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A Note on Post-Stratification When Analyzing Binary Survey Data Subject to Nonresponse

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In this article we present some results regarding the effect of post-stratification when analyzing binary survey data subject to nonresponse. Assuming that nonresponse depends on the variable of interest which is missing for nonrespondents, we show that sometimes the relative reduction in the bias due to post-stratification can be estimated from the respondents alone. This provides a simple sensitivity analysis for the potential bias of the post-stratified estimator, under a departure from the nonresponse assumption required for its unbiasedness. We illustrate with an example from the Norwegian Labour Force Survey.

Key words: Post-stratification; nonresponse; nonignorable nonresponse.

1. Introduction

Post-stratification is widely used to reduce nonresponse bias for survey data (e.g., Oh and Scheuren 1983; Bethlehem 1988; Smith 1991). It removes nonresponse bias entirely if nonresponse is conditionally independent of the variable of interest (which is missing for nonrespondents) within each post-stratum. In this note we allow for nonresponse to depend on the interest variable, and present some results regarding the effect of post-stratification for binary data. Under the alternative assumption that nonresponse is conditionally independent of the auxiliary variable (used for post-stratification) given the variable of interest, we show that the relative bias-reduction due to post-stratification can be estimated from the respondents alone. This provides a simple means of assessing the potential bias of the post-stratified estimator, under one possible departure from the basic assumption required for its unbiasedness.

The alternative assumption we make is a special case of nonignorable nonresponse (Rubin 1976), under which the mean of the variable of interest differs from the respondents to the nonrespondents within each post-stratum. Fay (1986) and Little and Rubin (1987) discuss more general approaches to estimation in the presence of nonignorable nonresponse. In this note we extend results of Thomsen (1973, 1978) on bias and variance under nonresponse, and show that the relative reduction in the bias due to post-stratification is in certain cases approximately equal to the relative reduction in the variance, and is given by the square of the correlation coefficient between the auxiliary and interest variable among the respondents. We illustrate these results with data of the Norwegian Labour Force Survey (LFS).

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2. Bias Due to Nonignorable Nonresponse

In this section we first derive expressions for the bias of both the observed sample mean and the post-stratified estimator. We assume simple random sampling throughout the text. These results provide alternative expressions to those in Thomsen (1973). We then show that the bias of the post-stratified estimator may be estimated under the simple nonignorable nonresponse assumption.

Denote by $U = \{1, \ldots, N\}$ the population, and by $s = \{1, \ldots, n\}$ the sample. Assume that we are to estimate the population mean of a binary variable, denoted by $\bar{Y}$; and that auxiliary information is available in the form of a second binary variable denoted by X. In addition, denote by R the response variable such that $R = 1$ indicates response and $R = 0$ nonresponse. Denote by $q_{ij} = N_{ij}/N$ the population proportion of $(X, Y) = (i, j)$ for $i, j = 0, 1$, and r_{ij} the nonresponse rate within the population group $(X, Y) = (i, j)$. The population and the expected sample have the following distribution:

	$Y=1$		$Y=0$	
	$R=1$	$R=0$	$R=1$	$R=0$
$X=1$	$q_{11}(1-r_{11})$	$q_{11}r_{11}$	$q_{10}(1-r_{10})$	$q_{10}r_{10}$
$X=0$	$q_{01}(1-r_{01})$	$q_{01}r_{01}$	$q_{00}(1-r_{00})$	$q_{00}r_{00}$

The population mean $\bar{Y}$ is given by $p = q_{11} + q_{01}$, and the marginal proportion of $X = 1$ by $q = q_{11} + q_{10}$. Given nonresponse, i.e., $s = (s_r, s_{mis})$ where s_r denotes the response group and s_{mis} the nonresponse group with respective sizes n_r and $n - n_r$, the observed sample mean is given by $\bar{y} = [n_r(1,1) + n_r(0,1)]/n_r$, where $n_r(i,j)$ denotes the size of the subsample $(X, Y) = (i, j)$ within the response group s_r, and

$$\begin{aligned} E(\bar{y} - p | n_r) &= \frac{\sum_{i,j} q_{i1} q_{j0} (r_{j0} - r_{i1})}{\sum_{i,j} q_{ij}(1 - r_{ij})} = \frac{p\left(\sum_i q_{i0} r_{i0}\right) - (1-p)\left(\sum_i q_{i1} r_{i1}\right)}{E(n_r)/n} \\ &= E(\bar{y} - p) \end{aligned} \quad (1)$$

