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A Parametric Method for Census Based Estimation of Child Mortality

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Abstract: A parametric method for estimating child mortality from reports concerning children ever born and surviving children is presented. In contrast with previously proposed methods, it facilitates use of single-year age reports by mothers on the survival of their children. In addition, the new method makes it possible to incorporate *a priori* knowledge of child mortality and fertility in the estimation process. The new method is illustrated by means of an application to data from the 1976 Western Samoa census.

Key words: Brass-type estimation of child mortality; indirect estimation of child mortality; parametric models of mortality and fertility.

1. Introduction

Brass (1961) was among the first to devise a method for estimating childhood mortality in developing countries with incomplete vital registration. His method uses census returns on the number of children ever born and surviving children tabulated by five-year age groups of mothers. Such data are often referred to as child survivorship data. Presumably, it was the intention that the method should be used on a makeshift basis until vital registration was made complete. However, because the improvements in vital registration in developing countries have been rather modest, the method has continued to play a salient role in estimating child mortality. Since it is reasonable to assume that most developing countries will not achieve complete vital registration during the remaining part of this century, there is ample reason to believe that the observation plan of children ever born and surviving children will be used during the coming years for estimating child mortality.

Because the retrospective observation plan of children ever born and surviving children confounds mortality and fertility, the child mortality estimates are always affected by fertility. In an effort to increase the precision in estimated child survival, relative to that of previously proposed methods, the paper addresses the problem of incorporating prior knowledge of fertility and child mortality in the estimation process.

The new method in this paper is based on Brass's original idea but deviates from his method, and from previous modifications of it (Brass 1975; Feeney 1976, 1980; Hill and Trussell 1977; Kraly and Norris 1978; Palloni 1979, 1980; Sullivan 1972; Trussell 1975; United Nations 1990) in that it relies on parametric models of child mortality and fertility. To advance the discussion so that the essential differences between the Brass

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method (or its previous modifications) and the method proposed in this paper are brought out, the paper begins with a discussion of how to capture the survival function for children from the reported proportions of deceased children. We then turn to a brief discussion of the Brass method and show how the introduction of parametric models of child survival and fertility lead to a new estimation technique. Although the present paper is intended principally as a methodological note, an application to data from the 1976 Western Samoa population census is given for illustrative purposes. The paper concludes with a discussion that focuses partly on theoretical, partly on practical issues.

2. Modeling the Proportions of Deceased Children Reported by Women in a Census

In what follows, it is assumed that the population is closed to migration and that mortality and fertility are stationary. Let f be the fertility function for women, s the survival function for both sexes and $W(x)$ the age density for females. This means that for an infinitesimal age increment dx, $W(x)dx$ is the expected number of females between ages x and $x + dx$, the probability for a woman aged x to give birth to a child before age $x + dx$ is $f(x)dx$ and that the probability for a child to survive from birth until age a is $s(a)$. The expected number of children born by these women when they were between ages x-a and $x - a + da$ is $W(x - a)f(x - a)da$. All of these children, however, cannot be reported since some of the women have died during the past a years. Instead, the expected number of children to be reported is $W(x)f(x - a)da$. Letting a_0 denote the starting age of the fertility schedule given by f and by extending the consideration to the whole of the reproductive sub-interval a_0 to x, the expected number of children to be reported by women aged x is

$$B(x) = W(x) \int_0^{x-a_0} f(x - a)\,da \qquad (1)$$

and the corresponding number of surviving children to be reported is

$$S(x) = W(x) \int_0^{x-a_0} f(x - a)s(a)\,da. \qquad (2)$$

From (1) and (2) it follows that the proportion of children ever born who will be reported as dead by women aged x is

$$H_x = \frac{\int_0^{x-a_0} f(x - a)q(a)\,da}{\int_0^{x-a_0} f(x - a)\,da} \qquad (3)$$

where $q(a) = 1 - s(a)$. Henceforth, q is referred to as the mortality function.

Assuming that $q(a)$ is continuous for strictly positive a, the mean value theorem for integrals and (3) imply that there is an age $\xi_x = \xi(f, q, x)$, $0 < \xi_x < x - a_0$, so that

$$q(\xi_x) = H_x. \qquad (4)$$

It is this elementary property that justifies estimation of childhood mortality from child survivorship data.

