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Should Stores Be Open on Sunday? The Impact of Sunday Opening on the Retail Trade Sector in New Brunswick

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Should stores be open on Sunday? This is a current issue for Canadians as the de-regulation of shopping hours is, depending on the province, either a recent phenomenon or being considered. This article focuses on the case of Sunday opening in the retail trade sector in New Brunswick and highlights some important aspects of that matter. One issue is whether Sunday opening of retail outlets increases total sales. A second issue is whether Sunday opening causes a redistribution of sales between trade groups in the retail trade sector. A third is whether Sunday opening causes a redistribution of the sales among the days of the week. These questions are answered using intervention analysis and trading-day regression. For the modelling of the latter, this article presents an alternative technique.

Key words: Easter effect; trading-day variations; intervention analysis; Fourier transform.

1. Introduction

In Canada, the provincial legislations used to prohibit the opening of retail trade stores on Sunday. This situation has changed substantially in recent years. For instance in 1991, the province of New Brunswick allowed Sunday opening for the first time in the months of November and December. In 1992, this province extended Sunday opening to the months of September and October; and in 1996, to August as well. In the spring of 1996, the New Brunswick statistical agency asked Statistics Canada to perform an assessment of the impact of Sunday opening on sales.

This article considers three issues related to Sunday opening. The first issue is whether or not the opening of stores on Sunday in selected months increased the level of retail sales in New Brunswick for those months. The second issue examined is whether Sunday opening caused a redistribution of sales between *trade groups* (kind of stores). The third issue is whether Sunday opening induced a redistribution of sales among the days of the week.

The article examines two time series, Total Retail Trade and Department Stores sales in New Brunswick from 1981 to 1996, using intervention analysis and trading-day regression. Intervention analysis, developed by Box and Tiao (1975), can be applied to measure the effect of Sunday opening on the sales volume. Trading-day regression (Young

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1965, Bell and Hillmer 1983, Dagum, Quenneville, and Sutradhar 1992) can be used to estimate the relative importance of the days of the week in monthly flow series.

Section 2 describes how the Sunday opening effect, the trading-day variation, and other effects usually present in time series are modelled. The section also introduces a convenient trigonometric reparametrization of the trading-day coefficients that has the potential to reduce the number of parameters needed. Section 3 presents the overall model with the estimates and the interpretation of the parameters. Section 4 presents concluding remarks.

2. Components of the Model

This section formulates a model for the Total Retail Trade and Department Stores sales series. The model addresses the issues raised in the Introduction. The first subsection presents the transformations made in the data. The second subsection introduces the deterministic components of the model. Finally, the third subsection gives the overall model and proposes the Fourier transform of the trading-day coefficients.

2.1. Transformation of the data

Figure 1 displays the two retail trade series published for New Brunswick, namely the sales by Department Stores and the Total Retail Trade sales. Tables 1 and 2 display the values of the series, as they were available in February 1997 in the Statistics Canada time series database (CANSIM No. D 658198 and D 658182). Each series will be denoted by z_t^*, $t = 1, \ldots, 192$, where $t = 1$ stands for January 1981 and $t = 192$ for December 1996.

First, the data is adjusted for the length-of-month (lom) variation. Each monthly observation of a sales series corresponds to the sum of the daily sales in that month. Since months have a variable length of 28, 29, 30 or 31 days, the lom-adjusted observation, $z_t = 30.4375\ z_t^*/N_t$ where N_t is the number of days in the month, represents sales of a month of 30.4375 days (365.25/12). The lom-adjusted series is displayed in Figure 2. The effect of this adjustment is most noticeable for leap-year Februaries where the increases from Januaries to Februaries are now more regular than those in Figure 1.

A logarithmic (log) transformation is used to stabilize the variation around the yearly

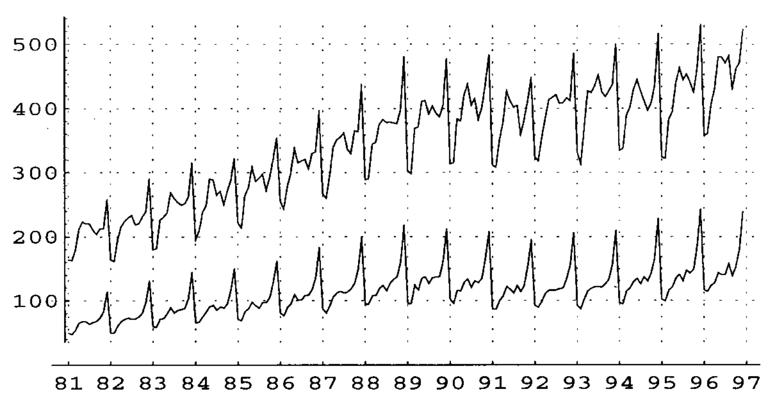

Fig. 1. Total Retail Trade sales (upper series) and Department Store sales (lower series) for New Brunswick, in millions of current dollars, from January 1981 to December 1996