While the first equation expresses the bias as a function of pairwise difference in response rates, the second one specifies the contribution of each subsample (X, Y).

Post-stratification further divides the response group into $s_r = (s_{r,1} s_{r,0})$ with respective sizes $n_{r,1}$ and $n_{r,0}$. The post-stratified mean is $\bar{y}_{pst} = q n_r(1,1)/n_{r,1} + (1-q) n_r(0,1)/n_{r,0}$, and

$$E(\bar{y}_{pst} - p | (n_{r,1}, n_{r,0})) = \frac{q_{11} q_{10} (r_{10} - r_{11})}{E(n_{r,1})/n} + \frac{q_{01} q_{00} (r_{00} - r_{01})}{E(n_{r,0})/n} = E(\bar{y}_{pst} - p) \quad (2)$$

Equations (1) and (2) provide alternative expressions to those in Thomsen (1973). If q is unknown and is estimated by $\hat{q}^* = n_1/n$ where n_1 is the size of the sample post-stratum $X = 1$, the result remains valid under suitable regularity conditions. Notice also that while the values of (1) and (2) are unknown in general, one sometimes can be quite certain about their signs. For instance, if it is known that, conditional to $X = i$, $Y = 1$ leads to lower nonresponse rate, then $\bar{y}_{pst}$ is positively biased according to (2).

With auxiliary information X available, nonresponse is ignorable (given X) if R is independent of Y given X, whereas it is nonignorable if R remains dependent on Y despite knowledge of X. In the present notation, ignorable nonresponse implies $r_{i0} = r_{i1}$ for $i = 0, 1$. It follows from (2) that the post-stratified mean is then unbiased, whereas the sample mean remains biased.

A simple nonignorable nonresponse mechanism involves assuming that R is independent of X given Y, which implies that $r_{i0} = r_0$ and $r_{i1} = r_1$ for $i = 0, 1$. It follows from (1) that the bias in $\bar{y}$, denoted by b_{srs}, is now given by

$$b_{srs} = \frac{(r_0 - r_1)p(1-p)}{E[n_r]/n} = \frac{r_0 - r_1}{(1-r_0)(1-r_1)} \cdot \frac{1}{n}\left\{\frac{E(n_r(-,1))E(n_r(-,0))}{E[n_r]}\right\} \tag{3}$$

where $n_r(-,j) = n_r(1,j) + n_r(0,j)$ for $j = 0, 1$; whereas the bias in $\bar{y}_{pst}$, denoted by b_{pst}, is obtained from (2) as

$$b_{pst} = \frac{r_0 - r_1}{(1-r_0)(1-r_1)} \cdot \frac{1}{n}\left\{\frac{E(n_r(1,1))E(n_r(1,0))}{E(n_{r,1})} + \frac{E(n_r(0,1))E(n_r(0,0))}{E(n_{r,0})}\right\} \tag{4}$$

In other words, the ratio of the biases, denoted by $\gamma = b_{pst}/b_{srs}$, can be estimated from the response group alone. Since $\bar{y} - \bar{y}_{pst}$ is an estimate of $b_{srs} - b_{pst}$, a bias-correcting estimator can be given by $\bar{y}_{adj} = -\bar{y}\hat{\gamma}/(1-\hat{\gamma}) + \bar{y}_{pst}/(1-\hat{\gamma})$.

