized by fixed, job-specific hours of work, wage rates and non-pecuniary attributes, such as the nature of tasks to be performed, location of the work place, working environment, etc. In this model, observed hours of work and disposable income, are thus interpreted as hours of work and consumption that follow from the chosen job. The quantitative choice restriction an agent in the labor market faces is represented by a distribution function of offered hours of work (offered by the firms).

Important contributions that address the problem of rationing of hours of work are Altonji and Paxson (1988), Ilmakunnas and Pudney (1990), Van Soest, Woittiez and Kapteyn (1990), Tummers and Woittiez (1991), Dickens and Lundberg (1993), and Bloemen (2000; 2008). The type of model presented here is capable of representing the distribution of preferences on one hand, and economic and other type of choice restrictions, on the other. Possible restrictions on for example part-time hours of work will in this setting be interpreted as less part-time than full-time jobs in the choice set. Finally, the alternative modeling framework we propose is rather practical and user-friendly, in the sense that it is very easy to simulate behavioral effects from policy reforms. Thus, we believe that our alternative framework offers additional advantages to the conventional discrete choice approach that should be of particular relevance for practitioners.

The rest of the paper is organized as follows. In the next section we discuss the theoretical and practical advantages of the model proposed in this paper. Section 3 offers a brief presentation of other approaches. In Section 4 we describe in detail our alternative job choice model. Section 5 provides evidence of the out-of-sample prediction properties of the model and in section 6 we report uncompensated wage elasticities and simulation results for the effect of selected policy reforms. The reforms we consider are the Norwegian tax reform of 2006, and a particular reform of the regulation of the restriction on hours of work. The latter reform is implemented through the offered distribution of hours of work, mentioned above. Thus, this particular reform is a good illustration of the potential of our framework and it highlights how counterfactual simulations of new types of constraints can be addressed. Section 7 concludes.
2. The background

Recent surveys of labor supply modeling, see Blundell and MaCurdy (1999), Blundell, MaCurdy and Meghir (2006) and Meghir and Phillips (2010), focus on both structural labor supply analyses, as well as analyses based on more quasi-experimental designs. In this paper we shall only be concerned with structural analysis, i.e., the formulation and justification of models based on explicit representations of preferences and budget restrictions, and possibly other choice restrictions, and consequently enable the researcher to simulate behavioral effects from counterfactual policy interventions. Such effects are at the core of the discussion on welfare effects of policy changes. We also restrict our discussion to static analysis.

An overwhelming majority of approaches in the static structural labor supply literature is based on variations of the standard textbook theory of consumer demand. As mentioned in the introduction, the obvious fact that agents have preferences over qualitative job attributes, and face important quantitative restrictions is neglected in most studies. A number of researchers have extended the standard labor supply model to the case with nonlinear (piece-wise linear) budget constraints that possibly imply a non-convex budget set. This type of budget sets follow from tax systems found in many countries, where deduction rules for different type of taxes imply that marginal taxes may not be monotonously increasing with income, but may in fact decrease in specific income intervals. Burtless and Hausman (1978) and Hausman (1979, 1980, 1985) and others, such as Blomquist (1983), have made important contributions to the modeling of labor supply in this type of situations. This approach is usually called the "Hausman approach". The Hausman model only extends the standard labor supply model in that it accommodates more realistic budget constraints. Blundell, Duncan and Meghir (1998) is also based on a continuous approach, but relies on an identification strategy based on comparing labor supply over time (covering several tax reforms) for different groups defined by cohort and education level.

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2 With respect to analyses employing reduced form specifications and exploiting some type of exogeneously induced variation, the literature has recently witnessed a number of analyses using tax reform as “natural experiments” and obtaining measures of elasticities of taxable income, following Feldstein (1995); see the survey by Saez, Slemrod and Giertz (2010). Chetty (2009) argues that such studies very efficiently provide information about welfare effects of taxation and cannot therefore simply be discarded for not addressing key questions.

3 MaCurdy, Green and Paarsch (1990) show that the econometric model may impose parametric restrictions that constrain estimates of substitution and income effects in applied work. Mroz (1987) also reviews the specifications employed in the (early) literature and finds that results are sensitive to the methodological choices, such as the measurement of wages. Similarly, Bloemen and Kapteyn (2008) report rather mixed experiences with the Hausman approach: Bloemen and Kapteyn (2008) demonstrates that even in the single agent case it is almost impossible to write down the true likelihood function of the empirical model given standard assumptions about unobservables, and considerable expertise and computer time is required to estimate this type of model.

4 To overcome the integrability problem at kinks Blundell et al. condition out observations close to kinks.
Unfortunately, the Hausman approach, as well as other versions of the standard approach, including the conventional discrete choice models, are silent about the potential importance of job attributes for labor supply behavior. Furthermore, these models cannot accommodate observed peaks at part time and full time hours, which is a typical feature of the hours of work distribution in most countries. Earlier attempts to model rigidities in hours of work and variations in the distribution of hours have taken different forms. For instance, in the discrete choice model of Ilmakunnas and Pudney (1990) different individuals are restricted in various degrees in their opportunities of choosing part-time and full-time work. They utilize actual information on which individuals that are constraints in the access to work, and conduct policy simulations with and without rationing regulations. Dickens and Lundberg (1993) assume a standard labor supply model with a particular rationing device that is somewhat similar in spirit to the one of Ilmakunnas and Pudney (1990), although the final quantitative specification of the model is entirely different. Similarly, Van Soest, Woittiez and Kapteyn (1990) and Tummers and Woittiez (1991) apply various specifications to account for hours of work restrictions.

During the last 15 years the adaptation of discrete choice models has become increasingly popular, mainly because this approach simplifies drastically the implementation of complicated nonlinear budget constraints, for instance for spouses in two-adult households, cf. Van Soest (1995), and because simulations of alternative policies can rather straightforwardly be carried out. The work of Blundell et al. (2000), Van Soest, Das and Gong (2002), Creedy, Kalb and Scutella (2006), Haan and Steiner (2005), Labeaga, Oliver and Spadaro (2008) are good examples of how empirical discrete choice labor supply models can be estimated and simulated for the purpose of assessing the effect of counterfactual tax reforms. The ability of discrete choice models to handle rather complex decision processes is also substantiated by the use of such models to analyze labor supply jointly with welfare programme participation, or employment of non-parental care services (for preschool children). Examples of this type of work are provided by Hoynes (1996), Keane and Moffitt (1998), Brewer et al. (2006), and Kornstad and Thoresen (2007).

Among some researchers there seems to be a belief that the use of discrete choice models in the context of labor supply represents a somewhat crude approximation (Heim, 2009). The reason is that the “true” choice setting is viewed as a continuous one, and consequently the discretization of the choice set of hours of work that follows by implementing a discrete choice model will induce approximation (measurement) errors. Further, it is believed by some researchers that commonly maintained assumptions about the error terms in
the utility function yielding labor supply choice probabilities that satisfy the Independence from Irrelevant Assumption (IIA) property, are unrealistic. In our opinion, such attitudes are are unjustified. The discrete approximation to what is believed to be the true continuous setting is hardly an important point. First, it may be argued (as we do) that the true choice setting is in fact a discrete one. Second, with today’s computer capacity one may use a very fine-meshed partition: there is hardly any limit to the number of discrete alternatives that can be applied.\(^5\) As regards the distributional assumptions of the error terms and the IIA property we shall postpone the discussion of these aspects to section 4 below.

