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Empirical Bayes Estimation of U.S. Undercount Based on Artificial Populations

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Abstract: Estimators of undercount are difficult to assess and compare because true population counts are not available. Isaki et al. (1988) made the comparison by constructing an artificial population where "true" population counts were known. We show that the synthetic estimator they used is a special case of an empirical Bayes estimator of undercount, derived from a compound-distribution model for the undercount mechanism. The validity of this model, for the artificial population, can then be examined.

Key words: Census counts; compound-distribution model; dual-system estimation; mean squared error; measures of improvement; post-enumeration survey; synthetic estimator; empirical Bayes estimator.

1. Introduction

The ability of a nation to count itself accurately is of paramount importance in "chronicl[ing] its past, describ[ing] its present, and illuminat[ing] its future" (part of the U.S. Census Bureau's mission statement; Bureau of the Census Strategic Planning Committee, 1985). Any errors in census counts need to be identified and their effect evaluated. If the various errors were homogeneous across the nation, the effect would be minimal. Experience with past censuses in the U.S.A. has indicated a consistent undercount that is differential according to age, race, and sex, and perhaps also urban/rural and geographical factors (Tukey 1981; Ericksen and Kadane 1985). This is a problem shared by all countries who census their population (over 200 in the 1975-1984 decennium), although few do anything to assess the coverage of their counts. Of those that do, Australia, Canada, Israel, and the United Kingdom produce post-censal estimates that are used by the government in one official way or another. The U.S.A. has not in the past adjusted its census counts to account for undercoverage. However, those cities and states that stand to lose most from not adjusting are attempting, through

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various means, to require the U.S. Bureau of the Census to adjust the 1990 census counts.

Census undercount is defined simply as the difference between the true count and the census count, expressed as a percentage of the true count. Since the true count is unknown, the assessment of undercount from the census is impossible without some source of extra information. This may come in the form of birth-death records, or administrative lists such as tax records, or a survey taken after the census. To obtain this information, the U.S. Census Bureau conducts a post-enumeration survey (PES) based on matching people counted in the census and people counted in the survey. Using statistical capture-recapture methods, the true population counts can be estimated (e.g., Wolter 1986).

Various undercount estimators based on PES data have been developed, but how can the performance of these estimators be compared? One way is to build a statistical model and compute the mean squared errors of the estimators based on the model (Ericksen and Kadane 1985; Cressie 1989). Another way is to simulate undercount using a statistical model and compare the estimates to the known simulated population counts (Schirm and Preston 1987). The first method is highly model dependent, although diagnostics can be used to check the fit of the model; the second method loses the veracity of how real data interrelate between and within (not necessarily contiguous) small areas.

A third method, the method featured in this article, is to construct an *artificial* population by replacing the undercount variable with a known variable that is thought to correlate highly with it. Isaki et al. (1988) use census *substitutions*, the number of persons imputed into housing units, as a proxy for persons missed in the census. We shall do the same, concentrating on their artificial population *AP3*.

In order to reduce variation of the undercount estimators, the 1990 Post Enumeration survey (PES) attempts to stratify the nation so that the undercount is as homogeneous as possible for areas within each stratum. The stratification with which we shall work is called *Syn 2* by Isaki et al. (1988), and consists of 96 strata defined by geographic regions, racial composition, and urbanicity.

The estimators we shall evaluate here are based on a compound-distribution model explained briefly in Section 2. Section 3 describes the artificial population AP3 and the Syn 2 strata construction. An empirical Bayes analysis of these data is given in Section 4. Section 5 contains discussion and conclusions.

2. The Compound-Distribution Model

We consider the true population count in any well-defined stratum of the U.S. to be unknown. After observing the corresponding census count, the uncertainties about the true count are updated. In other words, all inference will be performed *conditional* on the observed census counts.

