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Finite Sample Revision Variances for ARIMA Model-Based Signal Extraction

Tucker McElroy[1] *and Richard Gagnon*[1]

We consider properties of revisions to mean squared optimal concurrent estimates of unobserved components, e.g., seasonal adjustments or trends, obtained by ARIMA model-based signal extraction methods like those used by SEATS. Concurrent estimates, i.e., the estimates for the most recent month (or quarter), are updated whenever future observations become available, and the difference between the concurrent estimate and the mean squared optimal update is called the revision. It is therefore of interest to measure the variance of the revisions, which generally increase in size as more data becomes available; this is because the signal extraction mean squared error decreases as more observations are added. In this article we compute the variance of the revision based on a finite sample of data, where the revision lead, i.e., the amount of additional data that becomes available, is potentially infinite. The revision variance for the infinite revision lead represents the maximal revision variance possible, and is useful as a normalization. Our focus is on presenting practical, easily coded algorithms for computation of the revision variances. These make possible a finite sample-based alternative to SEATS's "percentage reduction in the standard error of the revision after additional years," which assumes that estimates are obtained using an infinite past. We present numerical comparisons between SEATS's diagnostic and our finite sample analogue.

Key words: Signal extraction; ARIMA models; seasonal adjustment.

1. Introduction

In the production of economic data at an official statistical agency, such as the U.S. Census Bureau, "on-line" or contemporaneous publications are inevitably updated as more data becomes available. But this updating can be encountered in a much wider venue. For example, suppose that an analyst produces three-step-ahead forecasts of a dynamic variable of interest; one and two periods into the future the forecast can be updated, reducing its error, and three periods into the future one knows the actual value, reducing the error to zero. Seasonal adjustment can be viewed as a signal extraction problem, and contemporaneous (referred to as concurrent) estimates often involve a higher degree of error, which is typically reduced as more data becomes available. The current practice at the U.S. Census Bureau is to update initial seasonal adjustment estimates a year later, when 12 more values have been obtained. After a certain point, determined by how rapidly the signal extraction filter coefficients decay, the revision becomes negligible.

[1] U.S. Census Bureau, 4600 Silver Hill Rd., Washington, DC 20233-9100, U.S.A. E-mails: tucker.s.mcelroy@census.gov and richardegagnon@aol.com
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There is a great deal of interest in knowing the variance of the error induced by revising the estimates, since this quantity indicates the extent to which signal extraction estimates can be expected to change when new data becomes available. Note however that, this revision variance is often only a small contributor to the overall signal extraction error. Knowledge of the revision error process can allow one to quantify confidence in the reliability of concurrent filtered estimates, and also gauge ahead of time exactly how many revisions are necessary to achieve a given tolerance of error. It is challenging to achieve precise results in a nonparametric framework, but in a model-based signal extraction scenario exact mathematical results can be obtained and applied. While this is the primary advantage of a model-based approach, the corresponding limitation is that the results are model dependent (and thus are wrong to the extent that the model is wrong). These results (and how to implement them) are the primary focus of the article.

As a secondary application, we consider the revision error problem for growth rates of a signal of interest. The growth rate is typically expressed as the percent change of the signal considered at two nearby time points, e.g., the signal at the present time point compared to the signal at the previous time point. If we apply a logarithmic transform to the raw data (which is not uncommon for economic series), this percent change is approximately a difference in the log scale. These growth rates are perceived by many consumers of seasonally adjusted data to be a valuable characteristic of the series, and hence it is desirable to quantify uncertainty about these rates. This can be done by computing revision variances for the rates. Below, we show that simple extensions of the revision error methodology for signal extraction can handle growth rates as well.

Pierce (1980) obtained some of the first results on revision variances, showing that optimal forecast and backcast extension of the data minimizes mean squared revisions. More recently, Planas and Depoutot (2002) described how to construct approximating filters that reduce the length of the revision period. However, these papers generally assume that a semi-infinite sample of data is available. We proceed a bit more generally, allowing for a finite sample (which is more realistic) and obtaining more precise results. We first treat the case of revising with a finite amount of additional data; for this purpose, it is natural to follow the matrix-based signal extraction theory of Bell and Hillmer (1988) and McElroy (2008). We also wish to compare these revisions to the "ultimate" revision, which is theoretically obtained when we have an infinite amount of additional data. In order to develop the appropriate formulas, we must project onto a semi-infinite sample, and so we use the approach of Bell and Martin (2004). Section 2 describes the general approach to computing the revisions, while Section 3 describes the requisite algorithms for the semi-infinite sample case in considerable detail. These methods require the calculation of partial fraction decompositions, and the Appendix discusses some computationally efficient approaches. In Section 4 we present some examples which serve to illustrate how the revision variances are computed and what information they provide. Section 5 concludes.

2. Revisions

In this section we develop expressions for the revisions and their variances. In order that the discussion here be self-contained, we include in Section 2.1 some background material

on signal extraction based on a finite sample of data (the case of signal extraction from a semi-infinite sample is discussed in Section 3). Then Section 2.2 provides the basic revision variance formulas, and Section 2.3 discusses how these can be calculated in practice. Both these latter sections also deal with revision variances for growth rates. We note that revision variances are an increasing function of the revision lead, which reflects the idea that as more data becomes available, a greater revision of the signal extraction estimate is to be expected.

2.1. Background on Finite Sample Signal Extraction

The following material can be found in an expanded form in McElroy (2008). We consider the additive decomposition of our data vector $Y = (Y_1, Y_2, \ldots, Y_n)'$ into signal S and noise N:

$$Y = S + N$$

Following Bell (1984), we let Y_t be an integrated process such that $W_t = \delta(B)Y_t$ is stationary, where B is the backshift operator and $\delta(z)$ is a polynomial with all roots located on the unit circle of the complex plane (also, $\delta(0) = 1$ by convention). This $\delta(z)$ is referred to as the differencing operator of the series, and we assume it can be factored into relatively prime polynomials $\delta^S(z)$ and $\delta^N(z)$ (i.e., polynomials with no common zeroes), such that the series

$$U_t = \delta^S(B)S_t \quad V_t = \delta^N(B)N_t \tag{1}$$

are stationary, mean zero time series. Note that $\delta^S = 1$ and/or $\delta^N = 1$ are included as special cases (in these cases either the signal or the noise or both are stationary). We let d be the order of δ, and d_S and d_N are the orders of δ^S and δ^N; since the latter operators are relatively prime, $\delta = \delta^S \cdot \delta^N$ and $d = d_S + d_N$.