3. Traditional labor supply models

3.1. The standard textbook approach

Typically, the standard approach to labor supply modeling, see Blundell and MaCurdy (1999), is to choose a specification of an individual labor supply function (hours of work function) consistent with the maximization of a quasi-concave utility function in disposable income and leisure, subject to the economic budget constraint. For simplicity, consider the case with convex budget sets with constraint approximated by a suitable smooth representation. Suppose, for example, that the chosen labor supply function has the structure

\[
(3.1) \quad h = \alpha + \beta \tilde{w}(h) + X \gamma + \delta \tilde{I}(h) + \varepsilon,
\]

when

\[
(3.2) \quad \alpha + \beta \tilde{w}(0) + X \gamma + \delta \tilde{I}(0) + \varepsilon > 0,
\]

and \( h = 0 \) otherwise, where \( \tilde{w}(h) \) is the marginal wage rate, \( \tilde{I}(h) \) is so-called virtual non-labor income, \( X \) is a vector of individual characteristics that affect preferences, \( \varepsilon \) is a random error term that is generated by, for example, a normal distribution, \( N(0, \sigma) \), and \( \alpha, \beta, \gamma, \sigma \) and \( \delta \) are unknown parameters. The inequality in (3.2) represents the condition for working. In general, when the tax system is non-linear, the marginal wage rate and virtual income depend on hours of work and they are therefore endogenous. In addition, there is the possibility of a corner solution, represented by (3.2). As a result, one cannot estimate the model by using OLS. Additional complications follow from the fact that the wage is not observed for those who do

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\(^5\) From a theoretical point of view it is interesting that in the case the set of discrete alternatives is infinite the corresponding choice probability distribution will be a continuous one, see Dagsvik and Strøm (2006).
not work. Now, suppose that the parameters of this labor supply function have been estimated. Then, in order to derive the hours of work relation when (3.2) holds, given the wage and non-labor income, one needs to solve for $h$ in the non-linear equations given by (3.1) and (3.2). Let us denote by $h_i = F(w_i, I_i, X_i, T_i, \varepsilon)$ the resulting labor supply function of worker $i$, obtained by solving for hours of work, where $w_i$ is the wage rate, $I_i$ is non-labor income and $T_i$ represents the tax system for worker $i$. The conditional distribution of labor supply (hours of work), given individual characteristics ($w_i, I_i, X_i$), can be estimated by integrating out the unobserved error term $\varepsilon$. Since it is typically difficult to obtain an analytical formula, one can simulate the conditional distribution by drawing $R$ i.i.d. error terms $\{\varepsilon\}$ from the normal distribution, $N(0,\sigma)$, and compute this conditional distribution as

\begin{equation}
(3.3) \quad P(h_i \leq y | w_i, I_i, X_i, T_i) = \int_{F(w_i, I_i, X_i, T_i, \varepsilon) \leq y} F(w_i, I_i, X_i, T_i, \varepsilon) d\Phi(\varepsilon)
\end{equation}

\begin{equation}
\approx \sum_{F(w_i, I_i, X_i, T_i, \varepsilon) \leq y} F(w_i, I_i, X_i, T_i, \varepsilon) \frac{1}{R}.
\end{equation}

The summation in last part of (3.3) is taken over all $k$ such that supplied hours are less than or equal to $y$. The empirical counterpart of the left-hand side of (3.3) is the fraction of workers with characteristics $(w_i, I_i, X_i, T_i)$ that supply hours of work of less than or equal to $y$. The unconditional labor supply distribution can be obtained by

\begin{equation}
(3.4) \quad P(h \leq y) = E(P(h_i \leq y | w_i, I_i, X_i, T_i)) \approx \frac{1}{N} \sum_{i \in \Omega} P(h_i \leq y | w_i, I_i, X_i, T_i),
\end{equation}

where $\Omega$ denotes a representative micro-population of size $N$. In principle, this can be done with general utility specifications and corresponding labor supply functions, but this will in most cases be rather cumbersome in practice. The reason for this is that the class of utility functions that implies explicit labor supply functions, such as the one in (3.1), is rather limited and therefore, in more general cases, one is forced to work with non-linear specifications. Thus, even when the budget constraint is simplified to ensure a convex budget set, the estimation and simulation of labor supply responses are not straightforward.
3.2. The conventional discrete choice model

In this section we shall review briefly the discrete choice approach to labor supply. Conventional discrete choice labor supply models, are typically formulated within a conditional logit model type of technology (McFadden, 1973). Let \( U(C, h) \) denote the agent’s utility function of real disposable income and hours of work, \((C, h)\), and assume that

\[
U(C, h) = v(C, h) + \eta(C, h)
\]

where \( v(C, h) \) is a positive deterministic term that represents the mean utility across observationally identical agents and \( \eta(C, h) \) is a random term that is not correlated with the structural term \( \eta(C, h) \), and with c.d.f. \( \exp(-\exp(-x)) \), defined for real \( x \). Moreover, \( \eta(C, h) \) and \( \eta(C', h') \) are independent for \((C, h) \neq (C', h')\). The budget constraint is given by

\[
C = f(hw, I),
\]

where \( I \) is nonlabor income and \( f(\cdot) \) is the function that transforms gross income into after-tax household income. The function \( f(\cdot) \) can in principle capture all details of the tax and benefit system. Furthermore, we assume that the set \( D \) of feasible hours of work is a finite set. If (3.6) is inserted into (3.5) we obtain that

\[
\tilde{U}(h, w) = U(f(hw, I), h) = v(f(hw, I), h) + \eta(f(hw, I), h) = \log \psi(h, w) + \tilde{\eta}(h, w),
\]

where \( \psi(h, w) = \exp(v(f(hw, I), h)) \), and \( \eta(f(hw, I), h) = \tilde{\eta}(h, w) \). For simplicity we have suppressed nonlabor income in the notation. By well known result from the theory of discrete choice it now follows that the probability \( p(h \mid w) \), that the agent shall supply \( h \) hours of work, given \( D \), the budget constraint in (3.6) and the wage rate and nonlabor income \((w, I)\), is equal to

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6 In the terminology of Resnick (1987) this c.d.f. is called the type III (standard) extreme value distribution, or Gumbel distribution. Other authors call this c.d.f. the type I extreme value distribution. Note that the error terms being distributed according to the type III extreme value distribution \( \exp(-\exp(-x)) \), for real \( x \), in an additive formulation of the utility function is equivalent to the error terms being distributed according to the c.d.f. \( \exp(-1/x) \), for positive \( x \), in a multiplicative utility formulation. This follows immediately by taking the exponential function of the multiplicative utility function in (4.1).
In empirical applications the structural part of the utility function, \( v(C,h) \) is assumed to have a convenient functional form and is allowed to depend on individual covariates, see for example Van Soest (1995) and Van Soest, Das and Gong (2002). Unfortunately, and similarly to the standard model above, the model given in (3.8) is unable to fit the data well in most cases due to observed peaks at full time and part time hours of work. Researchers have therefore replaced the systematic utility term \( v(C,h) \), by a “modified” utility term \( v(C,h) + \gamma(h) \), where \( \gamma(h) \) is equal to one for hours of work different from part time and full time hours of work. For example, \( \gamma(h) = a \) for \( h \) equal to full time hours and \( \gamma(h) = b \), for \( h \) equal to part time hours, where \( a \) and \( b \) are constants that are estimated from data. Under the modified specification the choice model takes the form

\[
(3.9) \quad p(h \mid w) = \frac{\psi(h,w)\exp(\gamma(h))}{\psi(0,0) + \sum_{x \in D} \psi(x,w)\exp(\gamma(x))}.
\]