Table 1. Total Retail Trade sales, in millions of current dollars, from January 1981 to December 1996

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1981	163.7	163.0	179.2	211.3	223.6	220.4	220.6	210.3	203.8	212.3	212.1	258.2
1982	164.1	161.1	196.2	215.8	224.6	229.6	233.1	218.6	220.4	230.8	238.9	290.3
1983	179.5	181.4	226.2	230.6	237.1	268.8	259.5	253.5	249.4	251.3	261.9	315.5
1984	195.2	208.6	238.0	248.5	290.3	288.8	265.0	271.3	249.6	270.7	287.9	322.1
1985	221.9	213.0	264.2	276.7	308.5	286.2	291.2	297.5	271.5	292.1	320.8	354.5
1986	254.8	242.7	278.1	299.0	337.8	315.3	318.2	320.5	305.4	329.7	331.9	396.8
1987	267.0	259.4	295.7	339.7	351.5	355.4	362.2	336.3	329.4	365.4	363.4	437.9
1988	289.0	290.3	344.5	347.0	375.2	383.0	378.1	378.7	377.7	376.2	397.7	481.0
1989	302.8	297.8	369.1	371.0	410.8	412.3	391.0	404.2	392.9	387.0	404.7	477.7
1990	313.4	315.0	383.9	380.5	421.7	437.8	405.1	415.4	382.0	401.3	437.2	483.4
1991	311.7	307.8	351.5	377.6	427.4	411.7	401.9	404.6	358.8	382.4	411.0	449.0
1992	324.4	317.5	354.8	384.6	413.5	417.0	421.4	407.5	408.4	416.2	411.8	485.9
1993	331.8	313.2	369.6	427.4	424.5	435.2	452.1	425.6	417.8	427.2	436.2	500.3
1994	333.9	337.6	388.8	402.0	429.0	444.8	426.8	411.8	396.5	408.2	435.5	517.2
1995	323.5	321.9	383.3	394.8	440.4	463.2	444.0	453.4	440.6	424.9	461.6	531.2
1996	357.0	360.8	407.5	430.5	480.8	480.0	470.7	482.0	428.9	462.1	471.1	523.5

Table 2. Department Store sales for New Brunswick, in millions of current dollars, from January 1981 to December 1996

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1981	49.6	47.3	53.8	65.0	67.3	67.4	63.2	66.2	67.7	73.0	83.4	114.3
1982	49.6	49.4	60.7	67.2	70.8	73.1	70.9	71.3	74.8	80.6	97.7	131.6
1983	59.2	58.6	71.0	72.1	79.6	89.1	80.8	85.0	86.2	88.0	103.4	145.1
1984	65.4	66.4	74.5	82.3	90.7	92.7	85.1	90.6	88.5	94.5	116.9	150.2
1985	70.5	68.6	82.9	87.4	97.7	92.2	88.1	97.2	96.5	104.7	129.1	162.2
1986	80.6	76.0	89.2	94.9	109.5	100.8	100.8	108.1	108.4	118.0	135.6	183.6
1987	86.9	80.7	92.2	106.0	112.3	114.5	112.0	113.3	118.3	127.0	148.8	200.7
1988	94.1	94.7	107.6	108.2	118.5	123.8	115.0	126.7	133.3	137.2	160.1	218.0
1989	95.6	95.5	124.5	115.7	135.2	137.1	126.6	135.9	136.9	137.5	161.8	212.3
1990	102.0	95.6	115.9	114.7	129.0	133.6	120.5	131.0	128.2	135.0	166.2	208.5
1991	86.8	86.6	100.0	107.1	121.9	117.5	111.2	123.7	113.8	123.4	153.6	195.8
1992	92.3	89.4	98.7	111.2	116.0	116.1	116.1	118.3	119.2	134.2	154.6	206.7
1993	91.9	87.1	102.2	114.9	118.9	121.3	121.2	120.7	126.1	133.1	156.6	209.8
1994	96.5	94.8	114.5	119.0	129.4	134.7	126.2	137.4	130.2	142.7	167.0	229.1
1995	101.9	99.5	116.2	120.8	135.2	141.0	130.6	147.8	142.6	148.6	184.2	243.4
1996	115.9	113.9	124.0	127.3	143.6	140.5	140.5	158.0	138.3	153.0	179.5	239.7