3. A Brief Outline of the Brass Method

3.1. *Child survival estimated from reporting women in five-year age groups*

In the approach given by Brass and Coale (1968, pp. 104–122), it was assumed that child survivorship data only were given by five-year age groups of women. In order to work with the standard five-year age groups 15–19, 20–24, . . . , 60–64, indexed i = 1, . . . , 10, respectively, D_i denotes the corresponding proportion of deceased children and P_i the mean parity for females in age group i. More specifically, P_i is the average number of children ever born reported by women in age group i. In what follows, we disregard survival reports from women aged 30 and over. The main reason for this is that they refer to a period long before the census and that, from a practical point of view, it becomes unrealistic to assume that mortality and fertility have remained stationary for such a long time.

If we assume, as Brass did, that the age distribution of females is uniform over each five-year age group then it follows that the mean parity for women in the ith five-year age group is

$$P_i = \frac{1}{5} \int_{t_i}^{t_i+5} F(x)\, dx$$

where $t_i = 10 + 5i$, and $F(x) = \int_{a_0}^{x} f(u)\, du$. It will also be seen that the proportion of deceased children to be reported by women in the ith five-year age group then becomes

$$D_i = \frac{\int_{t_i}^{t_i+5} \int_{0}^{x-a_0} f(x - a)q(a)\, da\, dx}{\int_{t_i}^{t_i+5} \int_{0}^{x-a_0} f(x - a)\, da\, dx}$$

where $t_i = 10 + 5i$.

Using the mean value theorem for double integrals with the assumption that f and q are continuous, it follows that there is an age $\zeta_i = \zeta(f, q, i)$ so that $q(\zeta_i) = D_i$. It is clear, therefore, that the proportion of deceased children reported, e.g., by women aged 20–24 years is equal to the probability of dying before a certain age, ζ_2, say. Estimating childhood mortality from proportions of deceased children given by five-year age groups of women, then, is a matter of finding the age ζ_i that corresponds to D_i. It will be noted, however, that ζ_i obviously depends on the fertility of women and the mortality of children.

3.2. *The development of Brass multipliers*

To estimate the probabilities $q(i)$, Brass introduced multipliers $M(\mu, i)$ which depend on the mean age of the fertility schedule μ and the age group i. Denoting the observed proportions of deceased children by $\hat{D}_i$, the estimation procedure is

$$\hat{q}(i) = M(\mu, i)\, \hat{D}_i,\ i = 1, 2, \text{and } 3.$$

The multipliers $M(\mu, i)$ have been constructed on the basis of two assumptions. First, it is assumed that for childhood ages, mortality functions are proportional in the sense that

$$q(x) = \rho q_s(x) \quad (5)$$

where $q_s(x)$ is a completely specified standard mortality function and ρ is a constant. The mortality function q_s, then, generates a family of related mortality functions. It will be noted that ρ cancels in (3) so that the multipliers are independent of ρ, that is, of the level of child mortality. Second, it is assumed that the fertility curve is given by

$$\phi_x = C(x - a_0)(33 + a_0 - x)^2,$$

$$a_0 \leqslant x \leqslant a_0 + 33 \quad (6)$$

which is known as the Brass fertility polynomial (Retherford 1979). Here C is a parameter which determines the total fertility rate. Notice also that C cancels in (3) so that the multipliers are independent of the level of fertility. For this polynomial the mean age of the fertility schedule is $\mu = a_0 + 13.2$ with fixed variance $\sigma^2 = 43.6$. It establishes a one-to-one relationship between, on the one hand, the ratios of mean parities $R_1 = P_1/P_2$ and $R_2 = P_2/P_3$ and, on the other, μ or a_0. Corresponding to a standard mortality function q_s (which was believed to be typical of developing countries) and different choices of a_0 in (6), the multipliers were calculated as $M(a_0, i) = q_s(i)/D_i$.

Brass has tabulated $M(a_0, i)$ as a function of the ratios of mean parities R_1 and R_2. In a practical situation, multipliers are chosen on the basis of the observed ratios $\hat{R}_1 = \hat{P}_1/\hat{P}_2$ or $\hat{R}_2 = \hat{P}_2/\hat{P}_3$. These ratios are accepted as measures of the central location of the fertility schedule underlying the reported proportions of deceased children. In applications of the method, it is mostly $\hat{R}_2$ which is used for derivation of the multipliers. The reason for this is that R_1 is often greatly affected by reporting errors and it does not correlate very well with the mean of the underlying fertility schedule (Hartmann 1989a).