After the introduction of the modified systematic term of the utility function the model can be made to fit the data quite well. However, there remains the problem of how this practice should be justified. One possibility is to interpret \( v(C,h) + \gamma(h) \) as the “true” representation of the systematic part of the utility function. But this implies that we assume that agents may have non-monotone utility in hours of work. In particular, agents will have higher utility for part time and full time hours of work than for other hours. Although one cannot rule out that individuals may have particular preference for full time hours of work, for example, due to social conventions and habits, it seems more plausible to interpret the peaks found in the data as due to constraints on hours of work. Indeed, some researchers have focused on this latter interpretation. Unfortunately, this interpretation lacks foundation because the modeling framework above has no theoretical rationing device that can rationalize quantity restrictions, apart from the assumption that the set of possible hours is discrete and finite. This is of course an immediate implication of the fact that the type of discrete labor supply model above is based on the standard theoretical framework in which the worker is able to choose leisure and disposable income combinations freely, subject to budget constraints and available set of
discrete hours. The discrete approximation $D$, of what is believed to be the ideal continuous choice set is not essential here. Indeed, by increasing the number of alternatives in $D$ one can make the approximation as close as desired to a continuum. More fundamentally, one can also show that there exists a continuous version of the model in (3.8) (or in 3.9), see Appendix B.

4. The job choice model

We shall now review essential features of our maintained model; estimation results are provided in Appendix A. A more rigorous exposition and justification of this type of model is given by Dagsvik and Strom (2006), and Dagsvik and Jia (2011). As mentioned in the introduction, this model departs in an essential way from previous approaches in that we focus on a more realistic description of the choice environment in which job choice is the fundamental decision variable. In this section we will first present the basic modeling framework, and then show how a model for joint labor supply for married couples can be specified given this framework.7

A job is characterized with fixed (job-specific) working hours and other non-pecuniary attributes. Let $U(C, h, z)$ be the (ordinal) utility function of the household, where $C$ denotes household consumption (disposable income), and $h$ is hours of work. The positive indices, $z = 1, 2, \ldots$, refer to labor market opportunities (jobs) and $z = 0$ refers to the nonmarket alternative. For a market opportunity (job) $z$, associated hours of work and wage rate are assumed fixed and equal to $(H(z), W(z))$. In this paper, we will assume that the hours of work and wage rate take only discrete values in a given set. The utility function is assumed to have the structure

\begin{equation}
U(C, h, z) = v(C, h) + \varepsilon(z),
\end{equation}

for $z = 0, 1, 2, \ldots$, where $v(\cdot)$ is a suitable deterministic function and $\{\varepsilon(z)\}$ are random taste shifters that are i.i.d. with c.d.f. $\exp(-\exp(-x))$ defined for real $x$. The random taste shifters are assumed to account for unobservable individual characteristics and nonpecuniary job-type attributes that affect utility, and hence will vary both across households and job opportunities.

Similarly to the previous section, the particular distribution of the taste shifters $\exp(-\exp(-x))$ is consistent with the property that the choice of jobs satisfies the assumption of independence from irrelevant alternatives (IIA), Luce (1959); we will return to this issue

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7 Bloemen (1990) discuss different specifications let the number of job offers depend on individual characteristics, also allowing wage rates to vary with respect to working hours (in one specification).
shortly below. For given hours and wage rates, $h$ and $w$, the economic budget constraint is given by (3.6). With the same notation as in the previous section, we realize that the term $\psi(h,w)$ is now is to be interpreted as the representative utility of jobs with hours of work $h$, a given wage rate $w$ and nonlabor income.

Agents in the labor market are likely to face restrictions on the set of available market opportunities. This is because there are job types for which the worker is not qualified and there may be variations in the set of job opportunities for which he or she is qualified. In addition, due to competition in the labor market, the most preferred type of job for which a worker is qualified may not necessarily be available to her or him. Let $B(h,w)$ denote the agent’s set of available jobs with hours of work and wage rate $(h,w)$; that is, this set contains those jobs $z$ for which $H(z) = h$ and $W(z) = w$. Let $m(h,w)$ be the number of jobs in $B(h,w)$. There is only one nonmarket alternative, so that $m(0,0) = 1$. The choice sets $\{B(h,w)\}$ are unobserved to the researcher. Here we treat the terms $\{m(h,w)\}$ as deterministic, which means that we neglect possible unobserved heterogeneity in choice sets. See Dagsvik (1994), Dagsvik and Strøm (2006) and Dagsvik and Jia (2011) for discussions on how the approach can accommodate stochastic choice sets.\(^8\)

Further, let $\varphi(h,w)$ denote the probability that the agent chooses a particular job with offered hours $h$, wage rate $w$, given nonlabor income and individual characteristics. Let $G$ be the set of possible wage rates of, $D$ be the maximal set of possible positive hours of work. Analogously to the previous section it follows from standard results in discrete choice theory that the agent will choose job $z$ in $B(h,w)$ if the utility of this job, $v(f(hw,I),h) + \varphi(z)$, is higher than, or equal to the utility of all other jobs that are available, or, what is equivalent, equal to the highest utility that can be attained given the choice restrictions. The corresponding probability that the agent shall choose this job can then be expressed as

\[
(4.2) \quad P\left(v(f(hw,I),h) + \varphi(z) = \max_{x \in D \setminus \{0\}, y \in G, k \in \Omega} \left(\max_{x \in D} \left(v(f(xw,I),h) + \varphi(k)\right)\right)\right) = \frac{\sum_{y \in G, x \in D} \sum_{z \in B(x,y)} \varphi(h,w)}{\sum_{y \in G, x \in D} \sum_{z \in B(x,y)} \varphi(x,y) + \varphi(0,0)}.
\]

\(^8\) There may several reasons for treating choice sets as random, for instance that choice sets are unobserved for the researcher and that agent has limited capacity to identify and take choice sets into account (a type of bounded rationality).
We recognize the expression on the right hand side as the representative utility of job \( z \) divided by the sum of the representative utilities across all available alternatives. The probability \( \varphi(h, w) \) is the probability that the agent shall choose any jobs with hours of work and wage rate \((h, w)\), that is, the probability that the agent shall choose any job within \( B(h, w) \). This probability is therefore obtained by summing the choice probability above over all alternatives within \( B(h, w) \), that is,

\[
\varphi(h, w) = \sum_{z \in B(h, w)} \sum_{x, y \in \mathcal{D}} \psi(h, w) + \psi(0, 0) = \frac{\psi(h, w)m(h, w)}{\psi(0, 0) + \sum_{x, y \in \mathcal{D}} \psi(x, y)m(x, y)},
\]

for positive \( h \) and \( w \), where we recall that \( \psi(h, w) = \exp(v(hw, I, h)) \). When \( h = 0 \) we get

\[
\varphi(0, 0) = \frac{\psi(0, 0)}{\psi(0, 0) + \sum_{x, y \in \mathcal{D}} \psi(x, y)m(x, y)},
\]

for \( h = 0 \). The resulting expression is a choice model that is analogous to a multinomial logit model with representative utility terms \( \{\psi(h, w)\} \), weighted by the frequencies of available jobs, \( \{m(h, w)\} \). Note that it is a consequence of our distributional assumptions of the stochastic error terms in the utility function that the respective numbers of available latent jobs, \( \{m(h, w)\} \), represents a set of sufficient statistics for the corresponding choice sets.