Suppose that there are $j = 1, \ldots, J$ strata, and $i = 1, \ldots, I$ areas; for the purposes of this article, $J = 96$ and $I = 51$ (the number of states, including Washington, D.C.). Think of stratum j as fixed. Then, as i ranges from $1, \ldots, I$, a sequence of subareas is generated; the subarea indexed by "ji" refers to that part of the ith area that has stratum j in it. For the Syn 2 stratification (Section 3) there are 451 state-strata combinations with nonzero census counts. At the state level, a zero census count is taken to mean that the stratum within the state in question does not exist. This is a reasonable assumption, but at lower levels of aggregation (e.g., the block level) one

might want to do otherwise. In all that is to follow, only areas with nonzero census counts are considered.

Define

$$Y_{ji} \equiv \text{true count in the } j\text{th stratum of area } i \tag{2.1}$$

$$C_{ji} \equiv \text{census count in the } j\text{th stratum of area } i \tag{2.2}$$

$$F_{ji} \equiv Y_{ji}/C_{ji}; \quad i = 1, \ldots, I; \quad j = 1, \ldots, J. \tag{2.3}$$

Suppose for the moment that we know the ratios $\{F_{ji}: j = 1, \ldots, J\}$ for the ith area. Then, from the census counts, the true count Y_i can be calculated:

$$Y_i = \sum_{j=1}^{J} F_{ji} C_{ji}. \tag{2.4}$$

The F_{ji} are often called *adjustment factors*. The strata are constructed so that these adjustment factors $\{F_{ji}: i = 1, \ldots, I\}$ are as homogeneous as possible within strata.

Based on the results presented in Cressie (1989), we propose the following model

$$F_{ji} \sim \mathrm{N}(F_j, \tau_j^2/C_{ji}); \quad i = 1, \ldots, I; \quad j = 1, \ldots, J \tag{2.5}$$

where "$\sim$" denotes "is distributed as," $\mathrm{N}(\mu, \sigma^2)$ is a normal distribution with mean μ and variance σ^2, and all the distributions are assumed independent. We shall refer to (2.5) as a *compound* or mixing distribution. The normality assumption is made for convenience and can be omitted if it is specified instead that linear estimators will be used (Cressie 1989). Here, F_j is a fixed but unknown mean to be estimated, and $\tau_j^2 = \mathrm{var}(C_{ji}^{1/2} F_{ji})$ is a parameter we shall call the (standardized) *stratum variance*.

In the PES, the $\{F_{ji}\}$ are observed imperfectly. Let the observation be X_{ji} (e.g., X_{ji} is the ratio of the PES capture-recapture estimator to census count, for the jth stratum in the ith area), and, conditional on F_{ji}, we propose the model

$$X_{ji} \sim \mathrm{N}(F_{ji}, \sigma_j^2/C_{ji}); \quad i = 1, \ldots, I; \quad j = 1, \ldots, J \tag{2.6}$$

where F_{ji} is the unknown mean parameter to be predicted, and all the distributions are assumed independent. Here, $\sigma_j^2 = \mathrm{var}(C_{ji}^{1/2} X_{ji})$ is the parameter we shall call the (standardized) *sampling variance*. Should the variance assumption in (2.6) (a consequence of probability-proportional-to-size sampling) not be appropriate, the theory that follows is still applicable but the algebra becomes more complicated (Cressie 1989, section 6).

Assuming squared error loss, the optimal estimator of F_{ji} is the Bayes estimator (e.g., Lindley and Smith 1972)

$$E(F_{ji}|X_{ji}) = F_j + D_j(X_{ji} - F_j) \tag{2.7}$$

where

$$D_j = \tau_j^2/(\tau_j^2 + \sigma_j^2). \tag{2.8}$$

To convert (2.7) into an empirical Bayes estimator, the unknown model parameters $\{F_j\}$ and $\{D_j\}$ have to be estimated from the data $\{X_{ji}\}$. This we do in Section 4, but first we describe the construction of the artificial population and how to emulate PES sampling on it.

