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An Index of Relative Crop Yields

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Abstract: An index for comparing the level of crop production in Austria, France, and Hungary is described and applied. The calculation of the index is based on yields and crop areas. We regard the yields as production indicators and use crop areas as weights. The index can be used in both spatial and temporal analyses, to show the changes among countries over time.

Key words: Index; crop yields; intercountry comparisons, Austria, France, and Hungary.

1. Introduction

Interest in agricultural production can focus on its volume, level, or dynamics. When analyzing the volume of agricultural production, one faces the problems presented by aggregation. If these problems can be solved, the dynamical analysis is practically problem free. In the regular Food and Agriculture Organization (FAO) and United Nations (UN) publications, there are fairly long time series on agricultural production which in itself speaks for the feasibility of dynamic analysis.

In general, the level of production can be measured in two different ways. The first is to calculate a given product's total production over a specified period, usually a year, and then compare it to one or more factors of production (land, labour, and capital). Clearly it is meaningless to simply add the physical volumes of the different crops or commodities without setting these physical units in relation to something else. One naturally thinks of setting these physical units in relation to a monetary measure, but then there are problems involved in defining a feasible monetary measure.

Attempts to define a feasible money measure have received a great deal of attention recently. One example is the International Comparison Project (ICP) that is already into its fifth stage of development. On the other hand, I am reluctant to use prices because of the differences in price setting between market and planned economies, and between countries with an abundance of land and agricultural products and countries where these resources are scarce.

One way of circumventing the above problems is, in the case of crop production, to consider yields per hectare. I propose to calculate an index for crop yields that provides an indication of the relative level of crop production. This index has recently been applied in two studies of the Central Statistical Office of Hungary to compare crop yields in Hungary

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with those in France and Austria. No attempt to summarize a complicated concept in a single statistic can be entirely successful, and our measure is no exception. No statistic can be better than the data on which it is based, and because of the difficulties of data collection, the statistic may be somewhat inaccurate.

Nevertheless, this particular index suffers less from conceptual discrepancies than do price/value aggregates. Furthermore, people are more familiar with crop yield measures than with energy or protein content based measures.

A detailed explanation of the differences indicated by the relative crop yield index should take into consideration environmental conditions, technical inputs, farm structure, etc. To evaluate the effects of these factors requires multivariate techniques, for example, factor analysis, cluster analysis, and multivariate scaling.

2. The Construction of the Index Number

The index used here for characterizing the relative level of crop production is the *average of individual indices*. The individual indices are obtained by dividing the yields of the country in question with those of the partner country. First the individual indices are weighted with crop areas in the given country, i.e., a Paasche type formula is calculated using the arithmetical mean:

$$I_1 = \frac{\Sigma n_1 \; (x_1 / x_0)}{\Sigma n_1} \tag{1}$$

where n = the crop area,
x = the yield,
subscript 1 = the country in question (usually the one that prepares the comparison),
subscript 0 = the partner country (the one that is the basis of comparison).

The individual indices are then weighted with the crop areas of the partner country, i.e., a Laspeyres type formula is calculated using the harmonical mean:

$$I_0 = \frac{\Sigma n_0}{\Sigma n_0 : (x_1 / x_0)} \; . \tag{2}$$

Finally their quadratic mean, i.e., the Fisher ideal index is calculated:

$$I = \sqrt{I_1 \; I_0} \; . \tag{3}$$

3. Methodological Remarks

3.1. *The choice of weights*

When calculating the average form of a price or volume index, i.e., using the weighted price or volume relatives, it is common practive to use $q{\cdot}p$ as a weight. Here the individual indices are weighted with the crop area instead of the production $n{\cdot}x$ which would be equivalent to the value $q{\cdot}p$. It should be stressed that $\Sigma n_1 x_1$ and $\Sigma n_0 x_0$ are meaningless expressions if using production as a weight, for example, in the denominator of (1) and in the numerator of (2). In other words, the physical production data of different crops are not *additive*. Thus, the index of yield cannot be obtained by dividing the production index $\Sigma n_1 x_1 / \Sigma n_0 x_0$ by the area index $\Sigma n_1 / \Sigma n_0$ (the factor test).