Unfortunately, the \( \{m(h, w)\} \) are not directly observable, but under specific assumptions, one can identify \( m(h, w) \) and \( \psi(h, w) \) and estimate their parameters. For the sake of interpretation, and with no loss of generality, let

\[
\theta = \sum_{x \in \mathcal{G}, x \in \mathcal{D}} m(x, y), \quad \text{and} \quad g(h, w) = m(h, w) / \theta.
\]

The interpretation of \( g(h, w) \) is as the fraction of available jobs (available to the agent) with offered hours of work and wage rates equal to \((h, w)\), whereas the parameter \( \theta \) is the total number of jobs available to the agent. We shall call \( \theta g(h, w) \) the opportunity measure and \( g(h, w) \) the opportunity distribution. When inserting the opportunity measure into the expressions for probabilities, we obtain
\[ \psi(h, w) = \frac{\psi(h, w)g(h, w)\theta}{\psi(0, 0) + \theta \sum_{y \in G, x \in D} \psi(x, y)g(x, y)}, \]

and

\[ \psi(0, 0) = \frac{\psi(0, 0)}{\psi(0, 0) + \theta \sum_{y \in G, x \in D} \psi(x, y)g(x, y)}. \]

The opportunity distribution has the interpretation as the offered distribution of hours and wages, similarly to the formulation of Tummers and Woittiez (1990), Dickens and Lundberg (1993) and Bloemen (2000; 2008). The interpretation of \( \theta \) can be extended to include fixed cost; see Cogan (1981). To realize this, assume that a positive parameter \( c \), representing the utility (disutility) of fixed cost, enters multiplicatively in the utility function given in (2.1) for positive hours of work. Then, evidently, the structure of the choice probabilities above remains the same apart from \( \theta \) which now transforms to \( \theta c \).

In order to obtain a tractable model, and to facilitate identification, the structure of \( g(h, w) \) is further simplified. With no loss of generality we can write

\[ g(h, w) = g_1(h)g_2(w|h), \text{ where } g_2(w|h) \text{ can be interpreted as the conditional probability mass function of offered wage rates given the available jobs with hours } h, \text{ and } g_1(h) \text{ is the probability mass function of the offered hours.} \]

The "empirical" counterpart of \( g_1(h) \) is the fraction of available jobs with hours \( h \). Similarly, the "empirical" counterpart of \( g_2(w|h) \) is the fraction of available jobs with wage \( w \) among the available jobs with hours \( h \). A key feature of our formulation is the assumption that \( g_2(w|h) = g_2(w) \), where \( g_2(w) \) is independent of \( h \). The mean of the distribution \( g_2(w) \) is allowed to vary across unobserved individual qualification characteristics, represented by a random effect, that is, we assume that \( W(z) = \bar{w}\eta \xi(z) \), where \( \bar{w} \) is a term that depends on observed individual characteristics, \( \eta \) denotes the random effect and \( \{\xi(z)\} \) represents the unobserved pure job-specific terms of the wages. The feature that there is unobserved heterogeneity in wages across individuals as well as across jobs rises serious serious identification problems in practice, given typical data.

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\(^9\) Our specification of the empirical model represents an extension of Dagsvik and Strom (2006), but differs from the one assumed by Aaberge, Dagsvik and Strom (1995), and Aaberge, Colombino and Strom (1999). In their analyses it was assumed that wages were job-specific, and that the variation of wages across workers could be captured by observed covariates.
sets available. We have there for not been able to identify and estimate the opportunity Thus, for \( h > 0 \) we can distribution of wages, and the corresponding joint distribution of hours of work and wages. Instead we have estimated the marginal distribution of hours of work, given the mean wage \( \bar{w} \), and given the distribution of the random effect \( \eta \). To this end we need to derive the conditional choice probability for hours of work given the random effect. This can be readily done by using the results in (4.5) and (4.6), although solely in an approximate way, namely as

\[
(4.7) \quad \varphi(h | \eta) = \sum_{w \geq 0} \varphi(h, w | \eta) \approx \frac{\theta \psi(h, \bar{w}, \eta) \theta h(h)}{\psi(0, 0) + \theta \sum_{x \in D} \psi(x, \bar{w}, \eta) \theta h(x)}.
\]

To obtain the result in (4.7) we have assumed the approximation

\[
E(\psi(h, W(z)) | \eta) \approx \psi(h, \bar{w}) \eta.
\]

This approximation will be close as long as the wage variance across jobs, conditional on the random effect, is small. The corresponding unconditional choice probability is obtained by taking expectations with respect to \( \eta \), that is,

\[
\varphi(h) = E_\eta \varphi(h | \eta).
\]

Introducing random effects in the wage equation loosens the somewhat restrictive form of the conditional logit model given by the assumption that error terms are independent and have equal variance (i.i.d.), also known as the independence of irrelevant alternatives (IIA) property. Note that the basic underlying intuition of the IIA assumption is that the agent’s ranking of job opportunities from a subset, \( B \) (say), within the choice set of feasible jobs with given job-specific hours of work and wage rate, does not change if the choice set of feasible jobs is altered.\(^{10}\) The random effect of the wage equation has a similar effect as the random coefficient model (Train, 2003) in that it relaxes the IIA property. See also Haan (2006) on this issue. This type of random effect specification to account for unobserved inter-individual heterogeneity in wages has been used by Dagsvik and Strøm (2006), Dagsvik and Jia (2006) and Kornstad and Thoresen (2007).\(^{11}\)

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\(^{10}\) It is commonly argued that the IIA assumption implies that the odds ratio does not depend on other alternatives.

\(^{11}\) In Dagsvik and Jia (2011) there is also a discussion of the effects of the partitioning of choice sets, which may alleviate the IIA restriction.
5. Out-of-sample prediction properties

As is shown in Dagsvik and Jia (2011) the model fits the data quite well. Another test of model performance is to examine the extent to which the model is able to predict out-of-sample labor supply behavior. In the first exercise, we use data from the same source as our sample for estimation, namely the Labor force survey of 2003 (Statistics Norway, 2009), merged with the income information (Statistics Norway, 2010) for the same year. The advantage of using this sample is that we can construct all variables in the same way as we did for the sample used to estimate the model and apply the same sample-selection rules. During these exercises we keep the parameters of the opportunity measures fixed. Thus, the predictions from the model are, apart from trends in the mean wages, conditional on a stationary choice environment.

For the second prediction we compare model results with income data for 2003. The only sample selection criterion imposed for this simulation is the requirement that the individuals should be wage earners between 26 and 62 years of age. In this data set, we have detailed income data but no information about hours of work. Thus, we only compare the actual and predicted distributions of different income variables.