As in Bell and Hillmer (1988), we assume Assumption A of Bell (1984) holds for the component decomposition, and we treat the case of a finite sample with $t = 1, 2, \ldots, n$ with $n > d$. Assumption A states that the initial d values of Y_t, i.e., the variables $Y_* = (Y_1, Y_2, \ldots, Y_d)$, are independent of $\{U_t\}$ and $\{V_t\}$. For a discussion of the implications of this assumption, see Bell (1984) and Bell and Hillmer (1988). A further assumption that we make is that $\{U_t\}$ and $\{V_t\}$ are uncorrelated with one another.

Now we can write (1) in a matrix form, as follows. Let Δ be an $(n-d) \times n$ matrix with entries given by $\Delta_{ij} = \delta_{i-j+d}$ (the convention being that $\delta_k = 0$ if $k < 0$ or $k > d$).

$$\Delta = \begin{bmatrix} \delta_d & \ldots & \delta_1 & 1 & 0 & 0 & \ldots \\ 0 & \delta_d & \ldots & \delta_1 & 1 & 0 & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & \delta_d & \ldots & \delta_1 & 1 \end{bmatrix}$$

The matrices Δ_S and Δ_N have entries given by the coefficients of $\delta^S(z)$ and $\delta^N(z)$, but are $(n - d_S) \times n$ and $(n - d_N) \times n$ dimensional, respectively. This means that each row

of these matrices consists of the coefficients of the corresponding differencing polynomial, horizontally shifted in an appropriate fashion. Hence

$$W = \Delta Y \quad U = \Delta_S S \quad V = \Delta_N N$$

where $Y = (Y_1, Y_2, \ldots, Y_n)'$, and W, U, and V are analogously defined. Then it is possible to write the estimate $\hat{S}$ as a linear matrix operating on Y:

$$\hat{S} = FY$$

The error covariance matrix, i.e., the covariance matrix of $\hat{S} - S$, is denoted by M; both F and M are given in McElroy (2008). The formula for M is

$$M = \left(\Delta_S' \Sigma_U^{-1} \Delta_S + \Delta_N' \Sigma_V^{-1} \Delta_N\right)^{-1} \tag{2}$$

where Σ_X denotes the covariance matrix for any random vector X.

2.2. Revision Variances

We now derive the formula for the revision variance, displayed in (5) below. For further clarity, let $\hat{S}_{t|_1^n}$ denote the estimate of S_t based on data $Y_1, Y_2, \ldots, Y_n$, where $1 \leq t \leq n$. (It is possible to extend the discussion to the case $t > n$ as well, using some extensions of the formulas in McElroy (2008), but this case will not be pursued here.) In our matrix notation, this corresponds to the tth row of the matrix F applied to Y, where F is n-dimensional and Y has n components. Then the hth revision is defined as

$$\hat{S}_{t|_1^{n+h}} - \hat{S}_{t|_1^n} \tag{3}$$

for any $h > 0$. Note that $\hat{S}_{t|_1^{n+h}}$ can be written as an $n+h$-dimensional matrix F acting on $(Y_1, \ldots, Y_{n+h})'$. Now although the process in (3) above has zero mean, its variance depends on t, so we compute its variance in an indirect manner. Write

$$\hat{S}_{t|_1^n} - S_t = \left(\hat{S}_{t|_1^n} - \hat{S}_{t|_1^{n+h}}\right) + \left(\hat{S}_{t|_1^{n+h}} - S_t\right) \tag{4}$$

and note that the quantities on the right hand side are uncorrelated with each other, since the first is a linear combination of the data $Y_1, Y_2, \ldots, Y_{n+h}$ which is always uncorrelated with the error process of the minimum mean squared error estimate of S_t based on data up through time $n+h$. Therefore,

$$\mathbb{E}\left[\left(\hat{S}_{t|_1^n} - S_t\right)^2\right] = \mathbb{E}\left[\left(\hat{S}_{t|_1^{n+h}} - \hat{S}_{t|_1^n}\right)^2\right] + \mathbb{E}\left[\left(\hat{S}_{t|_1^{n+h}} - S_t\right)^2\right]$$

Denoting the mean squared error of $\hat{S}_{t|_1^n}$ by $D_{t|_1^n}$, we have the revision variance equal to

$$R_t(h) = D_{t|_1^n} - D_{t|_1^{n+h}} \tag{5}$$

which is always a positive quantity. Since $D_{t|_1^{n+h}}$ must be a nonincreasing function in h (since more data is being added), the revision variance $R_t(h)$ is a nondecreasing function of the revision lead. Note that $R_t(h)$ also depends on n, but this dependency has been suppressed from the notation. The squared root of this revision variance is the standard

error of the revision, and can be used in conjunction with the data distribution to produce a confidence interval for $\hat{S}_{t|_1^{n+h}}$ centered at $\hat{S}_{t|_1^n}$. For a normal distribution, an approximate 95 percent interval is given by

$$\hat{S}_{t|_1^n} \pm 2\sqrt{R_t(h)}$$

By plotting this as a function of h, we obtain an envelope that will contain the revised estimate about 95 percent of the time; this gives an idea of how much the estimate may change in the future. Note that, since the signal extraction mean squared error formulas only depend on the models that are assumed for S_t and N_t, for a given t and n we can calculate $R_t(h)$ for any future horizon h, without the need for additional data.

As another application, suppose that we are interested in the revision of an estimated growth rate. For some $p > 0$, the growth rate at time t could be measured via $S_t - S_{t-p}$ (assume that we have already log-transformed the data), which would be estimated by

$$\hat{S}_{t|_1^n} - \hat{S}_{t-p|_1^n}$$

assuming that $t - p \geq 1$. Hence the revision, based on h additional data points, is

$$\left(\hat{S}_{t|_1^{n+h}} - \hat{S}_{t-p|_1^{n+h}}\right) - \left(\hat{S}_{t|_1^n} - \hat{S}_{t-p|_1^n}\right)$$

Now using a decomposition of the revision similar to (4), the revision variance for the growth rate is

$$\tilde{R}_t(h) = \tilde{D}_{t|_1^n} - \tilde{D}_{t|_1^{n+h}}$$

where $\tilde{D}$ denotes the variance of the signal extraction error for the growth rate. Explicit formulas for these quantities are provided in Section 2.3 below.