There are other issues that should also be addressed. Had production been used as a weight, the crops of higher yield would receive, in my opinion, unreasonably large weights. In short, there is no reason to give greater weight for, say, sugar beets purely because its yield, and in turn its production which is counted in roots, may be ten times higher than that of wheat, which is counted in grains. Theoretically it does not seem logical or fair to use the figure to be weighted (x) as its own weight, at least implicitly in the form $n x$.

It must be said that area weights have a number of deficiencies. Some countries collect data on sown area, others on harvested area. The difference, however, is insignificant in normal years. National practices differ regarding the use of gross or net areas. The net area consists of only that proportion of the gross area actually cultivated. Countries providing data on gross area consider the difference small compared to other errors. International bodies, for example, the FAO, ECE and the Study Group on Food and Agricultural Statistics in Europe do their best to standardize the concepts to give conceptually uniform and thus comparable figures.

Attention must be paid if using area weights for crops other than field crops. Vegetables grown mainly or totally under protective cover may significantly distort the comparison with those grown mainly or totally in the open. The comparison of orchards is preferably limited to commercial orchards (compact plantations) and to the area of trees of productive age.

In principle, the index presented here can also be used for livestock products. Had the specific output figures of animal husbandry been available, which they are not, the weights could have been derived from the coefficients of conversion into livestock unit.

3.2 *The index formulae*

Formula (1) is a weighted arithmetical mean and was referred before as the Paasche type formula. The general practice of calculating the Paasche index, i.e., the weighted average of price or volume relatives, applies to the harmonical mean. It should be noted that changing the order of calculating the arithmetical and harmonical means in (1) and (2) has only a slight effect on the value of the final formula (3) even in the case of an extreme, reverse distribution of weights.

If one calculates the arithmetical mean in (1), both the numerator and the index itself become *interpretable.* In the numerator the production of the country in question is divided by the yield of the partner country as $n_1 (x_1/x_0) = n_1 x_1/x_0$. Since dividing production by yield provides the crop area, it is easy to see that the numerator is equal to the area necessary to cultivate that particular crop in the country in question. This is conditional on the yields of the partner country. The denominator is the crop area in the given country, so it is equally clear that the I_1 index indicates the proportion of the country's area necessary given the yields of the partner country. Those who use the statistics may prefer an index that is simple to interpret. Changing the order of calculating arithmetical and harmonical means would result in fictitious data: $n_1 x_0$ in (1) and $n_0 x_1$ in (2).

In relation to formula (1), it should be stressed that the formula indicates the relative level of crop production in the country in question by the structure of its cropped area.

To understand the use of formula (2) let us approach it from another angle. Calculation by formula (1) for the partner country would be as follows:

$$\frac{\Sigma n_0 (x_0/x_1)}{\Sigma n_0} .$$

Taking its reciprocal value in order to maintain the same relation in formula (1) and (2) we obtain:

$$\frac{\Sigma n_0}{\Sigma n_0 (x_0/x_1)} = \frac{\Sigma n_0}{\Sigma n_0 / (x_1/x_0)} .$$

Thus we have obtained formula (2) which indicates the relative level of crop production in the country in question by the structure of cropped area of the partner country. Formula (2) calculated in the form of the harmonical mean has the *practical* advantage of allowing the use of the x_1/x_0 values calculated previously.

The values of I_1 and I_0 indicate the relative level of crop production for a given country but also reflect the difference in the cropped areas when comparing countries. Consequently the values may be, and sometimes actually are, different from each other. This is rather inconvenient since when comparing two countries it is expected that indices, using the same notations as before, should, in 1/0 and 0/1 relations, be reciprocal. Formula (3) meets the *reciprocity* criterion, a desirable property for practical purposes (the factor reversal test). Reciprocity is obtained by calculating the quadratic mean, but it would not be obtained by calculating the arithmetical mean. In other words, the "reality" may be somewhere between the values of I_1 and I_0 and, if so, the Fisher "ideal" index is then a suitable compromise.