Two parameters are important when using the model estimated for one year (the base year) to predict labor market behavior in another year (the simulation year): these are the wage growth rate and the inflation rate, both measured from base year to simulation year. We use the observed wage growth rate from 1997 to 2003 and the wage regression for the base year to generate the wage rate in the simulation year. It is also necessary to adjust incomes in the simulation year by using the inflation rate to compute real income in the base year for undertaking the model simulations.

Although the out-of-sample predictions for labor supply behavior are not as good as the within-sample predictions, our model continues to predict the proportions in each category well. As expected, the discrete choice model predicts poorly in this case. Figures 1 and 2 illustrate the observed and predicted distributions of labor supply for married couples based on the first sample, which is described above.

Further, Figure 3 shows the observed and predicted disposable income distribution based on the income and property statistics. In this case, our model predicts well, which indicates good performance. However, we need to be careful when interpreting this result. In addition to depending on the labor supply model (conditional on the wage rates), the distribution of disposable income depends heavily on the wage rate equations (conditional on...
wage rates). We have found that the shape of the distribution of disposable income appears to be quite robust with respect to moderate changes in the distribution of hours of work. Thus, it seems that the distribution of the error term in the wage rate equation is of crucial importance in this context. For example, the standard discrete choice model yields a similar distribution of disposable income to the one obtained from our model. Thus, a poor fit of the distribution of disposable income is not necessarily a sign of a poorly fitting underlying behavioral model, but could result from misspecified wage rate equations. We believe that the assumption of normally distributed error terms in the log wage rate equations may be restrictive. In simulations not reported, it is found that the wage rate equations are not capable of fully reproducing the right tails of the distribution of observed wages in the 2003 sample. This is not surprising because it is well known that the right tail of the lognormal distribution is not heavy enough to capture the right tails of most income distributions. In fact, a closer look at Figure 3 reveals that the right tail of the empirical density seems fatter than the tail of the corresponding simulation.

Figure 1. Predicted and observed distributions of hours of work for married males, 2003

![Figure 1. Predicted and observed distributions of hours of work for married males, 2003](Image)
Figure 2. Predicted and observed distributions of hours of work for married females, 2003

Figure 3. Observed and predicted density of disposable income for married couples, LOTTE 2003
6. Elasticities and selected policy reforms

6.1 Elasticities

In Table 1 below we report selected wage elasticities for married couples obtained from model estimates, see Dagsvik and Jia (2011) for further details. These are aggregate elasticities and they are computed as follows: for each household we simulate the changes in the probability of working and the expected hours of work, $\sum h \varphi(h)$, resulting from a 10 per cent increase in the wage rate (own wage or spouse’s wage). Subsequently, we aggregate over all households in the sample, which yields and aggregate measure of the change in fractions of female and males that will work and the change in mean hours of work after the reform. To obtain the corresponding elasticities we multiply by 10 and divide by the initial aggregate fractions that work and initial work and mean hours of work, respectively.

Table 1 shows both own and cross wage uncompensated elasticities as well as wage elasticities of the probability of working and mean hours of work for wives and husbands.

Table 1. Uncompensated wage elasticities for married couples

<table>
<thead>
<tr>
<th></th>
<th>Female</th>
<th>Male</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base value</td>
<td>Own wage elasticity</td>
</tr>
<tr>
<td>Probability of working</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole sample</td>
<td>0.89</td>
<td>0.333</td>
</tr>
<tr>
<td>Lowest decile</td>
<td>0.87</td>
<td>0.420</td>
</tr>
<tr>
<td>2nd to 8th decile</td>
<td>0.90</td>
<td>0.332</td>
</tr>
<tr>
<td>Highest decile</td>
<td>0.92</td>
<td>0.249</td>
</tr>
<tr>
<td>Mean hours of work, conditional on working</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole sample</td>
<td>1601</td>
<td>0.279</td>
</tr>
<tr>
<td>Lowest decile</td>
<td>1581</td>
<td>0.289</td>
</tr>
<tr>
<td>2nd to 8th decile</td>
<td>1602</td>
<td>0.279</td>
</tr>
<tr>
<td>Highest decile</td>
<td>1618</td>
<td>0.272</td>
</tr>
<tr>
<td>Unconditional mean hours of work</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whole sample</td>
<td>1444</td>
<td>0.612</td>
</tr>
</tbody>
</table>
In general, Table 1 shows that the uncompensated wage elasticities are moderate. For married women the own wage elasticity of the probability of working is 0.33, which means that if the wage rate for females increase by 5 per cent, the fraction of married women that participates in the labor force would increase by 0.017, that is from 0.89 to 0.907. Given work, the wage elasticity of mean hours of work for the wifes is 0.28. Note also that the wage elasticities of mean hours, given participation is approximately constant across decile groups.

There is a number of surveys that report labor supply elasticities, see for example Killingsworth and Heckman (1986), and Blundell and MaCurdy (1999). These surveys report very large variations across studies. It is however not without problems to compare elasticities because some of the models are nonlinear and the elasticities will therefore depend on the distribution of the exogenous households characteristics in the sample. In particular, the aggregate wage elasticities reported in Table 1 depend on the distribution of household characteristics.\(^{12}\)

6.2. Simulations of fiscal reforms

Labor supply effects of the 2006 tax reform

Norway has a “dual income tax” system, enacted in a 1992 tax reform\(^ {13}\) which consists of a combination of a low proportional tax rate on capital income and progressive tax rates on labor income. The system proliferated throughout the Nordic countries in the early 1990s. The Norwegian version had a flat 28 percent tax rate levied on corporate income, capital and labor income coupled with a progressive surtax applicable to labor income. These separate schedules for capital and labor income created obvious incentives for taxpayers to recharacterize labor income as capital income. The 1990s saw increasing pressure on the dual income tax system, resulting in numerous “patches.” As these were not entirely successful, the reform of 2006 emerged as an attempt to create a system that would prevent taxpayers from transforming labor income into capital income to benefit from the lower flat rate applied to the latter.

\(^{12}\) As an illustration of the latter point, let \(P(X)\) denote the probability of working as a function of explanatory variables, represented by \(X\). Suppose this probability has the form of a logit model where log of wage enter as one of the explanatory variables. Then it follows readily that the wage elasticity can be expressed as \(\frac{\partial \log P(X)}{\partial \log W} = b(1 - P(X))\). This implies that the average wage elasticity across the sample is equal to \(b(1 - P)\), where \(P\) is the average probability of participation. Suppose furthermore that \(b = 3\). Provided \(X\) has a distribution that yield \(P = 0.89\) (which is equal to the fraction of married women that participates in our sample), the mean elasticity becomes equal to 0.33. If instead the distribution of \(X\) has a different distribution that for example yields \(P = 0.8\), then the mean elasticity becomes 0.6. This example demonstrates that even for a given model the elasticities may vary a lot and one must therefore be cautious when comparing elasticities for different models and different samples.