3. The Artificial Population, Stratification, and Sampling

3.1. *Artificial population*

The artificial population AP3 (Isaki et al. 1988) provides (census and "true") counts for all enumeration districts (EDs) of the United States; its construction is given below. There were approximately 300,000 EDs in the 1980 U.S. Census, averaging

about 800 people per ED. Counts at the ED level can be aggregated unambiguously to county, state, and national levels. Further, AP3 provides (census and true) counts within each ED across five age, three race, and the two sex categories, where the three race groups are black, Hispanic, and non-black non-Hispanic (called white in this article). Therefore, artificial undercount can be calculated for all areas of the U.S., large or small, and for 30 age/race/sex groups within these areas.

Within an ED, the true (AP3) count Y, census count C, and number of substitutions S, of a particular age/race/sex combination, are connected (up to integerization) by

$$Y = C + dS \tag{3.1}$$

where d is a demographic factor (obtained at the national level) for that particular age/race/sex combination. The demographic factors guarantee that the artificial undercounts match the observed undercounts at the national level; the 30 values of d ranged between -1.27222 and 7.84316. Substitutions are the result of imputing people into housing units because no census form for the housing unit was completed, or because the form had incomplete information, or because of machine failure, etc. The undercount is then

$$u = (Y - C)/Y = \left\{\frac{C}{dS} + 1\right\}^{-1}.$$

But, since substitutions are included in census counts, $0 \leqslant S \leqslant C$. Hence,

$$|u| \leqslant |d|/(1 + d) \tag{3.2}$$

provided $d > -1$. Also, the AP3 adjustment factor $F \equiv Y/C$ is bounded

$$1 - |d| \leqslant F \leqslant 1 + |d|. \tag{3.3}$$

Cressie and Dajani (1988) examine further the assumptions upon which the artificial population is based. They conclude that, at the state level, AP3 is a realistic proxy for the true population.

3.2. Stratification

The 96 adjustment strata (called Syn 2) were designed by Isaki et al. (1988), following suggestions by Tukey (1981). The United States is divided into distinct, homogeneous areas, cross-classified by census division, race, and the size of place where the ED is located (urban/suburban/rural).

Cressie and Dajani (1988) discuss this stratification from the perspective of the compound-distribution model presented in Section 2. They find that a stratum with a large average adjustment factor also has large within-stratum variation. That is, F_j and τ_j^2 in (2.5) are related. Figure 1 shows a plot of $(\tau_j^2)^{1/4}$ versus stratum mean $F_{j\cdot}$, which illustrates the mean-variance relationship; the fourth-root transformation was chosen because it gives an approximately linear relation, but it is not as strong as the log transformation. The quantities $F_{j\cdot}$ and τ_j^2 are weighted means and variances of $\{F_{ji}: C_{ji} > 0; i = 1, \ldots, I\}$, weighted according to $\{C_{ji}: C_{ji} > 0; i = 1, \ldots, I\}$; see (4.1) and (4.2). Thus, it is only possible to draw Figure 1 when the true (artificial) population is known.

3.3. Sampling

In practice, the way the U.S. Census Bureau estimates true population counts for both large and small areas is through a post-enumeration survey (PES) that produces dual-system estimates of the true U.S. population. Stratified sampling in the PES was emulated in the artificial population through artificial sampling of the EDs.