2.3. Implementation of Revision Variance Formulas

Let $M^{(n)}$ denote the n-dimensional covariance matrix of the error process associated with signal extraction matrix F, as given in (2). Then

$$D_{t|_1^n} = M_{tt}^{(n)}$$

and it follows that the revision error variance, for finite revision horizon h, is computed via

$$R_t(h) = D_{t|_1^n} - D_{t|_1^{n+h}} = M_{tt}^{(n)} - M_{tt}^{(n+h)}$$

Computation of M involves several matrix inversions, so this procedure is computationally cumbersome for large h or large n. One advantage is that revision errors at all filter points t can be obtained at one time, for fixed n and h.

For the case of growth rates, the revision variance $\tilde{R}_t(h)$ is also easily obtained. Let e_j denote a vector (of dimension n or $n + h$ depending on the context) with a 1 in the jth component, and zeroes elsewhere. Then the growth rate estimate can be written as

$$(e_t - e_{t-p})'FY$$

where F and Y have dimension n or $n+h$ as the case may be. It follows that the mean squared signal extraction error, where the "signal" is now the growth rate, is given by

$$(e_t - e_{t-p})' M (e_t - e_{t-p}) = M_{tt} + M_{t-p,t-p} - 2M_{t,t-p}$$

The above formula gives $\tilde{D}_{t|_1^n}$ and $\tilde{D}_{t|_1^{n+h}}$, so that

$$\tilde{R}_t(h) = M_{tt}^{(n)} + M_{t-p,t-p}^{(n)} - 2M_{t,t-p}^{(n)} - M_{tt}^{(n+h)} - M_{t-p,t-p}^{(n+h)} + 2M_{t,t-p}^{(n+h)}$$

We should also consider the case that $h = \infty$, when an infinite amount of future data is available to perform revisions. This is only an ideal situation, but the corresponding revision error $R_t(\infty)$ yields the maximal limiting value of this increasing function of h, and thus serves as a suitable benchmark for the finite-lead revisions. In the seasonal adjustment program SEATS (Maravall and Caporello 2004), the following measure is used:

$$1 - \sqrt{1 - R_t(h)/R_t(\infty)} \tag{6}$$

The interpretation of this is given as follows: suppose that (6) exceeds some threshold $1 - \alpha$; this implies that

$$\sqrt{R_t(\infty) - R_t(h)} < \alpha \sqrt{R_t(\infty)} \Leftrightarrow \sqrt{D_{t|_1^{n+h}} - D_{t|_1^{\infty}}} < \alpha \sqrt{D_{t|_1^{n}} - D_{t|_1^{\infty}}}$$

Now the quantity $D_{t|_1^{n+h}} - D_{t|_1^{\infty}}$ is the variance of the revision obtained when we have data up through time $n+h$ and we get infinitely more data; whereas $D_{t|_1^{n}} - D_{t|_1^{\infty}}$ is the variance of the revision obtained when we have data up through time n and we get infinitely more data. The former quantity is that portion of the revision variance $R_t(\infty)$ that remains after we have accounted for the revision variance $R_t(h)$; thus

$$\frac{D_{t|_1^{n+h}} - D_{t|_1^{\infty}}}{D_{t|_1^{n}} - D_{t|_1^{\infty}}} = \frac{R_t(\infty) - R_t(h)}{R_t(\infty)}$$

is the proportion of the infinite lead revision variance $R_t(\infty)$ that remains after accounting for the finite lead revision variance $R_t(h)$. If the squared root of this ratio is less than α, then most of the revisions have already occurred, and there is little additional benefit in revising previous filter estimates. The calculation of $R_t(\infty)$ is theoretically more complicated than the finite-lead revision variances, since it cannot be accomplished with filter matrices. The next section discusses various mathematical details needed to compute these infinite-lead revision variances, following and adapting the exposition of Bell and Martin (2004).

3. Filtering from a Semi-Infinite Sample

We first set out carefully the mathematics behind two-component partial fraction decompositions for power series. The basic context is a signal-noise component model

given by

$$\phi(F)Y_t = \theta(F)\epsilon_t$$

$$\phi^S(F)S_t = \theta^S(F)\xi_t$$

$$\phi^N(F)N_t = \theta^N(F)\eta_t$$

where the ϕ polynomials combine *AR* operators and differencing operators necessary to produce a stationary time series model. The θ polynomials are *MA* operators, acting on the white noise sequences ϵ,ξ, and η. Although it is unusual to write *ARIMA* models in terms of the forward shift operator $F = B^{-1}$, this is the form that is most convenient for stating the mathematical results that follow (see Box and Jenkins (l976) for material on "backwards models"). Later on we present some results for the more conventional *ARIMA* equations, where F is replaced by B. Now the asymmetric signal extraction filters are computed from various combinations of these model operators. Letting $\pi(z) = \phi(z)/\theta(z)$, the optimal signal extraction filter that uses m past observations and an infinite number of future observations is

$$\alpha_S^{(m)}(F) = \frac{B^m}{\sigma_\epsilon^2}\pi(F)[\pi(B)\gamma_S(F)F^m]_+ \tag{7}$$

where $[\cdot]_+$ is an indicator function for nonnegative powers of F and $\gamma_S(z)$ is the pseudo-autocovariance generating function for the signal. The formula (7) is analogous to a result cited in Bell and Martin (2004, Equation (10)), but they consider an infinite number of past observations and a finite number m of future ones; the modification to our case is trivial (later we will see that the corresponding error process is uncorrelated with all linear functions of the data, which is a verification that (7) is indeed the minimum mean squared error signal extraction filter given Assumption A of Bell (1984)). Now we have the following simplification, which is analogous to Bell and Martin (2004, Equation (12)):