It should be noted that one of the reasons for elaborating with the statistics D_i is that they smooth the random errors in the single-year reports H_x. Nevertheless, whether there is large or small random variation in the H_x, it ought to be possible to base the estimation of child mortality on these. The present paper, as already noted, focuses on this issue.

3.3. *The uncertainties of Brass estimation*

It is evident that several uncertainties apply to Brass estimation of child mortality. First, it will be noted that the estimation procedure confounds mortality and fertility. In fact, for any given five-year age group, the multiplying factors $M(a_0, i)$ are chosen solely on the basis of R_1 or R_2. Second, the estimates are obtained subject to the assumption that the true underlying age patterns of child mortality and fertility are similar to those underlying the multiplying factors $M(a_0, i)$. Third, due to the assumption of stationary mortality and fertility, the estimated probabilities $\hat{q}(i)$, $i = 1, 2$, and 3 do not refer to the time of the census unless the past changes in mortality and fertility have been rather modest. Fourth, it is assumed that the age distribution of women over each of the above mentioned five-year age groups is uniform.

Furthermore, it should be noted that because the retrospective observation plan is based on reports from mothers concerning their number of ever born children and surviving children, the resulting estimates of child mortality only concern the children for whom survival returns are obtained. This means that one cannot estimate the mortality of children whose mothers (a) were dead at the time of the census, (b) were not contacted by the census enumerators (underenumerated females), (c) refused to answer the census questions (female non-respondents) or (d) were abroad at the time of the census. To this must be added that it is assumed that child mortality is independent of the age of mother as well as of her parity and, of course, that the returns on children ever born and surviving children are accurate.

3.4. *Relaxing the uncertainties of Brass estimation*

The method in this paper aims at reducing some of the above mentioned uncertainties. For example, it allows the use of parametric representations of child mortality and fertility which the user has reason to believe are close to the ones underlying the reported child survivorship data. More specifically, the user can tailor the method to an *a priori* choice of age patterns of mortality and fertility. Moreover, the limitation imposed by working with five-year age groups is done away with. We begin the discussion by introducing parametric models of child mortality and fertility which can be used in (3).

4. Modeling Childhood Mortality

It has recently been shown (Hartmann 1989b) that

$$s(t; \theta) = \frac{1}{1 + \exp(2a)t^{2b}} \qquad (7)$$

with parameter vector $\theta = (a, b)$ gives a close representation of survival curves between the ages of 0 and 15. Consequently, we use $q(t; \theta) = 1 - s(t; \theta)$ as a parametric model of child mortality in (3). Several applications of $s(t; \theta)$ to empirical child survival curves are given in Hartmann (1980).

5. Modeling the Fertility Schedule

Although many attempts have been made to model the fertility schedule (Brass 1960; Coale and Trussell 1974; Hoem, Madsen, Lövgreen Nielsen, Ohlsen, Hansen, and Rennermalm 1981) the shifted gamma probability density function

$$g(x; \kappa) = \frac{c^k}{\Gamma(k)}(x - d)^{k-1} \times \exp(-c(x - d)),\ x \geqslant d \tag{8}$$

with parameter vector $\kappa = (c, k, d)$, has always performed as one of the most accurate models of observed normalized fertility schedules, that is, schedules for which the total fertility rate has been set to one. The mean and variance are $\mu = d + k/c$ and $\sigma^2 = k/c^2$, respectively. A model of an empirical fertility schedule with total fertility rate R is $R\,g(x; \kappa)$. However, when modeling f in (3) the total fertility rate cancels so that it is sufficient to model the normalized fertility schedule. A slightly simpler but in most cases adequate model is obtained by letting $d = 0$ in (8) so that

$$\psi(x; \lambda) = \frac{c^k}{\Gamma(k)} x^{k-1} \exp(-cx),\ x \geqslant 0 \tag{9}$$

with $\lambda = (c, k)$, models the normalized fertility schedule. Empirical studies suggest (Hartmann 1982) that for $k = 18$ in (9)

$$\gamma(x; c) = \frac{c^{18}}{17!} x^{17} e^{-cx},\ x \geqslant 0$$

gives a simple but adequate one-parameter model of the normalized fertility schedule.