\(^{13}\) See Sorensen (1994, 2005) for more on dual income tax systems.
Marginal tax rates on wages were cut to narrow the differences between the marginal
tax rates on capital income and labor income and to encourage labor supply. Figure 4 reflects
the principal features of the Norwegian labor income tax system: a two-tier surtax that
supplements a basic income tax rate of 28 percent plus a 7.8 percent social insurance
contribution. In 2004 the first tier of the surtax was applied at approximately NOK380,000
(USD59,200) at a rate of 13.5 percent, and the second tier of 19.5 percent applied to income
in excess of approximately NOK970,000 (USD151,100). In the 2006 reform, the maximum
marginal tax rate fell from 55.3 to 47.8 percent, but became effective at a lower level of
NOK800,000 (USD124,600). To sum up, the reform effected a dramatic realignment of the
maximum marginal tax rates on capital income in excess of the risk-free rate of return and
wage income, from 28 and 55.3 percent respectively in 2004, to 48.2 percent and 47.8 percent
in 2006. Such cuts might be expected to have substantial labor supply effects. In order to
mitigate the distributional problems associated with the reform, the government increased the
wage income standard deduction, which is constructed by multiplying wage income by a
factor (equal to 24 percent in 2004) subject to a maximum (NOK50,780 or USD7,900, in
2004, in terms of wage-adjusted 2006 kroner). In 2006 the multiplicative factor increased to
34 percent, and the maximum deduction increased to NOK61,100 (USD9,500).

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14 We use an exchange rate of one U.S. dollar for 6.418 Norwegian kroner (NOK), the average exchange rate in 2006.
15 All thresholds are adjusted to 2006 levels.
Figure 4. Statutory Marginal Tax Rates on Wage Income, 2004 and 2006. All Thresholds Adjusted to 2006 Level (1US$=NOK6.418)

Table 2 shows the labor supply effects of the reform. For the sake of completeness we have included the results for single men and women in addition to the results for married/cohabiting couples. According to our model estimates the wage elasticities for single persons are quite small, and we would expect to find small effects of the tax reform on labor supply for this group of households. In fact, the changes in both the participation rate and average hours of work when working are rather small for both men and women.

Table 2. Labor market participation and average hours of work, 2004 and 2006 tax rates

<table>
<thead>
<tr>
<th></th>
<th>2004 tax rates</th>
<th>2006 tax rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Married/cohabiting women</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of labor market participation</td>
<td>0.942</td>
<td>0.951</td>
</tr>
<tr>
<td>Average hours of work when working</td>
<td>1 548</td>
<td>1 561</td>
</tr>
<tr>
<td>Unconditional hours of work</td>
<td>1 457</td>
<td>1 483</td>
</tr>
<tr>
<td>Married/cohabiting men</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average hours of work when working</td>
<td>1 901</td>
<td>1 919</td>
</tr>
<tr>
<td>Single women</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of labor market participation</td>
<td>0.938</td>
<td>0.942</td>
</tr>
<tr>
<td>Average hours of work when working</td>
<td>1 638</td>
<td>1 643</td>
</tr>
<tr>
<td>Unconditional hours of work</td>
<td>1 537</td>
<td>1 546</td>
</tr>
<tr>
<td>Single men</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average hours of work when working</td>
<td>1 870</td>
<td>1 877</td>
</tr>
</tbody>
</table>
For persons living in partnership we find somewhat larger effects. The predicted changes in the distribution of hours of work are a consequence of two effects: i) both men and women are confronted with a new tax system yielding a different real wage rate net of taxes. In addition there is an effect due to the interaction of responses between persons in the couple, cf. the cross wage elasticities of Table 1. From Table 2 we see that the total effect is that both the unconditional and the conditional (given work) average hours of work increase to some extent. For married/cohabiting women the participation probability also increases, by about one percentage point. Given the larger wage elasticities for married/cohabiting women compared to their male partners, we would expect larger effects for women. However, the cross wage elasticities (measured in absolute value) are also larger for women than for men, and this fact suggests that the average labor supply effect of the tax reform is smaller for women than for men. Another point is that the effect of the tax reform also depends on the income level of the person. For instance, the changes in surtax only influence the behavior of high income persons, typically full-time workers. Since men in partnership often have stronger preferences for full-time or overtime work, and also have higher pre-tax wage rate, it seems reasonable to assume that the labor supply responses of the tax reform is larger for men than for their partners.

**Changes in the opportunity distribution of offered hours of work**

As discussed above, it is an important feature of our model that it enables us to represent restrictions on hours of work in a convenient way through the opportunity distribution, and moreover to simulate the effect on labor supply from changing the opportunity distribution. In Norway, a high proportion of married women are found in part time positions. In our sample, more than 35 percent of the married women are working between 20 and 30 hours a week, while only around 40 percent are working full time (37.5 hours a week). Many of those in part time jobs are employed in public sector, especially in public health care sector. While Norwegian working environment legislation opens up the possibility of voluntary part time jobs, an important reason of the high concentration of part time workers is due to institutional restrictions in the public health sector. Part time positions are sometimes the only positions offered by public care organizations, in particular in more rural areas of the country. Recently, there has been a heated discussion among the politicians and trade unions to replace the part time positions with full time positions. It has been argued that the rationing in the labor marked hits women in particular, and that it works against gender equality since women are “forced” to work at home instead of getting paid work. Other discussants are primarily
concerned about the cost of this type of reform. Prospects of future lack of labor supply, particularly in the health sector, have also been an important aspect of the discussion. A proper assessment of the potential labor supply effect of such an organizational change is therefore of considerable interest.

Recall that in our framework, \( g(h) \) is the fraction of available jobs (available to the agent) with offered hours of work equals to \( h \), whereas the parameter \( \theta \) is the total number of jobs available to the agent. Originally, there are two peaks in the estimated offered hour distributions \( g(h) \), where the part time peak can be regarded as generated from the institutional settings of the public care sector. The proposed reform can then represented by a removal of the part-time peak and an increase of the full-time peak (since the part time jobs are replaced by the full time ones) while keeping the total number of jobs unchanged. Under the new opportunity distribution, say \( g^*(h) \), one can apply the model to simulate the corresponding realized labor supply distribution. Figure 5 displays the results given the 2004 tax system. As we can see from the figure, there is a significant decrease of the share of married/cohabiting women who prefer to work part time, accompanied by an increase of the share of full time hours of work of about a similar magnitude. We also observe a slightly increase of share of women who do not work, which may indicate that the new opportunity distribution may be perceived as more restrictive by the agents compared to the old one.

**Figure 5. Change in the aggregated labor supply for married and cohabiting women**
However, one needs to be careful when interpreting these simulation results in the context of reforms specific to labor market sectors (such as the health care sector) since the model does not explicitly account for sector specific preferences and restrictions. Nevertheless, this simulation exercise clearly illustrates the advantage and potential of our modeling framework.

7. Summary
Specification of labor supply models is a complicated and controversial task, and there is no common understanding of what should be the preferred strategy. As we have discussed above the theoretical departure in the labor supply literature is often highly stylized. The importance of specification issues related to functional form and distribution of unobservables, is often neglected or at least played down. Since economic theory with few exceptions is silent about such issues researchers have in practical empirical research resorted to various ad hoc specifications. As a result, there seems to be no consensus in the research community as to what should be the “right” specification, and consequently there is a large variety of specifications in the literature. This may be the most important reason why labor supply wage elasticities in previous work are found to vary all over the map, see Blundell and MaCurdy (1999).