$$\pi(B)\gamma_S(F)F^m = \frac{\phi^S(B)\phi^N(B)}{\theta(B)}\frac{\theta^S(F)\theta^S(B)}{\phi^S(F)\phi^S(B)}F^m\sigma_\xi^2 = \frac{\phi^N(B)\theta^S(F)\theta^S(B)}{\phi(B)\phi^S(F)}F^m\sigma_\xi^2$$

$$= \sigma_\xi^2\left(\frac{d(F)}{\phi^S(F)} + \frac{c(B)}{\theta(B)}\right)$$

for appropriate polynomials d and c. This last equality above is the partial fraction decomposition, which is explored in the appendix below. It turns out that the polynomial $c(z)$ satisfies $c_0 = c(0) = 0$, i.e., there is no order zero coefficient. This implies that the last term involves only negative powers of F. Hence

$$[\pi(B)\gamma_S(F)F^m]_+ = \sigma_\xi^2\frac{d_S^{(m)}(F)}{\phi^S(F)}$$

where $d(F) = d_S^{(m)}(F)$ is given the subscript S to denote the signal, and the superscript m for the number of past observations required. Substituting into (7) yields

$$\alpha_S^{(m)}(F) = \frac{\sigma_\xi^2}{\sigma_\epsilon^2}\frac{\phi^N(F)d_S^{(m)}(F)}{\theta(F)}B^m$$

which is analogous to Bell and Martin (2004, Equation (14)). This is for the signal; similar calculations yield for the noise

$$\alpha_N^{(m)}(F) = \frac{\sigma_\eta^2}{\sigma_\epsilon^2}\frac{\phi^S(F)d_N^{(m)}(F)}{\theta(F)}B^m$$

Essentially the main challenge is to produce the polynomials $d_S^{(m)}$ and $d_N^{(m)}$ which is described in great detail in the appendix below.

Now for t within the data span, the error process for signal extraction is given by

$$\begin{aligned} S_t - \alpha_S^{(m)}(F)Y_t &= -\alpha_S^{(m)}(F)N_t + (1 - \alpha_S^{(m)}(F))S_t = -\alpha_S^{(m)}(F)N_t + \alpha_N^{(m)}(F)S_t \\ &= -\frac{\sigma_\xi^2}{\sigma_\epsilon^2}\frac{d_S^{(m)}(F)\theta^N(B)}{\theta(F)}B^m\eta_t + \frac{\sigma_\eta^2}{\sigma_\epsilon^2}\frac{d_N^{(m)}(F)\theta^S(B)}{\theta(F)}B^m\xi_t \end{aligned}$$

The processes given above are stationary, being *ARMA* filters of white noise. The autocovariance generating function is then given by (cf. Bell and Martin 2004, Equation (30))

$$\frac{\sigma_\xi^2\sigma_\eta^2}{\sigma_\epsilon^4\theta(B)\theta(F)}\left[\sigma_\eta^2 d_N^{(m)}(B)d_N^{(m)}(F)\theta^S(B)\theta^S(F) + \sigma_\xi^2 d_S^{(m)}(B)d_S^{(m)}(F)\theta^N(B)\theta^N(F)\right] \tag{8}$$

We are interested in the coefficient of F^0, which is the mean squared error of the signal extraction estimate. But this mean squared error can be expressed as the sum of the variances of the following two ARIMA processes:

$$\theta(B)U_t = \left(d_N^{(m)}(B)/(d_N^{(m)})_0\right)\theta^S(B)\zeta_t \quad \zeta_t \sim WN(0, (d_N^{(m)})_0^2\sigma_\eta^4\sigma_\xi^2/\sigma_\epsilon^4)$$

$$\theta(B)V_t = \left(d_S^{(m)}(B)/(d_S^{(m)})_0\right)\theta^N(B)\chi_t \quad \chi_t \sim WN(0, (d_S^{(m)})_0^2\sigma_\xi^4\sigma_\eta^2/\sigma_\epsilon^4)$$

Here $(d_N^{(m)})_0 = d_N^{(m)}(0)$ is the leading coefficient of the polynomial. The autocovariances of such ARIMA processes can be determined using standard algorithms (see Tunnicliffe-Wilson 1979). Next, we show that the stated signal extraction filters yield the minimum mean squared error linear estimates. Following Bell and Martin (2004), we have

$$e_{t|_{t-m}^{\infty}} = \hat{S}_{t|_{t-m}^{\infty}} - S_t = \left(\hat{S}_{t|_{-\infty}^{\infty}} - S_t\right) + \left(\hat{S}_{t|_{t-m}^{\infty}} - \hat{S}_{t|_{-\infty}^{\infty}}\right) = e_{t|_{-\infty}^{\infty}} + \left(\alpha_S^{(m)}(F) - \alpha_S^{(\infty)}(F)\right)Y_t$$

Now the error $e_{t|_{-\infty}^{\infty}}$ is the symmetric bi-infinite Wiener-Kolmogorov signal extraction error, and hence is orthogonal to any linear combination of the data, at all times. The corresponding filter is $\alpha_S^{(\infty)}(F) = \pi(F)\pi(B)\gamma_S(F)/\sigma_\epsilon^2$, and hence

$$\alpha_S^{(m)}(F) - \alpha_S^{(\infty)}(F) = -\frac{1}{\sigma_\epsilon^2}B^m\pi(F)[\pi(B)\gamma_S(F)F^m]_- = -\frac{1}{\sigma_\epsilon^2}B^m\pi(F)\frac{c_S^{(m)}(B)}{\theta(B)}\sigma_\xi^2$$

where $[\cdot]_-$ means to take only negative powers of F. Here $c_S^{(m)}$ corresponds to the polynomial c used earlier, but we use subscripts and superscripts to call attention to its dependence on the signal and m. Now the second equality follows because $c_S^{(m)}(0) = 0$.