One could check on the nonresponse assumption $r_{i0} = r_0$ and $r_{i1} = r_1$ for $i = 0, 1$ from a model point of view. By considering the sample as having been generated under the model where $P((X, Y) = (i,j)) = q_{ij}$ and $P(R = 0|(X, Y) = (i,j)) = r_{ij}$, we obtain the likelihood function proportional to $P((X, Y, R))$. It is important to keep in mind that a good fit alone is not enough to establish the validity of the model. For instance, the ignorable nonresponse model $r_{i0} = r_{i1}$ always fits the data perfectly, i.e., reproduces the data exactly. On the other hand, it is probably reasonable to accept a bad fit as convincing evidence *against* the nonresponse assumption.

3. Variance

Thomsen (1978) derived the approximate variances of $\bar{y}$ and $\bar{y}_{pst}$ under simple random sampling, which can be estimated from the observed sample regardless of the values of r_{ij}. It was noted that the variance reduction is often not noteworthy unless the population marginal proportions of the post-strata are known. In the present notation and ignoring the finite population correction factors, these are given by

$$Var(\bar{y}) = E(n_r(1,1) + n_r(0,1)) \cdot E(n_r(1,0) + n_r(0,0))/E(n_r)^3$$

$$Var(\bar{y}_{pst}) = q^2 E(n_r(1,1)) \cdot E(n_r(1,0))/E(n_{r,1})^3 + (1-q)^2 E(n_r(0,1)) \cdot E(n_r(0,0))/E(n_{r,0})^3$$

In particular, the ratio of the variances, denoted by $\eta = Var(\bar{y}_{pst})/Var(\bar{y})$, describes the effect of post-stratification on the variance.

It is interesting to notice that, under the simple nonignorable nonresponse assumption above, $\gamma = \eta$ provided $E(n_{r,1})/E(n_r) = q$, i.e., the ratio of the biases equals the ratio of the variances. Since $q = E(n_1)/n$, the equality holds approximately in cases where the nonresponse is not too severe. In addition, it is sometimes the case that $q \doteq p$, such as when X is provided by a similarly defined variable available from other sources or simply the variable Y some short while ago. If this approximate equality holds also within the response group, we obtain

$$\gamma \doteq \eta \doteq 1-\rho_r^2 \quad \rho_r = \frac{E(n_r(1,1))\cdot E(n_r) - E(n_r(1,-))\cdot E(n_r(-,1))}{\sqrt{\{E(n_r(1,-))\cdot E(n_r(0,-1))\}\{E(n_r(-1))\cdot E(n_r(-,0))\}}} \tag{5}$$

where ρ_r is the correlation coefficient between X and Y among the respondents. Having estimated (ρ_r, γ, η), one can easily check whether (5) holds in a given situation.

4. An Example: the Norwegian LFS

Post-stratification has long been applied in connection with the LFS in a number of countries. By exploiting the high correlation between the LFS employment/unemployment status and the register-based employment/unemployment status, which is available by linking every sampled subject to a population register constructed independently of the LFS, post-stratification can greatly reduce the variance of the level-estimators (e.g., Djerf 1997). Meanwhile, since one can be quite certain that the employment rate is lower among nonrespondents, even when conditional on each state of the register-based status, the nonresponse in the LFS is most likely nonignorable.

At the moment, the register-based employment/unemployment status used in Norway is restricted to ''employment or not,'' and does not provide further classification between ''job seeker'' and ''inactive unemployment.'' For illustrative purposes we shall therefore concentrate on the estimation of the total LFS employment here. Proceeding under the assumption that the LFS nonresponse (denoted by R) is independent of the register-based employment status (denoted by X) conditional on the LFS employment status (denoted by Y), we apply the results earlier to data of the first quarter in 1995 from the Norwegian LFS:

$X=1$			$X=0$		
$(Y,R)=(1,1)$	$(Y,R)=(0,1)$	$R=0$	$(Y,R)=(1,1)$	$(Y,R)=(0,1)$	$R=0$
12,881	1,158	518	1,829	6,726	796

First of all, a simple calculation of the estimated variances gives us $\hat{\eta}=0.494$, i.e., an estimated 50% reduction in variance due to post-stratification according to the register-based employment status, which is consistent with the findings presented in Djerf (1997). Also, $\hat{\rho}_r = 0.716$ and $1-\hat{\rho}_r^2 = 0.487 \doteq \eta$. All the estimates here are obtained by replacing $E(n_r(i,j))$ with $n_r(i,j)$.