In this paper we have discussed a particular approach based on the notion of choice among job types. While concerns of practicality often have been dominating the motivation for the conventional discrete choice labor supply models, we argue that the discrete choice approach can also be applied to accommodate other important aspects of labor market behavior hitherto neglected, namely that individuals indeed care about the nature and content of the jobs, and that the set of perceived jobs available to them may be limited. Although we as researcher do not observe the the choice of jobs, nor choice restrictions, the model derived for the choices variables that are observable (hours of work of the chosen jobs and the corresponding disposable income). As we have seen above our particular approach allows us to take into account restrictions on the hours of work in a convenient way.

Finally, in order to demonstrate the practical use of the modeling framework in the context of policy simulation analysis we have carried out analyses of selected reforms. In one of these reforms we show how one can account for changes in the opportunity distribution of offered hours of work.
References


Appendix A

Model specification and estimation results

We consider the conditional joint distribution of female and male labor supply given that the male works, since this case corresponds to the empirical application below.

Let \( U(C, h_F, h_M, z) \) denote the utility function of the household, where \( h_F \) and \( h_M \) are hours of work for the female and the male and \( z = (z_F, z_M) \) indexes the combination of jobs for the female and male in the household, respectively. Assume that \( U(C, h_F, h_M, z) = v(C, h_F, h_M)\epsilon(z) \), which is interpreted analogously to the single-individual case above. The budget constraint in this case can be written as

\[
(A.1) \quad C = f \left( h_Fw_F, h_Mw_M, I \right),
\]

where \( w_F \) and \( w_M \) are the respective wage rates for the female and male and \( f(\cdot) \) is the function that transforms gross income to disposable income for the household. Let \( \varphi(h_F, h_M | w_F, w_M) \) be the joint density of hours of work for the female and male in the household, given wage rates and nonlabor income. The empirical counterpart of this density is the fraction of couples in which the wife works \( h_F \) hours and the husband works \( h_M \) hours, within the subpopulation of couples with wage rates and nonlabor income equal to \( (w_F, w_M, I) \). We assume further that the choice sets of jobs offered to the female and to the
male, respectively, are independent. The latter assumption may be restrictive because husband and wife may face the similar constraints on the choice of jobs due to the structure of the local labor market.\footnote{It is however easy to allow choice sets of husband and wife to be correlated.}

Further, if we assume that there is less variance in the wage component, we can simplify the decision making,

\[
\sum_{y_F \in D_1, y_M \in D} \psi(h_F, h_M, y_F, y_M) g_{2F}(y_F) g_{2M}(y_M) \equiv \psi(h_F, h_M, \tilde{w}_F, \tilde{w}_M),
\]

where \(\tilde{w}_F\) and \(\tilde{w}_M\) denote the respective mean wage rates for the female. Then the joint distribution of hours of work for the couple, \(\varphi(h_F, h_M)\), can be written as

(A.2) \[
\varphi(h_F, h_M) \equiv \frac{\theta_F \psi(h_F, h_M, \tilde{w}_F, \tilde{w}_M) g_{1F}(h_F) g_{1M}(h_M)}{K},
\]

for \(h_F > 0\) and \(h_M > 0\), and similarly for the case where \(h_F = 0\), where

(A.3) \[
K \equiv \sum_{x > 0, x \in D} \psi(0, x, 0, \tilde{w}_M) g_{1M}(x) + \sum_{x_F \in D_1, x_M \in D} \psi(x_F, x_M, \tilde{w}_F, \tilde{w}_M) \theta_F g_{1F}(x_F) g_{1M}(x_M),
\]

and \(\theta_F\) represents the total amount of jobs available to the female.

Next, we describe how the model given by (A.2) is empirically specified in order to be estimated. We consider only households where the husband is working. Further, we assume that the densities of offered hours, \(g_k(h), k = F, M\), are uniform except for peaks at full-time and part-time hours. The part-time and full-time peak in the hours distribution captures institutional restrictions and technological constraints and hence market imperfections in the economy. We specify seven intervals for hours of work. The medians of the intervals are 315, 780, 1,040, 1,560, 1,976, 2,340 and 2,600. Thus, the set \(D\) consists of these points. The full-time peak occurs in the fifth interval, in which the median is 1,976 annual hours. The part-time peak is related to the third interval, which has a median of 1,040 annual hours. These intervals correspond to the institutional setting of what constitutes full-time and half-time annual hours of work. The wage rates are assumed to be individual specific and equal across jobs for a given individual.

To deal with the problem of wage rates being unobserved for those who do not work and of wage rates, possibly being correlated with the taste-shifters in the utility function, we estimate instrumental wage rate equations. For \(k = F, M\), we assume that the mean in the
offered wage distribution for gender $k$ is equal to $\bar{w}_k \eta_k$, where $\{\eta_k\}$ are random terms that account for unobserved differences in wage rates across workers, and we assume that $\log \eta_k$, $k = F, M$, are independent and normally distributed, $N(0, \sigma_k^2)$. Furthermore, we assume that $\log \bar{w}_k$ is a linear function of the amount of schooling, experience and experience squared. Experience is defined as age minus years of schooling minus seven. When the wage rate equations are inserted into the model for married couples and when the error terms in these equations are integrated out, we obtain from (4.7) the following empirical model for the joint labor supply density for married couples

\[
\varphi(h_F, h_M) = E\left[ \psi(h_F, h_M; \bar{w}_F \eta_F, \bar{w}_M \eta_M) g_F(h_F) g_M(h_M) \theta_F \right] / K(\eta_F, \eta_M),
\]

for $h_F > 0, h_M > 0$, and similarly for $h_F = 0, h_M > 0$, where $K$ is defined above. The expectation in (A.4) is taken with respect to the error terms in the wage rate equations. In practice, we compute the expectation by Monte Carlo simulation when estimating the model. The term $\theta_F$ is assumed to depend on the wage rates solely through the amount of schooling. Specifically, we assume that

\[
\log \theta_F = f_{F1} + f_{F2} S,
\]

where $S$ is the length of education. Furthermore, we specify $\nu(\cdot)$ to be of the form

\[
\log \nu(C, h_F, h_M)
= \alpha_2 \left( \frac{10^{-4} (C - C_0)}{\alpha_1} \right)^{\alpha_1} - 1 + \left( \frac{(L_F - L_0)^{\alpha_3} - 1}{\alpha_3} \right) (\alpha_5 + \alpha_6 \log A_F + \alpha_7 (\log A_F)^2 + \alpha_8 CU6 + \alpha_9 CO6)
+ \left( \frac{(L_M - L_0)^{\alpha_4} - 1}{\alpha_4} \right) (\alpha_{10} + \alpha_{11} \log A_M + \alpha_{12} (\log A_M)^2 + \alpha_{13} CU6 + \alpha_{14} CO6)
+ \alpha_{15} \left( \frac{(L_M - L_0)^{\alpha_4}}{\alpha_4} - 1 \right) \left( \frac{(L_F - L_0)^{\alpha_3}}{\alpha_3} - 1 \right)
\]

where $C_0$ and $L_0$ are subsistence levels and $A_k$, $k = F, M$, is age for gender $k$ divided by 10, $CU6$ and $CO6$ are the number of children below or equal to and above the age of six,
respectively, \( C \) is given by (4.7), \( L_k, k = F, M \), is leisure for gender \( k \), with

\[
L_k - L_0 = 1 - h_k / 3650,
\]

and \( \alpha_j, j = 1, 2, \ldots, 15 \), are unknown parameters. Note that we have subtracted from total annual hours a “subsistence” level, \( L_0 = 5,110 \) hours, which allows for sleep and rest. This corresponds to about 14 hours per day reserved for sleep and rest. We have chosen \( C_0 \) to be approximately NOK \( 40,000 \sqrt{N} \), where \( N \) is the number of persons in the household. Disposable income, \( C \), is measured as the sum of the annual wage incomes of the woman and her husband after tax, household capital income after tax and child allowances.