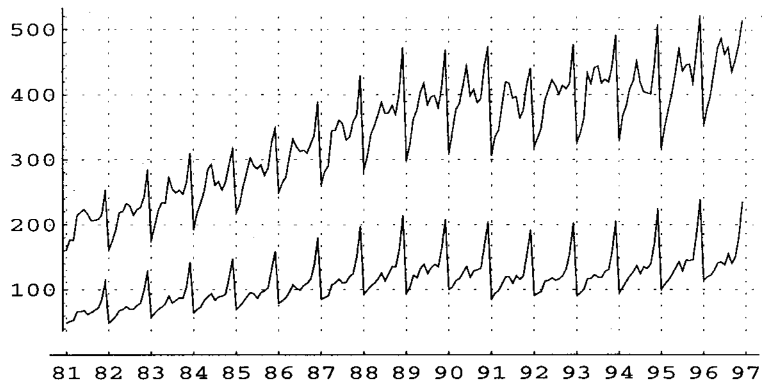

Fig. 2. Total Retail Trade sales (upper series) and Department Stores sales (lower series) adjusted for length-of-month variation

averages which is larger at higher levels of the series. Figure 3 displays the log of the lom-adjusted series. Note that annual differences of the log-data are approximations of annual growth rates: $\log(z_t) - \log(z_{t-12}) = \log(z_t/z_{t-12}) = \log[1 + (z_t - z_{t-12})/z_{t-12}] \approx (z_t - z_{t-12})/z_{t-12}$.

Because the log-lom-data display an upward trend as well as a strong seasonal pattern, both regular and seasonal differencing are applied to remove local linear trends present in these components. The differenced series is:

$$(1 - B)(1 - B^{12}) \log(z_t) = (\log(z_t) - \log(z_{t-1})) - (\log(z_{t-12}) - \log(z_{t-13}))$$

where B is the backshift operator ($B \log(z_t) \equiv \log(z_{t-1})$).

The differenced series still contains other deterministic effects which require explicit modelling.

2.2. *Deterministic components of the model*

In the case at hand, the other deterministic components are the Sunday opening effect, the trading-day variations, the Easter effect, the Goods and Services Tax (GST) effect and the effect of outlier observations.

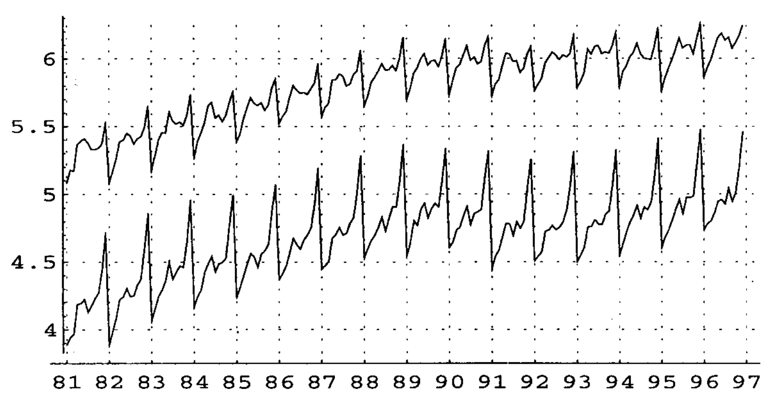

Fig. 3. Natural logarithm of lom-adjusted Total Retail Trade sales (upper series) and Department Stores sales (lower series)

Table 3. Values of the Sunday opening indicator variable I_t for the years and months indicated ($I_t = 0$ for all other periods)

Year	Aug	Sep	Oct	Nov	Dec
1991	0	0	0	1	1
1992	0	1	1	1	1
1993	0	1	1	1	1
1994	0	1	1	1	1
1995	0	1	1	1	1
1996	1	1	1	1	1

The Sunday opening effect refers to the impact of allowing the stores to open on Sunday on the level of sales. The effect is modelled by means of an intervention. It consists of including in the model a variable, I_t, that takes the value one for months when stores are open on Sunday and zero otherwise, as displayed in Table 3. The regression coefficient of I_t approximates the percent increase in the sales due to Sunday opening. In the discussion to follow, the periods when the stores are closed on Sunday will be referred to as the *regulated regime/months*; and those when they are allowed to open, as *de-regulated regime/months*.

Trading-day variation refers to the fluctuation in monthly sales caused by the fact that different days of the week have differing volumes of sales and that months do not have the same composition of days. Thus a month containing five of the relatively important days tends to display more sales than a month having only four of them. When log-data are used, the trading-day variations are estimated by regressing the monthly observations, $\log(z_t)$, on the variables $D_{i,t} = (N_{i,t} - N_{7,t})/N_t$, $i = 1, \ldots, 6$, where $N_{i,t}$, $i = 1, \ldots, 7$, is the number of occurrences of day i in month t and where $i = 1$ refers to Monday, $i = 2$ refers to Tuesday and so on. The monthly *trading-day component* is $\log TD_t = \sum_{i=1}^{6} \delta_i D_{i,t}$, where $\delta_1, \ldots, \delta_6$ are the regression coefficients. The term $\sum_{i=1}^{6} \delta_i D_{it}$ is equivalent to

$$\sum_{i=1}^{6} \delta_i D_{i,t} = \begin{cases} [\delta_j + \delta_{j+1} + \delta_{j+2}]/31 & \text{if } N_t = 31 \\ [\delta_j + \delta_{j+1}]/30 & \text{if } N_t = 30 \\ \delta_j/29 & \text{if } N_t = 29 \\ 0 & \text{if } N_t = 28 \end{cases}$$

where j is the day of the week on which the first day of month t falls, $\delta_7 = -\sum_{i=1}^{6} \delta_i$ and $\delta_{j+7} = \delta_j$.