Now, applying the results in Section 2, we obtain $(\bar{y}, \bar{y}_{pst}, \bar{y}_{adj}) = (0.651, 0.645, 0.640)$ with the known proportion registered employed $q = 0.613$ in the population, and $(\bar{y}, \bar{y}^*_{pst}, \bar{y}^*_{adj}) = (0.651, 0.642, 0.634)$ if $\hat{q}^* = 0.609$ is estimated from the sample. The estimated ratio of biases under the simple nonignorable model is $\hat{\gamma} = 0.487 = 1-\hat{\rho}_r^2$ in both cases. Notice that the difference between $\bar{y}_{pst}$ and $\bar{y}^*_{pst}$ is doubled into that between $\bar{y}_{adj}$ and $\bar{y}^*_{adj}$ through the term $1/(1-\hat{\gamma})$, which indicates the sensitivity of $\bar{y}_{adj}$ to the stochastic variation in the estimation of $(\gamma, b_{srs} - b_{pst})$.

We have evaluated the nonresponse assumption $r_{i0} = r_0$ and $r_{i0} = r_1$ for $i = 0, 1$ from a model perspective, as explained earlier. More explicitly, we calculated the maximum likelihood estimates (using EM algorithm), which gives us $(\hat{q}_{11}, \hat{q}_{01}, \hat{r}_1, \hat{r}_0) = (0.559, 0.078, 0.029, 0.099)$. The scaled deviance, i.e., twice the difference between the maximum attainable log-likelihood and the fitted log-likelihood, was zero so that these estimates also yielded a perfect fit to the data. Notice that, from the model perspective, we have $\bar{y}_{mod} = \hat{q}_{11} + \hat{q}_{01} = 0.637$. To check whether the perfect fit could be attained with any choice of X, we have also fitted the model where X was set to be sex instead. Using the known $q = 0.503$ in the population, we obtained $(\hat{q}_{11}, \hat{q}_{01}, \hat{r}_1, \hat{r}_0) = (0.363, 0.307, 0.082, 0.000)$ with scaled deviance 10.3, so we can be quite sure that the nonresponse assumption does not apply to sex. (Post-stratification with respect to sex gives $\hat{\eta} = 0.987$, i.e., with practically no effect on the variance.)

It should be noticed that, like the LFS in many other countries, the Norwegian LFS does not employ a strict simple random sampling design. Neither can one be sure of the simple nonresponse assumption here. For instance, it is probable that nonresponse is indeed more severe among the subsample $(X, Y) = (0, 0)$ than among $(X, Y) = (1, 0)$, in which case R is not strictly independent of X conditional on Y, although model fitting seems to suggest a very weak possible dependence in addition. We therefore do not recommend bias-correction *via* $\bar{y}_{adj}$ for the LFS. However, we suggest that it is likely that, using register-based employment status, post-stratification results in about 50% reduction, in both the variance and the bias caused by nonresponse, of which the latter reflects the ''nonignorability'' of the nonresponse.

5. Summary

We have shown that sometimes the bias-reduction due to post-stratification can be estimated from the respondents alone, even when nonresponse does depend on the variable of interest which is missing for nonrespondents. This provides a simple means of assessing the potential bias of the post-stratified estimator, under a specific departure from the assumption required for it to be unbiased. In reality both the sampling design and the nonresponse mechanism are likely to be more complex than the ones assumed here; and it is not proposed that the results for bias-correction be used directly. As a particular kind of sensitivity analysis, though, they may help us to assess the quality of estimates in surveys such as the LFS.

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