4. Application for Two Countries

Table 1 contains the original data published in the national statistical series on crop areas and yields in France and Hungary. The area figures are for the important crops in both these countries. The crop areas of 21 field crops common to France and Hungary shows the extent to which they comprise the total area of certain crops. The crops listed here and for which the indices are calculated represent 98 – 99 %, 92 – 97 % and 42 – 81 % of cereals, industrial crops, and fodder crops, respectively. The poor representation of fodder crops in France is due to the large area of temporary meadows which has no equivalent in Hungary.

Table 1. Crop Areas and Yields in France and Hungary, 1979–1981 Average

	Crop area (1 000 hectares)		Yield (tons/hectare)	
	France	Hungary	France	Hungary
	n_0	n_1	x_0	x_1
Wheat	4 471	1 187	4.92	4.04
Rye	120	72	3.04	1.60
Barley	2 671	265	4.12	3.19
Oats	523	45	3.53	2.61
Maize	1 773	1 248	5.44	5.52
Sorghum	75	7	4.50	1.80
Rice	6	16	4.01	2.14
Cereals total	9 801	2 856	–	–
Sugar beets	579	113	51.18	37.13
Tobacco, leaves	18	15	2.55	1.25
Sunflower seed	118	268	2.37	1.86
Rapeseed	367	47	2.37	1.53
Soybeans	11	22	1.79	1.78
Flax for fibre	7	7	6.33	7.42
Hemp for fibre	45	5	5.86	3.71
Industrial crops total	1 182	521	–	–
Potatoes	243	67	28.01	15.74
Lucerne	666	370	8.07	5.59
Red clover	212	54	6.69	3.65
Green maize	1 175	326	39.96	19.20
Sorghum for fodder	15	4	26.49	13.43
Cereal-legume mixture	13	37	29.06	9.54
Fodder beets	168	6	55.76	34.27
Fodder crops total	5 422	944	–	–

Table 2 shows the individual indices as well as I_1, I_0, and I. The values of I_1 tend to be consistently greater than those of I_0. This is explained by the weighting. When calculating the index by the structure of Hungary's cropped area, significantly larger weights were given to crops where Hungary's position is relatively better: to maize, sunflower and lucerne in cereals, industrial crops, and fodder crops. This results in a positive correlation between the relative areas and yields of individual crops within one country, although the correlation will probably be negative between the relative areas and prices/values. The larger the area, and in turn production, the lower the price of the crop. Apart from the differences in weights, Raj and Khamis (1958) proved the inequality $E(X)\,E(1/X) > 1$ for all positive non-constant random variables X. For the final I index, here referring to field crops in the last row of Table 2, Köves (1983, pp. 84 – 93) suggests the calculation of the two-stage Fisher formula. The calculation starts from the Fisher indices of the groups of crops which provide "improved" values, instead of the individual indices.

Table 2. Indices of the Relative Yields of Field Crops in Hungary, 1979–1981 Average (France = 100)

	$\frac{x_1}{x_0}$	I_1	I_0	I
Wheat	82			
Rye	53			
Barley	77			
Oats	74			
Maize	102			
Sorghum	40			
Rice	53			
Cereals		89	82	85
Sugar beets	73			
Tobacco, leaves	49			
Sunflower seed	79			
Rapeseed	65			
Soybeans	99			
Flax for fibre	117			
Hemp for fibre	63			
Industrial crops		76	70	73
Potatoes	56			
Lucerne	69			
Red clover	55			
Green maize	48			
Sorghum for fodder	51			
Cereal-legume mixture	33			
Fodder beets	62			
Fodder crops		58	54	56
Field crops		77	72	74

The calculation of indices was also carried out for an earlier period. Combining the results of these investigations makes possible the identification of the fields where Hungary came close to or fell behind France. Table 3 shows the changes over time of the relative level of field crop production in Hungary.