Isaki et al. (1988) and Huang (1987) explain how 1440 EDs were chosen from the approximately 300,000 possible EDs, by random sampling within strata. Then the

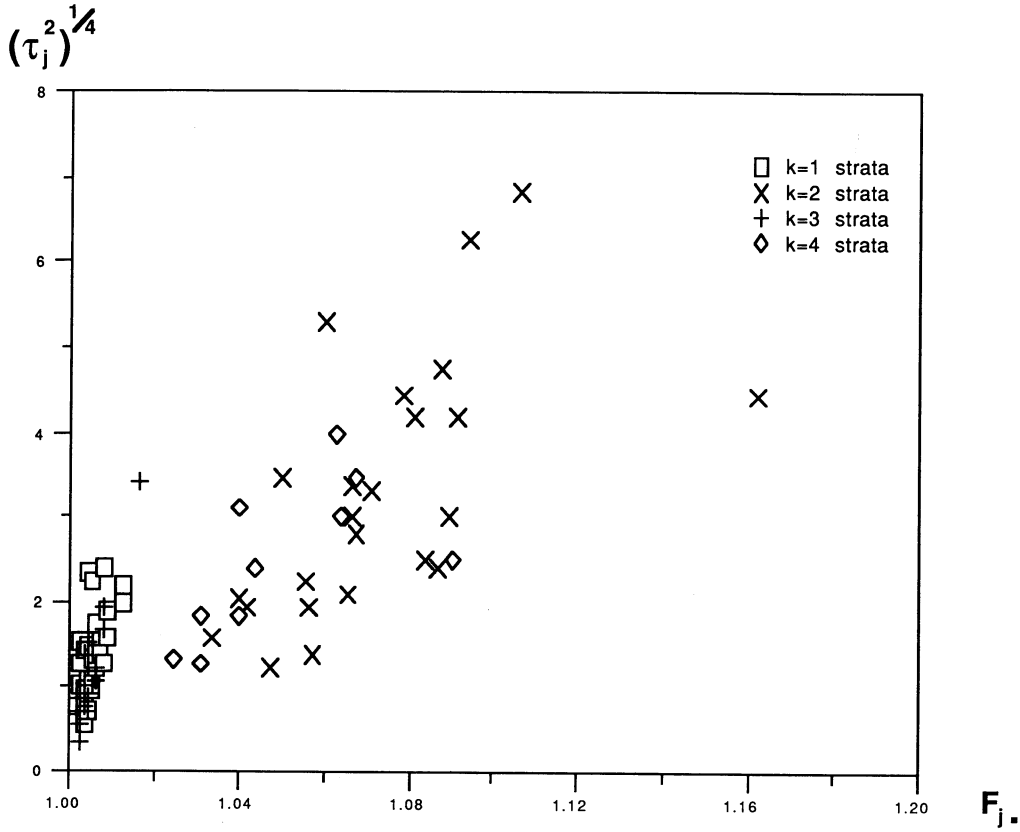

Fig. 1. Plot of (fourth-root) stratum variances versus stratum means. Membership in the size-4 grouping is given in the figure key; Central City White (k = 1), Central City Non-White (k = 2), Non-Central City White (k = 3), and Non-Central City Non-White (k = 4).

data are defined by

$$X_{ji} \equiv \left(\sum_{e \in E_{ji}} Y_e \Big/ \sum_{e \in E_{ji}} C_e \right); \quad c_{ji} \equiv \sum_{e \in E_{ji}} C_e$$

where the eth ED sampled has true count Y_e and census count C_e, and E_{ji} is a subset of the 300,000 EDs consisting of those sampled ED's in the (i, j)th state-stratum combination $(i = 1, \ldots, I; j = 1, \ldots, J)$. Notice that the only source of error in X_{ji} is, by definition, sampling error. Thus, the artificial-population approach ignores nonsampling error.

In order to obtain PES-like variances of synthetic adjustment factors, the whole sampling procedure was carried out 90 times. Some of the sampling variances for the 96 adjustment strata were very high. Two of the highest were of Hispanics in Southern central cities and for blacks in Chicago and Detroit. Two ways to lower variances would be either to change the stratification scheme (where one would perhaps separate Chicago and Detroit into two separate strata) or to alter the sampling plan to oversample EDs in known "problem" areas and undersample EDs in areas where problems in census coverage are not anticipated.