To control for selection bias when estimating the wage equations, we apply the estimation procedure proposed by Dagsvik and Strøm (2006). Conditional on the estimated parameters of the wage equations, the remaining parameters of the model are estimated in a second stage by using the maximum likelihood procedure. This two-stage procedure has the added advantage that it reduces the measurement error caused by a negative correlation between hours of work and wage rates.

**Table A1. Estimates of wage equations, females and males, 1997**

<table>
<thead>
<tr>
<th>Variables</th>
<th>Males Estimate</th>
<th>Males t-value</th>
<th>Females Estimate</th>
<th>Females t-value</th>
<th>Females (selection corrected) Estimate</th>
<th>Females (selection corrected) t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>4.08</td>
<td>135.1</td>
<td>4.10</td>
<td>132</td>
<td>4.11</td>
<td>109</td>
</tr>
<tr>
<td>Experience in years/10</td>
<td>0.22</td>
<td>12.2</td>
<td>0.143</td>
<td>8.6</td>
<td>0.141</td>
<td>7.8</td>
</tr>
<tr>
<td>((\text{Experience in years/10})^2)</td>
<td>-0.03</td>
<td>-10.1</td>
<td>-0.022</td>
<td>-6.6</td>
<td>-0.022</td>
<td>-6.1</td>
</tr>
<tr>
<td>Education in years</td>
<td>0.044</td>
<td>26.9</td>
<td>0.0388</td>
<td>23.1</td>
<td>0.0386</td>
<td>19.7</td>
</tr>
<tr>
<td>Married</td>
<td>0.05</td>
<td>6.02</td>
<td>-0.022</td>
<td>-2.67</td>
<td>-0.21</td>
<td>-2.37</td>
</tr>
<tr>
<td>Log(P)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance of error term</td>
<td>0.3029</td>
<td></td>
<td>0.2755</td>
<td></td>
<td>0.2755</td>
<td></td>
</tr>
<tr>
<td>No. observations</td>
<td>5,448</td>
<td></td>
<td>5,074</td>
<td></td>
<td>5,074</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.15</td>
<td></td>
<td>0.10</td>
<td></td>
<td>0.10</td>
<td></td>
</tr>
</tbody>
</table>
Table A2. Parameter estimates of the labor supply probabilities. Married couples, 1997

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Married Couples</th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>α₁</td>
<td>0.6643</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>α₂</td>
<td>1.8411</td>
<td>0.352</td>
</tr>
<tr>
<td></td>
<td>C₀</td>
<td>40,000√N</td>
<td></td>
</tr>
<tr>
<td>Female leisure</td>
<td>α₃</td>
<td>-0.8334</td>
<td>0.182</td>
</tr>
<tr>
<td></td>
<td>α₅</td>
<td>11.8387</td>
<td>1.888</td>
</tr>
<tr>
<td></td>
<td>α₆</td>
<td>-12.5285</td>
<td>1.945</td>
</tr>
<tr>
<td></td>
<td>α₇</td>
<td>5.2456</td>
<td>0.733</td>
</tr>
<tr>
<td></td>
<td>α₈</td>
<td>0.9682</td>
<td>0.168</td>
</tr>
<tr>
<td></td>
<td>α₉</td>
<td>0.5075</td>
<td>0.094</td>
</tr>
<tr>
<td>Male leisure</td>
<td>α₄</td>
<td>-1.8043</td>
<td>0.430</td>
</tr>
<tr>
<td></td>
<td>α₁₀</td>
<td>3.8929</td>
<td>1.112</td>
</tr>
<tr>
<td></td>
<td>α₁₁</td>
<td>-4.3054</td>
<td>1.142</td>
</tr>
<tr>
<td></td>
<td>α₁₂</td>
<td>1.6682</td>
<td>0.444</td>
</tr>
<tr>
<td></td>
<td>α₁₃</td>
<td>0.0547</td>
<td>0.051</td>
</tr>
<tr>
<td></td>
<td>α₁₄</td>
<td>0.0083</td>
<td>0.029</td>
</tr>
<tr>
<td>Leisure interaction</td>
<td>α₁₅</td>
<td>0.2047</td>
<td>0.147</td>
</tr>
<tr>
<td>Leisure subsistence</td>
<td>L₀</td>
<td>5,110</td>
<td></td>
</tr>
</tbody>
</table>

The parameters θ_F: logθ_F = f_F₁ + f_F₂S

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>f_F₁</td>
<td>-3.5041</td>
</tr>
<tr>
<td>Education</td>
<td>f_F₂</td>
<td>1.2389</td>
</tr>
</tbody>
</table>

Opportunity density of offered hours

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male full-time peak</td>
<td>2.3769</td>
<td>0.086</td>
</tr>
<tr>
<td>Female full-time peak</td>
<td>1.4380</td>
<td>0.296</td>
</tr>
<tr>
<td>Male part-time peak</td>
<td>1.0960</td>
<td>0.063</td>
</tr>
<tr>
<td>Female part-time peak</td>
<td>0.5622</td>
<td>0.067</td>
</tr>
</tbody>
</table>

Number of Observations | 2,511 |
Log likelihood | -5,706.5 |
McFadden’s ρ² | 0.44 |
Appendix B

A continuous logit labor supply model

We shall now demonstrate that a continuous random utility model that corresponds to the conventional discrete choice model discussed in section 3.2. The exposition here is based on Dagsvik (1994). Assume now that $D$ is a continuum on the positive part of the real line and let $B$ be the countable subset of $D$ consisting of an infinite number of independent and uniformly distributed hours $H(z)$, $z = 1, 2, \ldots$, on $D$. Thus, there is no essential difference between $B$ and $D$ as representations of the set of feasible hours. Furthermore, let $\{\eta(H(z)), z = 1, 2, \ldots\}$ be an enumeration of points of a Poisson process on $(0, \infty)$ with intensity measure $\varepsilon^2 d\varepsilon$. Then by Resnick (1987, p.135) it follows that the points $\{(H(z), \eta(H(z)), z = 1, 2, \ldots\}$, are the points of a Poisson process on $D \times (0, \infty)$, with intensity measure $\varepsilon^2 d\varepsilon dh$. Assume that that $\eta(0)$ is a positive random variable with c.d.f. $\exp(-1/x), x > 0$. Let $\bar{U}(h)$ be the utility of working $h$ hours and assume that

(B.1) \[ U(h) = \psi(h, w) \eta(h) \]

for positive $h$, and assume Let $p(h \mid w)$ be the probability that the agent chooses hours of work $h$, given that the choice is determined by maximizing the utility function in (B.1) subject to hours of work belonging to the set $B$. Dagsvik (1994) demonstrates that

(B.2) \[ p(h \mid w) = \frac{\psi(h, w)}{\psi(0, 0) + \int_{x \in D} \psi(x, w) dx} \]

Clearly, the model in (B.2) is a continuous version of the model discussed in section 3.2.