In the original scale, the trading-day component becomes $TD_t = \exp[\sum_{i=1}^{6} \delta_i D_{i,t}]$. Because the exponent term is usually close to zero, TD_t can be approximated by

$$TD_t \approx 1 + \sum_{i=1}^{6} \delta_i D_{i,t} = \begin{cases} [28 + (1+\delta_j) + (1+\delta_{j+1}) + (1+\delta_{j+2})]/31 & \text{if } N_t = 31 \\ [28 + (1+\delta_j) + (1+\delta_{j+1})]/30 & \text{if } N_t = 30 \\ [28 + (1+\delta_j)]/29 & \text{if } N_t = 29 \\ 1 & \text{if } N_t = 28 \end{cases}$$

which are similar to adjustment factors given in Young (1965).

The terms $(1 + \delta_i)$, $i = 1, \ldots, 7$, have been referred to as the seven daily weights (*Ibid.*). However, as it will be the case for the Department Stores series, one of the $\delta_i' s$ is smaller than -1, giving a negative daily weight. We therefore define the *daily weights* as $\psi_i = \exp(\delta_i)/(\sum_{i=1}^{7}\exp(\delta_i)/7)$. These weights are now all positive and have an arithmetic mean equal to 1, in accordance with the concept of *daily pattern*. Thus $\psi_4 = 1.20$ means that there is 20% more activity on Thursdays than on an average day; and $\psi_7 = 0.60$ indicates 40% less activity on Sundays than on average. The *percentage of weekly activity* taking place on day i is thus $(\psi_i/7) \times 100\%$.

In order to measure the effect of the de-regulation on the daily pattern, two sets of trading-day variables are needed. The first set of regressors, $D_{i,t}$ with coefficients δ_i, is defined over the whole series; and the second one, $D^*_{i,t}$ with coefficients δ^*_i, is only defined over the de-regulated months. The trading-day coefficients in regulated months are thus $\delta_1, \ldots, \delta_7$; and in de-regulated months, $\delta_1 + \delta^*_1, \ldots, \delta_7 + \delta^*_7$. Over the de-regulated regime, the daily weights are thus $\psi^*_i = \exp(\delta_i + \delta^*_i)/(\sum_{i=1}^{7}\exp(\delta_i + \delta^*_i)/7)$, and the percentage of weekly activity taking place on day i is $(\psi^*_i/7) \times 100\%$.

The Easter effect refers to the increase in sales caused by Easter, which can fall between March 22 and April 25. The Easter effect is modelled by incorporating a variable, E_t, whose values are displayed in Table 4 corresponding to the dates of Easter. The coefficient of E_t approximates the percent increase in sales associated with Easter. Our specification, following that of Bell and Hillmer (1983), assumes that the event uniformly affects sales in the 15-day period before the Easter Sunday, which includes the two preceding weekends. It distributes the Easter effect between March and April proportionally to the fraction of the 15-day period falling in the respective months. Obviously, this 15-day period is an average duration for the whole retail sector; in fact the period is likely to be longer for clothing (sales) than for flowers.

The GST effect refers to the impact of the Goods and Services Tax on the recorded value of retail sales. The effect is apparent in Figures 1 to 3, where prior to January 1991 sales figures included the ''hidden'' Manufacturers' Sales Tax (MST), whereas they now exclude both taxes, resulting in a drop in the level of the series in January

Table 4. Date of Easter and Easter-variable E_t for the years indicated ($E_t = 0$ for all other months)

Year	Date of Easter	March – value	April – value
1981	April 19	0	1
1982	April 11	5/15	10/15
1983	April 3	13/15	2/15
1984	April 22	0	1
1985	April 7	9/15	6/15
1986	March 30	1	0
1987	April 19	0	1
1988	April 3	13/15	2/15
1989	March 26	1	0
1990	April 15	1/15	14/15
1991	March 31	1	0
1992	April 19	0	1
1993	April 11	5/15	10/15
1994	April 3	13/15	2/15
1995	April 16	0	1
1996	April 7	9/15	6/15

1991. This change in the level of the series is modelled with a step variable, $S_t^{(121)}$, which takes the value -1 before January 1991 and zero otherwise. The coefficient of $S_t^{(121)}$ measures the percent level change in the series due to the removal of both taxes and corrects the series accordingly.

Finally, most time series contain some *outlier* observations. They can take the form of a single unusual observation or they can appear as a sudden change in the level of the time series. In this analysis, the technique of Chen and Liu (1993) was used to detect and correct for outliers. A level shift was found for both series in June 1983. This level shift is modelled by a step variable $S_t^{(30)}$ which takes the value -1 before that date and zero otherwise. The coefficient of $S_t^{(30)}$ is used to correct the series prior to June 1983 for this level shift.