Of the 90 sampling replicates, only one was used by Isaki et al. (1988) for synthetic estimation. For comparability, we shall use the same replicate.

4. Analysis of Sampled Undercount

4.1. Pooling strata to estimate variance parameters

The Bayes estimator given by (2.7) requires estimation of $\{\tau_j^2 : j = 1, \ldots, 96\}$, which can be achieved by pooling the 96 strata. Define a partition of the stratum index set $\{1, 2, \ldots, J\}$ into $A_1, \ldots, A_K$, where $\bigcup_{k=1}^{K} A_k = \{1, 2, \ldots, J\}$ and $A_k \cap A_l = \varnothing$ $(k \neq l)$. Then assume $\tau_j^2 = t_k^2$, for all $j \in A_k$; $k = 1, \ldots, K$. In other words, we model equal τ_j^2 in each of $A_1, \ldots, A_K$.

After some experimenting, we chose $K = 4$, the stratum groups being Central City White ($k = 1$), Central City Non-White ($k = 2$), Non-Central City White ($k = 3$), and Non-Central City Non-White ($k = 4$). Because the true population counts are available, we can check whether our pooling was sensible by plotting $(\tau_j^2)^{1/4}$, versus $F_{j\cdot}$; $j = 1, \ldots, 96$, where

$$\tau_j^2 \equiv \left\{\sum_{i=1}^{I} C_{ji}(F_{ji} - F_{j\cdot})^2 I(C_{ji} > 0)\right\} \Big/ \left\{\left(\sum_{i=1}^{I} I(C_{ji} > 0)\right) - 1\right\} \tag{4.1}$$

$$F_{j\cdot} \equiv \left\{\sum_{i=1}^{I} F_{ji} C_{ji} I(C_{ji} > 0)\right\} \Big/ \left\{\sum_{i=1}^{I} C_{ji} I(C_{ji} > 0)\right\}. \tag{4.2}$$

The plot is given in Figure 1, with the k in our size-4 grouping identified. Clearly, we have captured the large differences in variation from stratum to stratum, but there still remains a positive relationship between variance and mean within the groups, particularly for $k = 2$ (Central City Non-White) and $k = 4$ (Non-Central City Non-White).

4.2. Empirical Bayes estimators of adjustment factors

Define,

$$\hat{t}_k^2 = \frac{\sum_{j \in A_k} \left[\sum_{i=1}^{I} C_{ji}(X_{ji} - X_{j\cdot})^2 I(c_{ji} > 0) - \hat{\sigma}_j^2 \left\{\left(\sum_{i=1}^{I} I(c_{ji} > 0)\right) - 1\right\}\right]}{\sum_{j \in A_k} \left\{\left(\sum_{i=1}^{I} I(c_{ji} > 0)\right) - 1\right\}}$$

where $\{\hat{\sigma}_j^2 : j = 1, \ldots, 96\}$ is obtained from the 90 sampling replicates referred to in the previous section. Then, for $j = 1, \ldots, 96$, define

$$\tilde{\tau}_j^2 \equiv \max(0, \hat{t}_k^2); j \in A_k, \quad k = 1, \ldots, K. \tag{4.1}$$

Finally, the empirical Bayes estimator, which is motivated by the Bayes estimator (2.7), is

$$F_{ji}^{\text{eba}} \equiv X_{j\cdot}^{(s)} + \{\tilde{\tau}_j^2 / (\tilde{\tau}_j^2 + \hat{\sigma}_j^2)\}^{1/2} \times (X_{ji} - X_{j\cdot}^{(s)}) \tag{4.2}$$

and the synthetic estimator is

$$F_{ji}^{\text{syn}} \equiv X_{j\cdot}^{(s)} \tag{4.3}$$

where $X_{j\cdot}^{(s)}$ is a sample-based estimator of F_j, given by

$$X_{j\cdot}^{(s)} \equiv \left\{\sum_{i=1}^{I} X_{ji} c_{ji} I(c_{ji} > 0)\right\} \Big/ \left\{\sum_{i=1}^{I} c_{ji} I(c_{ji} > 0)\right\}. \tag{4.4}$$