Hence in analogy with Bell and Martin (2004, Equation (36))

$$e_{t|_{t-m}^{\infty}} = e_{t|_{-\infty}^{\infty}} - \frac{\sigma_\xi^2}{\sigma_\epsilon^2}\frac{c_S^{(m)}(B)}{\theta(B)}\epsilon_{t-m}$$

so that the second term only depends on the innovations ϵ_s at times before $t-m$. Now letting $W_t = \delta(F)Y_t$ be the differenced observed process, it follows that $\epsilon_{t|_{t-m}^{\infty}}$ is uncorrelated with W_s for $s \geq t-m$. By Assumption A, the initial values Y_* are orthogonal to $\{W_t\}$ and hence to $\{\epsilon_t\}$, and therefore the error process is orthogonal to the observations $Y_{t-m}, Y_{t-m+1}, \ldots$ This shows optimality of the filter.

Next, suppose that we have more conventional models for the components:

$$\phi(B)Y_t = \theta(B)\epsilon_t \tag{9}$$

$$\phi^S(B)S_t = \theta^S(B)\xi_t$$

$$\phi^N(B)N_t = \theta^N(B)\eta_t$$

This can be recast into the previous "backwards model" framework by letting $\tilde{g}(z) = z^b g(z^{-1})$ for any polynomial g of order b (this has the effect of reversing the coefficients). Using this device, (9) can be written as a backwards model by reversing the coefficients of all the polynomials and redefining new innovation sequences if necessary. Then the optimal signal extraction filter for (9), denoted by $\tilde{\alpha}_S^{(m)}(F)$, is given by

$$\tilde{\alpha}_S^{(m)}(F) = \frac{\sigma_\xi^2}{\sigma_\epsilon^2}\frac{\tilde{\phi}^N(F)\tilde{d}_S^{(m)}(F)}{\tilde{\theta}(F)}B^m$$

Here $\tilde{\phi}^N$ and $\tilde{\theta}$ are the reverse polynomials of ϕ^N and θ, respectively, and $\tilde{d}_S^{(m)}$ is obtained by solving the corresponding system of equations as in the backwards model case. It can be seen that $\tilde{d}_S^{(m)}$ is also the reverse polynomial of $d_S^{(m)}$. If we now compute the error process corresponding to the filter $\tilde{\alpha}_S^{(m)}$, we see that its autocovariance generating function is also given by (8); this uses the fact that $\tilde{g}(B)\tilde{g}(F) = g(B)g(F)$ for any polynomial g. In summary, the spectrum of the error process corresponding to the optimal signal extraction filter for the conventional model (9) is the same as the error spectrum in the backwards model, and (8) can therefore be used to compute autocovariances of the error process. In particular, the signal extraction mean squared error $D_{t|_1^{\infty}}$ is the lag zero term of (8) with the choice of $m = t-1$.

The autocovariance generating function (8) can also be applied to the revision variance for growth rates. It is required to compute the variance

$$\tilde{D}_{t|_1^{\infty}} = \mathbb{E}\left[\left((\hat{S}_{t|_1^{\infty}} - S_t) - (\hat{S}_{t-p|_1^{\infty}} - S_{t-p})\right)^2\right]$$

$$= \mathrm{Var}(\hat{S}_{t|_1^{\infty}} - S_t) + \mathrm{Var}(\hat{S}_{t-p|_1^{\infty}} - S_{t-p}) - 2\mathbb{E}\left[(\hat{S}_{t|_1^{\infty}} - S_t)(\hat{S}_{t-p|_1^{\infty}} - S_{t-p})\right]$$

Now the covariance term is equal to

$$\mathbb{E}\left[(\hat{S}_{t|_{p+1}^{\infty}} - S_t)(\hat{S}_{t-p|_1^{\infty}} - S_{t-p})\right] \tag{10}$$

which is true because the difference between them is equal to the covariance of $\hat{S}_{t-p|_1^\infty} - S_{t-p}$ with $\hat{S}_{t|_1^\infty} - \hat{S}_{t|_{p+1}^\infty}$, and this latter term is a linear function of $Y_1, Y_2, \cdots$. Now the covariance in (10) is seen to be the lag p autocovariance of the signal extraction error process with the filter $\alpha_S^{(t-p-1)}(F)$, and this can be extracted from (8) as well. If we denote the autocovariance function corresponding to the signal extraction error process for the filter $\alpha_S^{(m)}(F)$ by $\gamma_\epsilon^{(m)}$, then we have

$$\tilde{D}_{t|_1^\infty} = \gamma_\epsilon^{(t-1)}(0) + \gamma_\epsilon^{(t-p-1)}(0) - 2\gamma_\epsilon^{(t-p-1)}(p)$$

Since $\tilde{D}_{t|_1^n}$ would be computed with matrices, as described in Section 2, we can now obtain the growth rate revision variance $\tilde{R}_t(\infty)$.

4. Empirical Illustrations of Revision Variances

This article has so far discussed a method for computing revision variances, where the revision is defined by (3). This expression describes changes in the estimate of S_t when our sample expands from $Y_1, \ldots, Y_n$ to $Y_1, \ldots, Y_{n+h}$. However, a different (and less exact) approach is adopted in the ARIMA model-based seasonal adjustment program SEATS (see Maravall and Caporello 2004). Following the work of Pierce (1980) and Maravall (1986), SEATS utilizes a method for computing revision variances that is appropriate when there is an infinite amount of past data. In terms of our notation, SEATS computes the variance of $\hat{S}_{t|_{-\infty}^{n+h}} - \hat{S}_{t|_{-\infty}^{n}}$, where $0 < h \leq \infty$ (note that this variance will not depend on n). The assumption that an infinite amount of past data is available allows for the mathematical development of SEATS's approach to revision variances; however, it is an unrealistic assumption in practice. One motivation for the present work is to update the revision variance calculations of SEATS to handle finite samples.