6. A New Method

6.1. The characteristics of child survivorship data

Before the new method is discussed, it is instructive to evaluate the numerical consequences of (4). To this end, Table 1 gives parametrized model proportions of deceased children obtained from (3) by fitting $q(t; \theta)$ to West model life table 16 for both sexes (Coale and Demeny 1966) and the model fertility function $\gamma(x; c)$ with mean ages ranging from 24.7 to 29.7 (this range of mean ages is the same as is given by Brass (1975, p. 55). The estimated parameters in $q(t; \theta)$ are $\hat{a} = -1.1282$ and $\hat{b} = 0.1002$ (Hartmann 1980, p. 47). The ages ξ_x of children for which $H_x = q(\xi_x; \theta)$ are also given. For further details, see Hartmann (1982).

Women aged 16, with $\mu = 24.7$, report a proportion of 0.0958 dead children. Solving the equation $q(\xi_x; -1.1282, 0.1002) = 0.0958$ with respect to ξ_x gives $\xi_x = 1.06$ (Table 1). This shows that for such a choice of fertility and mortality functions, the proportion deceased children reported by women aged 16 years is nearly the same as infant mortality. However, with the same mortality function, but with $\mu = 29.7$, the proportion of deceased children for women aged 16 years is 0.0921 for which $\xi_x = 0.85$ (Table 1). The discrepancy between these solutions for ξ_x is explained by the effects of rapid versus slow childbearing during early fecund ages. Also, it will be seen (Table 1) that the proportions of deceased children reported by women with single-year ages 20, 21, 22, 23, and 24 and $\mu = 24.7$ equal the probabilities of dying before ages ξ_x with $1.94 \leqslant \xi_x \leqslant 3.16$ and that when $\mu = 29.7$

Table 1. Model proportions of deceased children corresponding to model fertility schedules $\gamma(x; c)$ with mean ages μ ranging from 24.7 to 29.7 years and child mortality corresponding to West model life table level 16 for both sexes

Age x	Mean age of model fertility schedule											
	24.7		25.7		26.7		27.7		28.7		29.7	
	$H_x(\mu)$	$\xi_x(\mu)$	$H_x(\mu)$	$\xi_x(\mu)$	$H_x(\mu)$	$\xi_x(\mu)$	$H_x(\mu)$	$\xi_x(\mu)$	$H_x(\mu)$	$\xi_x(\mu)$	$H_x(\mu)$	$\xi_x(\mu)$
15	0.0917	0.83	0.0912	0.81	0.0908	0.79	0.0880	0.66	0.0876	0.65	0.0872	0.63
16	0.0958	1.06	0.0952	1.02	0.0936	0.93	0.0931	0.91	0.0926	0.88	0.0921	0.85
17	0.0990	1.27	0.0979	1.19	0.0972	1.15	0.0967	1.12	0.0961	1.08	0.0943	0.97
18	0.1016	1.47	0.1009	1.41	0.1002	1.36	0.0989	1.26	0.0983	1.22	0.0977	1.18
19	0.1043	1.70	0.1032	1.60	0.1024	1.53	0.1017	1.48	0.1010	1.42	0.0997	1.32
15–19	0.0985	1.23	0.0977	1.18	0.0968	1.12	0.0957	1.05	0.0951	1.02	0.0942	0.97
20	0.1068	1.94	0.1058	1.84	0.1049	1.75	0.1038	1.65	0.1030	1.58	0.1023	1.52
21	0.1092	2.19	0.1081	2.07	0.1071	1.97	0.1062	1.88	0.1053	1.79	0.1043	1.70
22	0.1116	2.48	0.1104	2.33	0.1093	2.21	0.1083	2.09	0.1074	2.00	0.1065	1.91
23	0.1140	2.79	0.1127	2.62	0.1115	2.47	0.1104	2.33	0.1094	2.22	0.1085	2.12
24	0.1165	3.16	0.1151	2.95	0.1138	2.77	0.1125	2.59	0.1114	2.45	0.1105	2.34
20–24	0.1116	2.48	0.1104	2.33	0.1093	2.21	0.1082	2.08	0.1073	1.99	0.1064	1.90
25	0.1189	3.54	0.1174	3.30	0.1160	3.08	0.1148	2.91	0.1136	2.74	0.1124	2.58
26	0.1214	3.99	0.1197	3.68	0.1183	3.44	0.1169	3.22	0.1157	3.04	0.1145	2.86
27	0.1238	4.46	0.1221	4.12	0.1205	3.82	0.1191	3.58	0.1178	3.36	0.1166	3.17
28	0.1263	4.99	0.1245	4.60	0.1228	4.26	0.1213	3.97	0.1199	3.72	0.1186	3.49
29	0.1287	5.56	0.1268	5.11	0.1251	4.73	0.1235	4.40	0.1220	4.10	0.1206	3.84
25–29	0.1238	4.46	0.1221	4.12	0.1205	3.82	0.1191	3.58	0.1178	3.36	0.1165	3.16

ξ_x is a solution to the equation $H_x = q(\xi_x; -1.1282, 0.1002)$. The sex ratio at birth is assumed to be 1.05.