Table 3. Changes in the Relative Yields of Field Crops in Hungary (France = 100)

	$\frac{x_1}{x_0}$	I
	1979–1981 average minus 1969–1971 average (percentage points)	
Wheat	10	
Rye	– 6	
Barley	3	
Oats	16	
Maize	32	
Sorghum	– 5	
Rice	– 2	
Cereals		15
Sugar beets	5	
Tobacco, leaves	– 3	
Sunflower seed	11	
Rapeseed	– 19	
Soybeans	..	
Flax for fibre	– 9	
Hemp for fibre	– 7	
Industrial crops		– 1
Potatoes	6	
Lucerne	1	
Red clover	– 14	
Green maize	15	
Sorghum for fodder	..	
Cereal-legume mixture	..	
Fodder beets	– 5	
Fodder crops		0
Field crops		9

A possible interpretation of the figures in Tables 2 and 3 may be that:

– the smallest difference in the level of field crop production for the two countries is for cereals. Moreover, cereals is the crop where Hungary's position improved a lot, mainly because the improvement of its maize yield;

– there is a considerable, and not diminishing, difference in the production level of industrial and fodder crops in the two countries.

Comparing the difference in the level of crop production in France and Hungary and considering the change taken place during ten years, one is tempted to calculate the time necessary to catch up with France. I think such a calculation would be misleading, not because the result would be dispiriting, but because the idea implicitly postulates that all countries will be the same or will reach the same level in the future. Countries follow different growth paths and approach different plateaus of both total agricultural production and the production of individual crops. Hungary's advance in cereals and stagnation in industrial and fodder crops indicates the differences in the natural endowments of the two countries.

5. Application for More than Two Countries

The comparison may, of course, cover several countries. In this case a multipositional index of the Fisher type is to be calculated. This index, EKS, is named after its inventors, Éltetö and Köves (1964) and Szulc (1964). The EKS index is not very well known in Western Europe and the Americas (it was published in Hungarian and Polish). It is worthwhile to mention that a version of this index was used by the Statistical Office of the European Communities (SOEC).

In multilateral comparisons, the EKS index is calculated in the form of the geometric mean of Fisher indices:

$$I_{i/j} = \sqrt[r]{\prod_{k=1}^{r} (I_{i/j})^2 \, I_{i/k} \, I_{k/j} \ldots} \qquad (4)$$

where r = the number of countries in the comparison, and

subscripts i and j = the notations of the individual countries.

In essence the EKS index is a "correction" of the index of direct comparison. A larger weight is attached by using the quadratic form thus chaining the indices of all two-step indirect comparisons. The index satisfies the *transitivity* criterion, a desirable property for practical purposes (the circular test).

To make multilateral comparisons possible the extension of formula (3) to formula (4) solves a problem which has not been dealt with so far. The bilateral comparison can take into account only the crops common in the countries being compared. It provides a good or at least acceptable representation only for countries with similarly structured crop areas. The use of formula (4) allows for comparisons of countries or regions with different altitudes. In comparing, for instance, the Netherlands and Hungary, the maize yield of the latter can not be taken into account since this crop is not grown in the Netherlands, at least not as a grain. Extending the comparison, for instance, to the Netherlands, Hungary, and France, Hungary's maize yield as it appears in the Hungary/France relation, permits an adjustment to the direct Hungary/Netherlands index.

Table 4. EKS Indices of the Relative Yields of Cereals, 1979–1981 Average (Individual Indices for Wheat, Rye, Barley, Oats, and Maize are given in Brackets)

i \ j	Austria	France	Hungary
Austria	100	92 (77, 102, 84, 91, 129)	111 (94, 194, 109, 123, 128)
France	108 (130, 98, 118, 110, 77)	100	120 (122, 190, 129, 135, 99)
Hungary	90 (107, 51, 92, 82, 78)	83 (82, 53, 77, 74, 102)	100

Table 4 shows the results of the comparison of the level of cereal production in Austria, France and Hungary. The data for Austria come from Austrian official statistics. The possible interpretation of the figures in Table 4, limited to the main points, can be:

– the level of cereal production follows the order of France-Austria-Hungary;

– Austria and Hungary are lagging behind France by 8 and 17%, respectively;

– Hungary, however, being at the end of the list has an advantage in wheat and maize production.

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