The next pertinent question is: how much do SEATS's revision variances differ from the exact revision variances? If there is little discrepancy and the exact revision variances require much more computational time, then SEATS users may prefer the current (incorrect) output. The following empirical illustrations demonstrate how the two approaches generate divergent revision variances, and show how these discrepancies can depend upon sample size n, the model parameters, and the revision lead h. In order to fix ideas, we will consider the Airline Model (Box and Jenkins 1976) for monthly data, with a signal-noise decomposition into nonseasonal N_t (or "seasonally adjusted") and seasonal S_t:

$$(1-B)(1-B^{12})Y_t = (1-\theta B)(1-\Theta B^{12})\epsilon_t$$
$$(1-B)^2 N_t = \theta^N(B)\eta_t$$
$$(1+B+\cdots+B^{11})S_t = \theta^S(B)\xi_t$$

for independent white noise sequences ϵ_t, ξ_t, and η_t. These component models will be obtained via the method of canonical decomposition (Hillmer and Tiao 1982). Now because Θ is associated with B^{12}, it will have a more enduring impact than θ on signal extraction filter coefficients, so we focus on how the revision variances are affected by various values of Θ. In our study we consider samples of 5 through 11 years, where $\theta = .9$ and Θ takes on the values .6, .7, .8, and .9 (note that all the calculations are purely derived

from the model, and thus it is not necessary to consider simulations). We are interested in the relative revision variance measure given by (6) for h ranging from 1 to 5 years (for most cases, revision variances had essentially converged beyond 5 years revision lead). We can expect revisions to be greatest for the concurrent filter, so we let $t = n$. The results are displayed in the tables below.

Reading across the rows, we see that the measure (6) decreases in n, and gets fairly close to the SEATS value. This approximation is even tighter for higher revision leads when $\Theta = .6$ and .7. Reading down the columns, we see the expected behavior that most of the revisions have occurred by the fourth or fifth year; however, for larger values of Θ this convergence is considerably slowed. For $\Theta = .9$, all the values are under 50 percent. The largest discrepancies between SEATS and the finite sample approach occur for large Θ and small sample size; for $\Theta = .9$ and $h = 5$ years, the discrepancy can be almost 9 percent. Thus in general, the SEATS revision variances are too small, and the offset can be considerable when Θ is large or the sample size is small.

5. Conclusion

This article discusses the problem of revising concurrent signal extraction estimates, and how to explicitly compute the variance of the revisions. In the case that there is only a finite number h of additional data points, the procedure is a straightforward application of finite-sample signal extraction formulas. When there is an infinite amount of additional data, it is necessary to compute the variance of the corresponding error process. In order to compute the filter coefficients, it is necessary to perform a partial fraction decomposition and arrange the information appropriately. This semi-infinite sample revision error forms a normalization for the h-step ahead revision error, allowing one to gauge the ultimate stability of signal extraction estimates.

We have seen that in some cases there can be a considerable discrepancy between our exact method and that of SEATS; this justifies the additional computation time required to compute M in (2). Nevertheless, when the sample size is large and Θ is smaller, some users may judge that SEATS provides an adequate approximation to the revision variance. The advantage of such an approximation is that it is much faster to compute; the exact method requires the inversion of matrices with dimension of order n. The prototype program X-13 ARIMA-SEATS, which is a hybrid of X-12 ARIMA and SEATS, contains an implementation of the revision variance approach of this article. Typically, a run of a standard length series (between 10 and 15 years of monthly data) using the inexact method of SEATS takes a split second of time, whereas a run using exact finite sample techniques takes several seconds – even as long as 10 seconds. This would be prohibitively long for mass production. Therefore we propose the following scheme (suggested by a referee): compute analogs of Tables 1–4 for a grid of models, with various lengths and lead times, and store this as a lookup table. Then set up a tolerance threshold – say 80 percent – for the revision measure, such that identified models that fall under the threshold are required to use the exact method; the others can safely use the approximate SEATS approach. This would give a practical means of splitting the horns of the dilemma.

Table 1. Revision Measure for (.9, .6) Airline Model

	Finite Sample Method							
Lead	5 years	6 years	7 years	8 years	9 years	10 years	11 years	SEATS
1	.4015	.4006	.4001	.3999	.3999	.3999	.3999	.3999
2	.6412	.6404	.6401	.6399	.6399	.6399	.6399	.6399
3	.7848	.7842	.7840	.7840	.7839	.7839	.7839	.7839
4	.8709	.8705	.8704	.8703	.8703	.8703	.8703	.8703
5	.9225	.9223	.9223	.9222	.9222	.9222	.9222	.9222

Table 2. Revision Measure for (.9, .7) Airline Model

	Finite Sample Method							
Lead	5 years	6 years	7 years	8 years	9 years	10 years	11 years	SEATS
1	.3059	.3028	.3013	.3006	.3003	.3001	.3000	.2999
2	.5162	.5129	.5114	.5107	.5103	.5101	.5100	.5099
3	.6620	.6594	.6581	.6575	.6572	.6571	.6570	.6570
4	.7636	.7617	.7608	.7603	.7601	.7600	.7600	.7599
5	.8346	.8332	.8325	.8322	.8321	.8320	.8320	.8319

Table 3. Revision Measure for (.9, .8) Airline Model

	Finite Sample Method							
Lead	5 years	6 years	7 years	8 years	9 years	10 years	11 years	SEATS
1	.2180	.2111	.2069	.2044	.2027	.2017	.2011	.2000
2	.3831	.3744	.3690	.3657	.3636	.3623	.3615	.3600
3	.5108	.5022	.4970	.4937	.4916	.4903	.4895	.4880
4	.6108	.6032	.5985	.5955	.5937	.5925	.5917	.5904
5	.6897	.6832	.6792	.6767	.6751	.6741	.6735	.6723

Table 4. Revision Measure for (.9, .9) Airline Model

	Finite Sample Method							
Lead	5 years	6 years	7 years	8 years	9 years	10 years	11 years	SEATS
1	.1441	.1328	.1250	.1193	.1150	.1118	.1094	.1000
2	.2578	.2412	.2293	.2206	.2140	.2090	.2051	.1900
3	.3506	.3317	.3180	.3078	.3000	.2940	.2893	.2710
4	.4280	.4086	.3943	.3835	.3752	.3688	.3638	.3439
5	.4938	.4748	.4605	.4497	.4414	.4349	.4298	.4095