2.3. *Overall model*

The various components defined in Section 2.2 are included in the following model:

$$\log(z_t) = \eta I_t + \sum_{i=1}^{6} \delta_i D_{i,t} + \sum_{i=1}^{6} \delta_i^* D_{i,t}^* + \epsilon E_t + \gamma S_t^{(121)} + \zeta S_t^{(30)} + v_t \tag{1}$$

with

$$(1-B)(1-B^{12})v_t = (1-\theta_1 B)(1-\theta_2 B^{12})a_t \tag{2}$$

Parameters η of Equation (1) measures the effect of Sunday opening; δ_i, the regulated daily coefficients; δ_i^*, the change in the daily coefficients after de-regulation; ϵ, the effect of Easter; γ, the effect of the GST; and ζ, the effect of the identified unexplained level shift. Variable v_t of (1) stands for a disturbance, which follows a multiplicative seasonal autoregressive integrated moving average model: ARIMA$(0,1,1)(0,1,1)_{12}$ (Box and Jenkins 1976) given in Equation (2). This model accounts for the autocorrelation present in the time series; and in this case also includes the combined regular and seasonal differencing operators, $(1-B)(1-B^{12})$, used to model the trend and seasonal components. The a_t's of Equation (2) form a sequence of uncorrelated random variables with mean zero and unknown variance, θ_1 and θ_2 are the regular and the seasonal moving average parameters respectively, and B is the backshift operator defined in Section 2.1.

One weakness of Model (1) is that the trading-day component of the de-regulated regime requires the extra six coefficients, δ_1^* to δ_6^*, to be estimated from only 23 months (cf. Table 3), which leaves few degrees of freedom. In order to overcome this problem, we now present an alternative parametrization of the trading-day coefficients, based on trigonometric functions of time at the daily frequencies, $\lambda_j = 2\pi j/7, j = 1, 2, 3$. This specification is an adaptation of the well-known *trigonometric specification* of monthly seasonal patterns (e.g., Harvey 1989). The sine and cosine at frequency λ_1 describe a cycle of duration of seven days; at frequency λ_2, 7/2 days; and at frequency λ_3, 7/3 days. The coefficient for day i is reparametrized as

$$\delta_i = \sum_{j=1}^{3} [\alpha_j \cos(\lambda_j i) + \beta_j \sin(\lambda_j i)], \quad i = 1, \ldots, 7 \tag{3}$$

Table 5 presents the seven daily coefficients in terms of the six trigonometric parameters. Specification (3) also guarantees that the seven daily coefficients δ_1 to δ_7 sum to zero, which can be verified from Table 6.

Table 5. Symbolic values of the daily coefficients, δ_i, $i = 1, \ldots, 7$, in terms of the trigonometric parameters α_1, α_2, α_3, β_1, β_2, β_3

Daily coefficient	$\delta_i = \sum_{j=1}^{3} [\alpha_j \cos(\lambda_j i) + \beta_j \sin(\lambda_j i)]$					
$\delta_1 =$	$\cos\left[\frac{2\pi}{7}\right]\alpha_1$	$+\cos\left[\frac{4\pi}{7}\right]\alpha_2$	$+\cos\left[\frac{6\pi}{7}\right]\alpha_3$	$+\sin\left[\frac{2\pi}{7}\right]\beta_1$	$+\sin\left[\frac{4\pi}{7}\right]\beta_2$	$+\sin\left[\frac{6\pi}{7}\right]\beta_3$
$\delta_2 =$	$\cos\left[\frac{4\pi}{7}\right]\alpha_1$	$+\cos\left[\frac{8\pi}{7}\right]\alpha_2$	$+\cos\left[\frac{12\pi}{7}\right]\alpha_3$	$+\sin\left[\frac{4\pi}{7}\right]\beta_1$	$+\sin\left[\frac{8\pi}{7}\right]\beta_2$	$+\sin\left[\frac{12\pi}{7}\right]\beta_3$
$\delta_3 =$	$\cos\left[\frac{6\pi}{7}\right]\alpha_1$	$+\cos\left[\frac{12\pi}{7}\right]\alpha_2$	$+\cos\left[\frac{18\pi}{7}\right]\alpha_3$	$+\sin\left[\frac{6\pi}{7}\right]\beta_1$	$+\sin\left[\frac{12\pi}{7}\right]\beta_2$	$+\sin\left[\frac{18\pi}{7}\right]\beta_3$
$\delta_4 =$	$\cos\left[\frac{8\pi}{7}\right]\alpha_1$	$+\cos\left[\frac{16\pi}{7}\right]\alpha_2$	$+\cos\left[\frac{24\pi}{7}\right]\alpha_3$	$+\sin\left[\frac{8\pi}{7}\right]\beta_1$	$+\sin\left[\frac{16\pi}{7}\right]\beta_2$	$+\sin\left[\frac{24\pi}{7}\right]\beta_3$
$\delta_5 =$	$\cos\left[\frac{10\pi}{7}\right]\alpha_1$	$+\cos\left[\frac{20\pi}{7}\right]\alpha_2$	$+\cos\left[\frac{30\pi}{7}\right]\alpha_3$	$+\sin\left[\frac{10\pi}{7}\right]\beta_1$	$+\sin\left[\frac{20\pi}{7}\right]\beta_2$	$+\sin\left[\frac{30\pi}{7}\right]\beta_3$
$\delta_6 =$	$\cos\left[\frac{12\pi}{7}\right]\alpha_1$	$+\cos\left[\frac{24\pi}{7}\right]\alpha_2$	$+\cos\left[\frac{36\pi}{7}\right]\alpha_3$	$+\sin\left[\frac{12\pi}{7}\right]\beta_1$	$+\sin\left[\frac{24\pi}{7}\right]\beta_2$	$+\sin\left[\frac{36\pi}{7}\right]\beta_3$
$\delta_7 =$	α_1	$+\alpha_2$	$+\alpha_3$			