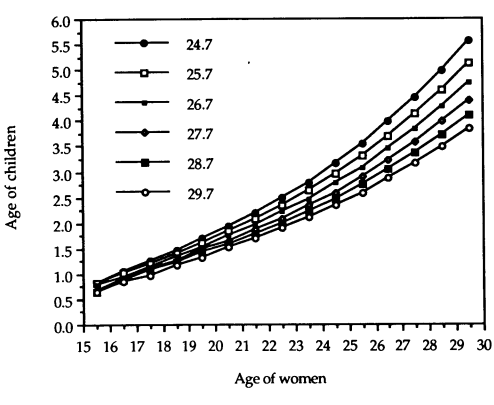

Fig. 1. -Plot of ξ_x against x

then $1.52 \leqslant \xi_x \leqslant 2.34$. In consequence, ξ_x is sensitive to μ. This sensitivity, however, is much more pronounced in the age group 25–29 than in the age groups 15–19 and 20–24. This is illustrated by Fig. 1 which shows a plot of ξ_x as a function of x when μ increases from 24.7 to 29.7.

It is evident, therefore, that the observation plan of children ever born and surviving children suggest that one should use the reports from women aged about 20 years for estimating child mortality. In particular, if the population of young women is so large that there is little or no random variation in $\hat{H}_x, x = 19, 20$, and 21, say, estimated child mortality based on these reports reflect current child mortality more so than if based on, e.g., the reports from women aged 25–29.

6.2. Estimating the mortality function

Regardless of how the problem of estimating child mortality from child survivorship data is approached, it is necessary to make model assumptions concerning underlying mortality and fertility functions. As to the childhood mortality function, the assumption (5) is both practical and realistic. With this assumption it follows from (3) that model proportions of deceased children, generated by the same age pattern of fertility, become proportional, that is,

$$H_x = \rho H_x^s \qquad (10)$$

where H_x^s is generated by q_s.

Assume now that we have $\hat{H}_{x_i}$ for the ages $x_1, \ldots, x_N$ of women, that the underlying fertility schedule is approximated by $\gamma(x; c^*)$ with mean μ^* (in which case $c^* = 18/\mu^*$), and that the underlying mortality function q is proportional, in the sense of (5), to a completely specified mortality function $q_s(t) = q(t; \hat{\theta})$. Estimation of q is then a matter of solving the system of equations

$$\hat{H}_{x_i} = \rho \frac{\int_0^{x_i+0.5} \gamma(x_i + 0.5 - a; c^*) q_s(a)\, da}{\int_0^{x_i+0.5} \gamma(x_i + 0.5 - a; c^*)\, da},$$

$$i = 1, \ldots, N$$

with respect to ρ. (The upper limit of integration is $x_i + 0.5$ because we refer to women aged x at last birth day and because the age distribution for women is considered uniform at each single-year age interval.) Since this may be a mathematically intractable problem, it is necessary to follow a slightly different route.

The two functions $q_s(t) = q(t;\hat{\theta})$ and $\gamma(x; c^*)$ can be used to compute model proportions of deceased children in agreement with (3). Denoting the resulting model proportions of deceased children by $H_x(\mu^*, \hat{\theta})$, it follows from (10) that

$$\hat{H}_{x_i} = \rho H_{x_i}(\mu^*, \hat{\theta}), \; i = 1, \ldots N.$$

Minimizing

$$\sum_{i=1}^{N} [\hat{H}_{x_i} - \rho H_{x_i}(\mu^*, \hat{\theta})]^2$$

with respect to ρ gives

$$\hat{\rho} = \sum_{i=1}^{N} [\hat{H}_{x_i} H_{x_i}(\mu^*, \hat{\theta})] \Big/ \sum_{i=1}^{N} [H_{x_i}(\mu^*, \hat{\theta})]^2 \qquad (11)$$

whereby the estimated mortality function is

$$\hat{q}(t) = \hat{\rho}\hat{q}_s(t) = \hat{\rho} q(t; \hat{\theta}).$$

Here it is evident that the estimated mortality function $\hat{q}(t)$ depends on the estimated, or guessed, μ^*, on the assumption that the age pattern of fertility is reasonably approximated by $\gamma(x; c^*)$, and on the validity of the *a priori* estimated mortality function $q_s(t) = q(t; \hat{\theta})$. Moreover, it should be noted that the main idea behind (11) is that the chosen model proportions of deceased children should be very similar to the observed ones so that $q_s(t)$ only has to undergo a slight adjustment to yield an estimate of the mortality function underlying the observed proportions of deceased children.