6. Appendix: Partial Fraction Implementation

We are primarily interested in obtaining the autocovariance functions $\gamma_\epsilon^{(m)}$ from the autocovariance generating function (8), and the polynomials $d_S^{(m)}(F)$ and $d_N^{(m)}(F)$ are obtained by the use of partial fraction decomposition methods. We now describe two alternative algorithms to accomplish this: the first algorithm safely handles the case

where the number of past observations $m >> 0$; the second algorithm handles the opposite case. Although many algorithms for partial fraction decompositions are available, it seemed useful to provide explicit details for the revision variance problem. Now we introduce the following notation for the degrees of model polynomials: based on (9), the model *AR* polynomial $\phi(z)$ has degree p, and is the product of the signal *AR* polynomial $\phi^S(z)$ with degree p_S and the noise *AR* polynomial $\phi^N(z)$ with degree p_N. (Of course these are generalized *AR* polynomials, in that they may include differencing operators with unit roots.) The model *MA* polynomial $\theta(z)$ has degree q, where the signal and noise polynomials $\theta^S(z)$ and $\theta^N(z)$ have degrees q_S and q_N, respectively, and where $\gamma_S(z)$ represents the signal autocovariance generating function. With this notation in hand, we discuss the two partial fraction decomposition algorithms below, and also describe a useful iterative procedure for determining $d_S^{(m+1)}(F)$ from $d_S^{(m)}(F)$. For ease of exposition, we restrict ourselves to the case of the signal S_t.

6.1. First Partial Fraction Decomposition

An alternative partial fraction decomposition to the one discussed in Section 3, for $m \geq p_N + q_S$, is as follows

$$\begin{aligned}
\pi(B)\gamma_S(F)F^m &= \sigma_\xi^2 \frac{\phi^S(B)\phi^N(B)\theta^S(F)\theta^S(B)}{\theta(B)\phi^S(F)\phi^S(B)} F^m \\
&= \sigma_\xi^2 \frac{\theta^S(F)[F^{p_N}\phi^N(B)][F^{q_S}\theta^S(B)]F^{m-p_N-q_S}}{\phi^S(F)\theta(B)} \\
&= \sigma_\xi^2 \frac{g(F)[F^q\theta(B)] + k(F)}{\phi^S(F)\theta(B)} = \sigma_\xi^2 \frac{g(F)F^q}{\phi^S(F)} + \frac{k(F)}{\phi^S(F)\theta(B)} \\
&= \sigma_\xi^2 \frac{g(F)F^q}{\phi^S(F)} + \frac{b(F)}{\phi^S(F)} + \frac{c(B)}{\theta(B)} = \sigma_\xi^2 \frac{g(F)F^q + b(F)}{\phi^S(F)} + \frac{c(B)}{\theta(B)} \\
&= \sigma_\xi^2 \frac{d_S^{(m)}(F)}{\phi^S(F)} + \frac{c(B)}{\theta(B)}
\end{aligned}$$

where the $g(F)$ polynomial (with degree $max\{m + q_S - q, 0\}$) and the $k(F)$ polynomial (with degree at most $q - 1$) are found using polynomial division. That is,

$$\frac{\theta^S(F)[F^{p_N}\phi^N(B)][F^{q_S}\theta^S(B)]F^{m-p_N-q_S}}{[F^q\theta(B)]} = g(F) + \frac{k(F)}{[F^q\theta(B)]}$$

Also, the $b(F)$ polynomial has degree $max\{q - 1, p_S\}$ and the $c(B)$ polynomial has degree q in order to satisfy

$$k(F) = \theta(B)b(F) + \phi^S(F)c(B) \tag{11}$$

The polynomial $d_S^{(m)}(F)$ is given by

$$d_S^{(m)}(F) = g(F)F^q + b(F) \tag{12}$$

and has degree $max\{m + q_S, q, p_s\}$ (again, suppose m is so large such that this holds). Now the polynomial Equation (11) for $k(F)$ in terms of unknown polynomials $b(F)$ and $c(B)$ represents a system of $max\{q-1, p_S\} + q + 1$ equations with $max\{q-1, p_S\} + 1$ unknown coefficients in $b(F)$ and with $q+1$ unknown coefficients in $c(B)$. Hence the number of unknown coefficients is one plus the number of equations, so that the coefficient $c_0 = c(0)$ is chosen such that $c_0 = 0$. The unknown polynomial coefficients are found by solving the following system of equations (in this case assuming $p_S > q$)

$$\begin{pmatrix} 0_{-q} \\ \vdots \\ 0_{-1} \\ k_0 \\ \vdots \\ k_{q-1} \\ \vdots \\ 0_{p_S} \end{pmatrix} = \begin{bmatrix} \phi_0^S & \cdots & 0 & \theta_q & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \phi_{q-1}^S & \cdots & \phi_0^S & \theta_1 & \cdots & \theta_q & \cdots & 0 \\ \phi_q^S & \cdots & \phi_1^S & \theta_0 & \cdots & \theta_{q-1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_{2q-1}^S & \cdots & \phi_q^S & 0 & \cdots & \theta_0 & \cdots & \theta_{p_S 0-q+1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & \theta_0 \end{bmatrix} \begin{pmatrix} c_q \\ \vdots \\ c_1 \\ b_0 \\ \vdots \\ b_{q-1} \\ \vdots \\ b_{p_S} \end{pmatrix} \tag{13}$$

Here $b_j, j = 0, \ldots, p_S$, denotes the coefficients of $b(F)$ and c_j, $j = 1, \ldots, q$, denotes the coefficients of $c(B)$. Also $\theta_j, j = 0, \ldots, q$, denotes the coefficients of $\theta(B)$ and $\theta_j = 0$ for $j > q$, whereas ϕ_j^S, $j = 0, \ldots, p_S$, denotes the coefficients of $\phi^S(F)$ and $\phi_j^S = 0$ for $j > p_S$. Since $\phi^S(z)$ and $\theta(z)$ can have no common zeroes by assumption, the above matrix is invertible by application of the Lemma on page 25 of Bell and Martin (2002). Hence, we can proceed as follows:

1. Determine $g(z)$ and $k(z)$ by the Euclidean division algorithm
2. Solve (13) for $b(z)$
3. Determine $d_S^{(m)}(z)$ using (12)

6.2. *Second Partial Fraction Decomposition*

An alternative partial fraction decomposition for $m < p_N + q_S$ is given by

$$\pi(B)\gamma_S(F)F^m = \frac{\sigma_\xi^2[\theta^S(F)F^m][\phi^N(B)\theta^S(B)]}{\phi^S(F)\theta(B)} = \frac{r(F,B)}{\phi^S(F)\theta(B)} = \frac{d(F)}{\phi^S(F)} + \frac{c(B)}{\theta(B)}$$

where $r(F,B)$ has degree $p_r = m + q_S$ in F and degree $q_r = p_N + q_S$ in B. Here $d(F) = d_S^{(m)}(F)$ has degree $p_d = max\{m + q_S, p_S\}$ and $c(B)$ has degree $q_c = max\{p_N + q_S, q\}$, and are defined such that they satisfy

$$r(F,B) = \theta(B)d(F) + \phi^S(F)c(B)$$

The above polynomial equation for $r(F,B)$ represents a system of $p_d + q_c + 1$ equations in $p_d + 1$ unknown coefficients for $d(F)$ and $q_c + 1$ unknown coefficients for $c(B)$.