Table 6. Numerical approximations to the daily coefficients, δ_i, i = 1, …, 7, in terms of the trigonometric parameters α_1, α_2, α_3, β_1, β_2, β_3

Daily coefficient	$\delta_i = \sum_{j=1}^{3}[\alpha_j \cos(\lambda_j i) + \beta_j \sin(\lambda_j i)]$					
$\delta_1 =$	$+0.62349\alpha_1$	$-0.22252\alpha_2$	$-0.90097\alpha_3$	$+0.78183\beta_1$	$+0.97493\beta_2$	$+0.43388\beta_3$
$\delta_2 =$	$-0.22252\alpha_1$	$-0.90097\alpha_2$	$+0.62349\alpha_3$	$+0.97493\beta_1$	$-0.43388\beta_2$	$-0.78183\beta_3$
$\delta_3 =$	$-0.90097\alpha_1$	$+0.62349\alpha_2$	$-0.22252\alpha_3$	$+0.43388\beta_1$	$-0.78183\beta_2$	$+0.97493\beta_3$
$\delta_4 =$	$-0.90097\alpha_1$	$+0.62349\alpha_2$	$-0.22252\alpha_3$	$-0.43388\beta_1$	$+0.78183\beta_2$	$-0.97493\beta_3$
$\delta_5 =$	$-0.22252\alpha_1$	$-0.90097\alpha_2$	$+0.62349\alpha_3$	$-0.97493\beta_1$	$+0.43388\beta_2$	$+0.78183\beta_3$
$\delta_6 =$	$+0.62349\alpha_1$	$-0.22252\alpha_2$	$-0.90097\alpha_3$	$-0.78183\beta_1$	$-0.97493\beta_2$	$-0.43388\beta_3$
$\delta_7 =$	$+1.00000\alpha_1$	$+1.00000\alpha_2$	$+1.00000\alpha_3$	$+0.00000\beta_1$	$+0.00000\beta_2$	$+0.00000\beta_3$

Substituting the trigonometric trading-day coefficients (3) in the *classical specification* $\sum_{i=1}^{6} \delta_i D_{i,t}$ of Equation (1), yields

$$\sum_{i=1}^{6} \delta_i D_{i,t} = \sum_{i=1}^{6} \left(\sum_{j=1}^{3} [\alpha_j \cos(\lambda_j i) + \beta_j \sin(\lambda_j i)] \right) D_{i,t}$$
$$= \sum_{j=1}^{3} \alpha_j \left(\sum_{i=1}^{6} \cos(\lambda_j i) D_{i,t} \right) + \sum_{j=1}^{3} \beta_j \left(\sum_{i=1}^{6} \sin(\lambda_j i) D_{i,t} \right)$$

The regressors for α_j and β_j are then $A_{j,t} = \sum_{i=1}^{6} \cos(\lambda_j i) D_{i,t}$, and $B_{j,t} = \sum_{i=1}^{6} \sin(\lambda_j i) D_{i,t}$. Our alternative trigonometric specification of the trading-day component is thus

$$\sum_{j=1}^{3} \alpha_j A_{j,t} + \sum_{j=1}^{3} \beta_j B_{j,t} = \sum_{i=1}^{6} \delta_i D_{i,t} \tag{4}$$

When the six trigonometric parameters, $(\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3)$, are included, the specification (4) is equivalent to the classical one and the estimated trading-day variations are identical. When the statistically nonsignificant parameters are dropped a more parsimonious model is achieved.