The flexibility of the method is that it is possible to make the assumption that the fertility schedule is approximated by a parametric representation given by $g(x; \kappa)$, $\psi(x; \lambda)$, $\gamma(x; c)$, ϕ_x, or by another convenient parametric fertility model. In particular, it may be possible to select a parametric representation of fertility with approximately

Table 2. Proportions of deceased children corresponding to Swedish male mortality 1901–10 and West model life table 16 (both sexes) and $\gamma(x; c)$ model fertility with mean 29.7

Age of women	Sweden, 1901–10	West 16
	$\hat{H}_x(29.7)$	$H_x(29.7)$
19	0.1026	0.0997
20	0.1055	0.1023
21	0.1078	0.1043
22	0.1104	0.1065

$\hat{\rho} = \Sigma_{i=19}^{22} [\hat{H}_x H_x(29.7)]/\Sigma_{i=19}^{22} [H_x(29.7)]^2 = 0.04402/0.04263 = 1.033\,\hat{q}(i) = 1.033\,q(i; \hat{\theta})$ with $\hat{\theta} = (-1.1282, 0.1002)$

the same R_2 and μ that are estimated, or guessed, for the hypothetical fertility schedule underlying the $\hat{H}_x$. In other words, the model proportions of deceased children appearing in (11) can be tailored to reflect available knowledge concerning current fertility of women. With respect to the *a priori* choice of the mortality function $q_s(t) = q(t; \hat{\theta})$, it is clear that $q(t; \theta)$ can be fitted to a child mortality experience which is believed to be a good approximation to that underlying the observed proportions of deceased children. Furthermore, as noted, one is not limited to only making use of $\hat{D}_i$, $i = 1, 2,$ and 3, but can take advantage of reports $\hat{H}_x$ for any convenient range of single-year ages of mothers.

7. Numerical Illustrations

7.1. A hypothetical example

As a first, but hypothetical, illustration, Table 2 gives proportions of deceased children $\hat{H}_x$, $x = 19$, 20, 21, and 22, generated from (3) by γ with $\mu = 29.7$ years and $s(t; \theta)$ fitted to Swedish male mortality during 1901-10 which yields $\hat{a} = -1.11955$ and $\hat{b} = 0.11472$ (Hartmann 1980, p. 55).

Table 3. Actual and estimated survival probabilities using the new method

Age of children	Actual $q(i)$	Estimated $\hat{q}(i)$
1	0.0963	0.0979
2	0.1110	0.1110
3	0.1206	0.1193

This is the mortality function we wish to capture by means of the new method. The first problem is to find a set of model proportions of deceased children which are similar to $\hat{H}_x$. Since the $H_x(29.7)$ in Table 1 are quite close to $\hat{H}_x$, we use these in (11) for estimation of ρ. We choose the ages 19, 20, 21, and 22 for estimation of ρ. This gives $\hat{\rho} = 1.033$ (see Table 2). The mortality function underlying the model proportions of deceased children in Table 1 is $q_s(t) = q(t, \theta)$ with $\hat{a} = -1.1282$ and $\hat{b} = 0.1002$. Hence, the estimated mortality function underlying the $\hat{H}_x$ is $\hat{q}(t) = 1.033\ q_s(t)$. It will be seen that the estimates of $q(1)$, $q(2)$, and $q(3)$ are quite accurate (Table 3).

7.2. An application to child survival data from Western Samoa 1976

The data in the present illustration come from the 1976 Western Samoa population census; the population is small and the conclusions drawn from the estimates are somewhat limited. In illustrating the method with these data, the focus is on applicational aspects rather than on the reliability of the derived estimates. Table 4 gives the reported numbers of ever born and surviving children.