Consequently, the coefficient $c_0 = c(0)$ is chosen such that $c_0 = 0$. The unknown coefficients for $d(F)$ and $c(B)$ are found by solving the following system of equations

$$\begin{pmatrix} r_{-q_c} \\ r_{-q_c+1} \\ \vdots \\ r_{-1} \\ r_0 \\ \vdots \\ r_{p_d-1} \\ r_{p_d} \end{pmatrix} = \begin{bmatrix} \phi_0^S & 0 & \cdots & 0 & \theta_{q_c} & 0 & \cdots & 0 \\ \phi_1^S & \phi_0^S & \cdots & 0 & \theta_{q_c-1} & \theta_{q_c} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_{q_c-1}^S & \phi_{q_c-2}^S & \cdots & \phi_0^S & \theta_1 & \theta_2 & \cdots & \theta_{p_d+1} \\ \phi_{q_c}^S & \phi_{q_c-1}^S & \cdots & \phi_1^S & \theta_0 & \theta_1 & \cdots & \theta_{p_d} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \phi_{p_d}^S & 0 & 0 & \cdots & \theta_1 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \theta_0 \end{bmatrix} \begin{pmatrix} c_{q_c} \\ c_{q_c-1} \\ \vdots \\ c_{-1} \\ d_0 \\ \vdots \\ d_{p_d-1} \\ d_{p_d} \end{pmatrix} \tag{14}$$

Here $r_j, j = 0, \ldots, p_r$, denotes the coefficients of $r(F, B)$ in F and $r_j = 0$ for $j > p_r$, whereas $r_{-j}, j = 1, \ldots, q_r$, denotes the coefficients of $r(F, B)$ in B and $r_{-j} = 0$ for $j > q_r$. Since the matrix is invertible, we can proceed as follows:

1. Write down $r(F, B)$
2. Solve (14) for $d(z)$

Note that this second alternative partial fraction decomposition can also be used to handle $m \geq p_N + q_S$; however, the first alternative partial fraction decomposition is faster in terms of software speed. As noted in Bell and Martin (2004), a direct partial fraction decomposition method that avoids solving linear systems will be more efficient.

6.3. Sequential Partial Fraction Decomposition

Given that $d_S^{(m)}(F)$ has been calculated via either of the two previous partial fraction decompositions, it is possible to sequentially calculate $d_S^{(m+1)}(F)$. Let $d_S^{(m)}(F)$ and $c^{(m)}(B)$ denote the polynomials obtained via partial fraction decomposition for the filter $\alpha_S^{(m)}$. Then we have

$$\begin{aligned} \pi(B)\gamma_S(F)F^{m+1} = [\pi(B)\gamma_S(F)F^m]F &= \left[\frac{d_S^{(m)}(F)}{\phi^S(F)} + \frac{c^{(m)}(B)}{\theta(B)}\right]F = \frac{d_S^{(m)}(F)F}{\phi^S(F)} + \frac{c^{(m)}(B)F}{\theta(B)} \\ &= \frac{d_S^{(m)}(F)F + c_1^{(m)}\phi^S(F)}{\phi^S(F)} + \frac{c^{(m)}(B)F - c_1^{(m)}\theta(B)}{\theta(B)} \\ &= \frac{d_S^{(m+1)}(F)}{\phi^S(F)} + \frac{c^{(m+1)}(B)}{\theta(B)} \end{aligned}$$

where $c_1^{(m)}$ is the coefficient associated with B^1 in $c^{(m)}(B)$. So we have the definitions

$$d_S^{(m+1)}(F) = d_S^{(m)}(F)F + c_1^{(m)}\phi^S(F)$$
$$c^{(m+1)}(B) = c^{(m)}(B)F - c_1^{(m)}\theta(B)$$

from which it follows, together with $c^{(m)}(0) = 0$ and $\theta(0) = 0$, that

$$c^{(m+1)}(0) = 0$$

In this manner, $d_S^{(m+1)}$ can be obtained from $d_S^{(m)}$. Using the above sequential decomposition, it is possible to calculate $d_S^{(m+k)}(F)$ from the partial fraction decomposition associated with $d_S^{(m)}(F)$, by sequentially calculating the partial fraction decomposition associated with $d_S^{(m+j)}(F)$ for each $j = 1, \ldots, k$.

6.4. *Discussion*

The second partial fraction decomposition method will work in all cases that need to be considered. The first decomposition, however, is somewhat faster and can be used when m is sufficiently large. Both approaches, as well as the iterative algorithm for d_S, have been implemented in X-13 ARIMA-SEATS. The computational cost of each is related to the size of the matrix that needs to be inverted; typically this is of the order T^3 operations, where T is the size of the matrix (assuming that we are not taking advantage of any special band structures). The first algorithm is therefore $O((q + p_S)^3)$ (the other aspects of the algorithm will take negligible time compared with the matrix inversion), whereas the second is $O(T^3)$ with $T = max\{p_N + q_S, q\} + max\{m + q_S, p_S\}$ which is dominated by m when m is large.

These methods serve as an illustration of how the autocovariance functions of Section 3 might be computed. Based on our experience with X-13 ARIMA-SEATS, these partial fraction decomposition routines perform well, being both flexible and fairly efficient. They are mainly provided here for the benefit of readers who wish some assistance with generating their own implementations of revision variance calculations.

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