Model (1) is then re-written as

$$\log(z_t) = \eta I_t + \sum_{j=1}^{3} \alpha_j A_{j,t} + \sum_{j=1}^{3} \beta_j B_{j,t} + \sum_{j=1}^{3} \alpha_j^* A_{j,t}^* + \sum_{j=1}^{3} \beta_j^* B_{j,t}^* + \epsilon E_t + \gamma S_t^{(121)}$$
$$+ \zeta S_t^{(30)} + \nu_t \tag{5}$$

where the parameters α_j^*, β_j^* and the regressors $A_{j,t}^*$, $B_{j,t}^*$ pertain to the trading-day component over the de-regulated months.

3. Interpretation of the Results

Two variants of Model (5) were fitted to each series, a *fully parametrized* variant and a *parsimonious* variant with the nonsignificant parameters dropped. Tables 7 and 8 display the respective parameter estimates, with their Student t-values, obtained with the PC SCA© Statistical System (1992). (In this analysis a parameter is nonsignificant when its absolute t-value is less than 2.)

In Table 7, the coefficient η estimated for the Sunday opening effect is not significant for Total Retail Trade, but it is significant for Department Stores. In other words, according to the data, de-regulation did not affect total sales in New Brunswick, but did positively affect sales in Department Stores. The impact, from the parsimonious model in Table 8, amounts to a significant 2.2% increase in monthly sales, or approximately \$3,500,000. These results suggest Sunday opening did not increase total sales in the province, but instead redistributed the sales among trade groups to the benefit of Department Stores, at the expense of the rest of the retail trade sector. This settles the first and second issues raised in the introduction.

As for the third issue, the results suggest a substantial impact of de-regulation on the daily pattern, especially for Department Stores. Table 7 also shows that the coefficients α_1^* and β_2^* are significant for both series, and that α_2^* and β_3^* are only significant for Department Stores. Despite its nonsignificance in the parsimonious model of Table 8,

Table 7. Parameter estimates of the fully parametrized model for Total Retail Trade (Total) and Department Stores sales

Parameter	Variable	Estimate		t-value	
		Total	Dept.	Total	Dept.
η	I_t	−.0010	.0207	−0.13	2.14
α_1	$A_{1,t}$	−.3038	−.2689	−13.12	−11.12
α_2	$A_{2,t}$	−.2860	−.2706	−4.63	−4.16
α_3	$A_{3,t}$	−.1146	−.2664	−1.63	−3.73
β_1	$B_{1,t}$	−.2306	−.1613	−9.97	−6.66
β_2	$B_{2,t}$	.0562	.1586	1.01	2.73
β_3	$B_{3,t}$	−.0069	.0261	−0.10	0.36
α_1^*	$A_{1,t}^*$	.1790	.3297	2.02	3.45
α_2^*	$A_{2,t}^*$	.1469	.5630	0.78	2.78
α_3^*	$A_{3,t}^*$	−.3231	−.1222	−1.32	−0.48
β_1^*	$B_{1,t}^*$	.0554	−.0713	0.64	−0.77
β_2^*	$B_{2,t}^*$	−.6209	−.5001	−3.49	−2.65
β_3^*	$B_{3,t}^*$	−.2340	−.8252	−0.80	−2.56
γ	$S_t^{(121)}$	−.0637	−.1305	−3.63	−6.44
ϵ	E_t	.0239	.0458	3.71	6.88
ζ	$S_t^{(30)}$	.0763	.0736	4.29	3.66
θ_1		.4480	.3572	6.47	5.06
θ_2		.8070	.6927	15.62	11.35

Table 8. Parameter estimates of the parsimonious model for Total Retail Trade and Department Stores sales

Parameter	Variable	Estimate		t-value	
		Total	Dept.	Total	Dept.
η	I_t		.0216		2.32
α_1	$A_{1,t}$	−.3048	−.2686	−13.09	−11.27
α_2	$A_{2,t}$	−.2642	−.2701	−4.69	−4.21
α_3	$A_{3,t}$		−.2914		−4.40
β_1	$B_{1,t}$	−.2251	−.1711	−10.49	−7.65
β_2	$B_{2,t}$		.1600		2.79
β_3	$B_{3,t}$				
α_1^*	$A_{1,t}^*$	.1040	.3017	1.40	3.37
α_2^*	$A_{2,t}^*$		.4896		2.83
α_3^*	$A_{3,t}^*$				
β_1^*	$B_{1,t}^*$				
β_2^*	$B_{2,t}^*$	−.4321	−.4707	−2.91	−2.56
β_3^*	$B_{3,t}^*$		−.7259		−2.69
γ	$S_t^{(121}$	−.0678	−.1313	−3.99	−6.60
ϵ	E_t	.0234	.0458	3.66	6.98
ζ	$S_t^{(30)}$	.9759	.0728	4.31	3.68
θ_1		.4605	.3592	6.96	5.23
θ_2		.8088	.6925	16.67	11.71

parameter α_1^* is retained for Total Retail Trade, because according to Table 6 it points to an increase in the Sunday activity which is known to have happened. This requires some explanation.