A close investigation of fertility in Western Samoa (Western Samoa 1979) suggested an estimate of $\mu = 29.6$ (Yap State 1988). The model proportions (Table 4) were derived by means of $\gamma(x; 0.6081)$ ($\hat{c} = 18/29.6$) and $q_s(x) = q(x; -1.6420, 0.1106)$. This

Table 4. Observed proportions of deceased children by single-year ages of women reported in the 1976 Western Samoa population census, and model proportions of deceased children

Age	Women	Children ever born	Surviving children	Observed proportion of deceased children	Model proportion of deceased children
20	1,380	471	451	0.0425	0.0381
21	1,116	697	667	0.0430	0.0394
22	1,172	1,092	1,049	0.0394	0.0397
23	985	1,223	1,177	0.0376	0.0407
24	939	1,419	1,355	0.0451	0.0416
20–24	5,592	4,902	4,699	0.0414	0.0401
25	770	1,554	1,495	0.0380	0.0433
26	839	1,964	1,864	0.0509	0.0446
27	831	2,273	2,154	0.0524	0.0456
28	826	2,486	2,371	0.0463	0.0464
29	723	2,538	2,410	0.0504	0.0475
25–29	3,989	10,815	10,294	0.0482	0.0457

Special tabulation produced by the Government of Fiji Electronic Data Processing Services. See also Western Samoa (1979). Although the proportions deceased children are given with four decimal places, only the two first digits should be seen as significant because of the smallness of the population.

mortality function was motivated by the results of a sample survey (Western Samoa 1979). In consequence, the model proportions of deceased children were computed to reflect current circumstances of child mortality and fertility. It will also be seen that the model proportions of deceased children are close to the observed ones (Table 4). Because of the relatively small number of women (Table 4), we use all the single-year reports between ages 20 and 29 for estimation of ρ. When these values are inserted in (11), we get $\hat{\rho} = 1.0442$.

This means that the estimated child mortality function is

$$\hat{q}(x) = 1.0442q(x; -1.6420, 0.1106).$$

The resulting $\hat{q}(x)$ for $x = 1, \ldots, 5$ are given in Table 5.

In passing, notice that we also could have used the data given by five-year age groups. For example, using the age group 20–24, we get $\hat{\rho} = \hat{D}_2/D_2 = 0.0414/0.0401 = 1.0324$. For the age group 25–29, we get $\hat{\rho} = \hat{D}_3/D_3 = 1.0547$.

This illustrates that, in a population larger than this, the method may enable one to see if different age groups of women report substantially different "levels" of child mortality. The present example would suggest that Samoan mothers aged 20-24 reported nearly the same level of childhood mortality as those aged 25–29.

The example also illustrates that if one is able to specify the underlying fertility function from other sources than $\hat{R}_1$ and $\hat{R}_2$, this can be used in the estimation process. It is important to note that the estimation process builds on a specification of a standard mortality function that is close to the

Table 5. Estimated child mortality function for Western Samoa 1976

$\hat{q}(1)$	$\hat{q}(2)$	$\hat{q}(3)$	$\hat{q}(4)$	$\hat{q}(5)$
0.0377	0.0437	0.0476	0.0506	0.0530

observed one so that the estimated ρ is close to one. In effect, it is necessary to make only a slight adjustment to the model mortality function q_s to arrive at an estimate of the underlying mortality function.

8. Discussion

Since the traditional methods of child survivorship estimation impose fixed choices of age patterns of child mortality and fertility, and are designed to work only with the five-year statistics D_i, there is obviously a need to develop a flexible technique which allows the researcher to use his or her own choices of mortality and fertility in the estimation procedure, and to select a reasonable range of single-year reports upon which to base the estimation.

Assuming that child survival is reliably reported and that the population of women is so large that there is no visible random variation in the $\hat{H}_x$, it is possible to estimate the underlying mortality function from a few single-year age reports $\hat{H}_x$, e.g., $\hat{H}_{19}$, $\hat{H}_{20}$, and $\hat{H}_{21}$. The assumption of stationary mortality and fertility is then relaxed to the extent of assuming that fertility has remained constant for a few generations of women and that their children have had the same mortality. In passing, it will be remembered that f and q in (3) are assumed continuous and this implies that, in practice, the population of reporting women should be large (so that random variation in the proportions of deceased children is minor). When the population of women is small, so that there is clearly visible random variation in the $\hat{H}_x$, it may be necessary to base the estimation of ρ in (11) on (graduated) child survival from a larger number of female generations, e.g., women aged 19, 20, . . . , 25. Hence, when working with small populations (or with survey data) it is necessary to make the (unrealistic) assumption of constant mortality and fertility during a period of 10 years or so before the census. It is, among other things, for this reason that it is particularly difficult to work with child survivorship data from small populations. For a recent application of the new method to a small Micronesian population, see Yap State (1988).