First, the Sunday coefficients are δ_7 in regulated months and $\delta_7 + \delta_7^*$ in de-regulated months. For Total Retail Trade sales, only α_1, α_2, β_1, α_1^*, and β_2^* are kept to model the trading-day coefficients; consequently, Table 6 gives that $\delta_7 = \alpha_1 + \alpha_2$ and $\delta_7^* = \alpha_1^*$. This shows that a positive value of α_1^* points to an increase in the Sunday activity.

Parameter β_2^* is significant and negative for both series. According to Table 6, it can be concluded that a negative value for β_2^* will effect a decrease in Thursday, Friday and Monday activities. For the series of Department Store sales, a positive value of α_2^* is also another indication of a significant increase in the Sunday activity. Finally, parameter β_3^* is related to the change in the highest frequency among all the days and an explanation of its significance will be discussed later.

Table 9 displays the daily weights and the percentage of activity on each day of the week. For Total Retail Trade, de-regulation increased the importance of Tuesday, Wednesday, Saturday and Sunday and decreased the importance of the other days. The changes are most important for Thursday, Friday and Saturday. For Department Stores, de-regulation increased the importance of Tuesday, Thursday, Saturday and Sunday and decreased the importance of the other days. The changes are most important for Friday and Saturday. For both series, the combined share of Saturday and Sunday increased and that of Thursday and Friday declined. However, in both cases the effect of de-regulation is surprisingly less important for Sunday than for most other days. This is especially true for Total Retail Trade, with a Sunday increment of only 1%. Furthermore, the regulated patterns were relatively smooth through the week, with a peak of activity on Thursday and Friday and a trough on Sunday, whereas the de-regulated patterns seem rather chaotic, up one day, down the next. This requires some explanations.

One explanation lies in the simplifying assumption of the models used to calculate the daily patterns. The model assumes that consumers adopt the de-regulated daily pattern as soon as stores open on Sundays, and immediately revert to the regulated pattern when stores close. Such sudden behavioural changes are doubtful. Given that consumers take time to adapt to a regime, it is likely that de-regulation triggered chaotic change in the daily patterns. This chaotic change is consistent with the significance of parameter β_3^* for Department Stores, which is associated with cycles lasting between two to three days.

Before concluding on the daily pattern, it is worth noting the parameter parsimony achieved by the trigonometric reparametrization of the trading-day coefficients. It takes only five parameters to model the twelve coefficients for Total Retail Trade and, nine for Department Stores.

Finally, a few more conclusions emerge from this analysis. As shown in Tables 7 and 8, the GST intervention (γ) is statistically significant for both series. The exclusion of both the MST and the GST has reduced the nominal level of Total Retail Trade sales in New Brunswick by 6.8%, and that of Department Stores sales by 13.1%. The Easter holiday effect (ϵ) is significant for both series. During the 15 days preceding Easter, Total Retail Trade sales increase by 2.3%, and Department Store sales by 4.6%. For reasons unknown to the authors, the level of the Total Retail Trade sales shifted up by 7.6% in June 1983, and that of Department Stores sales by 7.3%. Finally, although not shown, the values of

Table 9. Percentages of the weekly activity taking place on the days of the week and daily weights, for Total Retail Trade and Department Stores sales, over the regulated and de-regulated regimes

Days	Total				Department Stores			
	regulated		de-regulated		regulated		de-regulated	
	% activity	ψ_i	% activity	ψ_i^*	% activity	ψ_i	% activity	ψ_i^*
Monday, $i = 1$	10	0.698	7	0.475	16	1.124	7	0.489
Tuesday, $i = 2$	15	1.035	17	1.185	12	0.839	14	0.953
Wednesday, $i = 3$	14	0.961	17	1.192	13	0.886	8	0.569
Thursday, $i = 4$	17	1.169	11	0.737	19	1.320	24	1.672
Friday, $i = 5$	23	1.605	18	1.264	19	1.346	5	0.326
Saturday, $i = 6$	14	0.993	22	1.568	15	1.075	31	2.201
Sunday, $i = 7$	7	0.538	8	0.579	6	0.410	11	0.790

the Ljung-Box (1978) statistic (21.6 and 20.9 at lag 24) confirm that the estimated residuals a_t of both series form two uncorrelated sequences of observations.

4. Conclusions

This article investigated the impact on the retail trade sector of allowing Sunday shopping in the province of New Brunswick. The main conclusions of the analysis are as follows:

1. Sunday opening had no significant effect on the level of the Total Retail Trade sales but a positive and significant effect on Department Stores sales.
2. Because of this, a redistribution of sales between trade groups must have taken place within the retail sector to the benefit of Department Stores.
3. For both series, Sunday opening caused a substantial change in the daily pattern of sales; although the change was more pronounced for some days other than Sunday, the share of Saturday and Sunday increased and that of Thursday and Friday decreased.

In conducting this analysis, we also introduced a trigonometric reparametrization of the trading-day coefficients, which in the case considered requires fewer parameters than the classical approach.

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