Notwithstanding the assumption of stationary mortality and fertility, it is evident that the new approach can incorporate mortality transitions in the mortality and fertility functions to be used in (3). Hence, the approach given in this paper also facilitates the construction of model proportions of deceased children which reflect given choices of changing mortality and fertility. Because the present paper principally is intended as a methodological note which brings out the main features of a parametric approach to estimating child mortality from child survivorship data, we abstain from discussing this issue in detail.

9. References

Brass, W. (1960). The Graduation of Fertility Distributions by Polynomial Functions. Population Studies, 14, 148–162.

Brass, W. (1961). The Construction of Life Tables from Child Survivorship Ratios. In International Population Conference, New York, 1961. Liege: International Union for the Scientific Study of Population, vol. 1, 294–301.

Brass, W. (1975). Methods for Estimating Fertility and Mortality from Limited and Defective Data. Chapel Hill, NC: University of North Carolina.

Brass, W. and Coale, A.J. (1968). Methods of Analysis and Estimation. In The Demography of Tropical Africa, ed. William Brass. Princeton, NJ: Princeton University Press, 88–150.

Coale, A.J. and Demeny, P.D. (1966).

Regional Model Life Tables and Stable Populations. Princeton, NJ: Princeton University Press.

Coale, A.J. and Trussell, T.J. (1974). Model Fertility Schedules: Variations in the Age Structure of Childbearing in Human Populations. Population Index, 40, 185–258.

Feeney, G. (1976). Estimating Infant Mortality Rates from Child Survivorship Data by Age of Mother. Asian and Pacific Newsletter, 3, 12–16.

Feeney, G. (1980). Estimating Infant Mortality Trends from Child Survivorship Data. Population Studies, 34, 109–128.

Hartmann, M. (1980). Infant and Childhood Mortality. Working paper, University of Lund, Department of Statistics. CODEN: LUSADG/SAST-3074/1-65.

Hartmann, M. (1982). Some Reflections on the Use of Child Survivorship Data to Estimate Childhood Mortality. Working paper, University of Lund, Department of Statistics. CODEN: LUSADG/SAST-3092/1-97.

Hartmann, M. (1989a). Variations in the Age-Pattern of Fertility in Sweden Around 1986. Research and Development Report no. 9, Statistics Sweden, Stockholm.

Hartmann, M. (1989b). Modeling Childhood Mortality. Journal of Official Statistics, 5, 241–251.

Hill, K. and Trussell, J. (1977). Further Developments in Indirect Mortality Estimation. Population Studies, 31, 313–334.

Hoem, J.M., Madsen, D., Lövgreen Nielsen, J., Ohlsen, E-M., Hansen, H.O., and Rennermalm, B. (1981). Experiments in Modelling Recent Danish Fertility Curves. Demography, 18, 231–244.

Kraly, P.E. and Norris, D.A. (1978). An Evaluation of Brass Mortality Estimates Under Conditions of Declining Mortality. Demography, 15, 549–557.

Palloni, A. (1979). A New Technique to Estimate Infant Mortality with an Application for El Salvador and Columbia. Demography, 16, 455–474.

Palloni, A. (1980). Estimating Infant and Childhood Mortality Under Conditions of Changing Mortality. Population Studies, 34, 129–142.

Retherford, R.D. (1979). The Brass Fertility Polynomial. Asian and Pacific Census Forum, 5, 15–19.

Sullivan, J.M. (1972). Models for the Estimation of the Probability of Dying Between Birth and Exact Ages of Early Childhood. Population Studies, 26, 79–97.

Trussell, T.J. (1975). A Re-Estimation of the Multiplying Factors for the Brass Technique for Determining Childhood Survivorship Rates. Population Studies, 29, 97–107.

United Nations (1990). Step-by-Step Guide to the Estimation of Child Mortality. Population Studies, no. 107, New York: N.Y.

Western Samoa (1979). Census of Population and Housing, 1976. Analytical report, vol. II., Apia, Western Samoa: Department of Statistics.

Yap State (1988). Report on the 1987 Yap State Census of Population, vol. II., Yap, Colonia, Western Caroline Islands: Office of Planning and Budget.

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