Notice that when $\tilde{\tau}_j^2 = 0$, $F_{ji}^{\text{eba}} = F_{ji}^{\text{syn}}$; i.e., the synthetic estimator (4.3) is a special case of (4.2). Notice also that (4.2) and (4.3) differ slightly from the empirical Bayes and synthetic estimators presented in Cressie (1988a, 1988b, 1989), through (4.4). There, sampled $\{c_{ji}\}$ were unavailable, so they were replaced with $\{C_{ji}\}$ in (4.4), to define a model-based estimator of F_j. Isaki et al. (1988) used the synthetic estimator (4.3) in this artificial-population setting.

4.3. Comparing estimators of the true population

Let $\{F_{ji}^{\text{est}}\}$ be *any* estimator of $\{F_{ji}\}$. Define the ith area's true-count estimator Y_i^{est} to be

$$Y_i^{\text{est}} \equiv \sum_{j=1}^{J} F_{ji}^{\text{est}} C_{ji} I(C_{ji} > 0). \tag{4.5}$$

Since the true counts are available, it is possible to determine the "distance" from census counts $\{C_i : i = 1, \ldots, I\}$ to true counts $\{Y_i : i = i, \ldots, I\}$, which can then be compared to the "distance" from estimated counts $\{Y_i^{\text{est}} : i = 1, \ldots, I\}$ to true counts $\{Y_i : i = 1, \ldots, I\}$. Various suggestions have been made for measures of improvement. Cressie and Dajani (1988) used 14 such measures (including the first 10 of 11 used by Isaki et al. 1988) and replicate Isaki et al.'s results in places where the two studies overlapped.

For brevity, we shall concentrate on four measures of improvement, which yield an area's contribution to the total loss that reflects the size of its population (National Academy of Sciences 1985, recommendation 7.2, p. 282). That is, in these measures an undercount of 3% in California (population, approximately 23,700,000 in 1980) receives considerably more weight than an undercount of 3% in Delaware (population, approximately 600,000 in 1980). Such measures of improvement summarize a *national* concern for undercount. Define

$$M_1^{\text{est}} \equiv \sum_{i=1}^{I} (Y_i^{\text{est}} - Y_i)^2 / C_i \tag{4.6}$$

$$M_2^{\text{est}} \equiv \sum_{i=1}^{I} |Y_i^{\text{est}} - Y_i| / C_i^{1/2} \tag{4.7}$$

$$M_3^{\text{est}} \equiv \sum_{i=1}^{I} (Y_i^{\text{est}} - Y_i)^2 / Y_i \tag{4.8}$$

$$M_4^{\text{est}} \equiv \sum_{i=1}^{I} |Y_i^{\text{est}} - Y_i| / Y_i^{1/2} \tag{4.9}$$

where Y_i^{est} is given by (4.5), Y_i is given by (2.4), and $C_i \equiv \Sigma_{j=1}^{J} C_{ji}$; $i = 1, \ldots, I$. Here the areas under consideration are the states of the U.S.A. (including Washington, D.C.), so that $I = 51$. Weighting the differences by $Y_i^{-1/2}$ was considered by Isaki et al. (1988); Cressie (1989) suggested a modification that substitutes $C_i^{-1/2}$ for $Y_i^{-1/2}$. The empirical Bayes (eba) and synthetic (syn) estimators are compared to census counts (cen) via M_1, M_2, M_3, and M_4, in Table 1.

From the table, eba has a slight advantage over syn and both are considerably better than cen, the unadjusted census counts. We then asked: Which of the states were contributing substantially to $M_1, \ldots, M_4$ presented in Table 1? For every measure, California did poorly; for the eba (syn) estimator, the percentage contribution of California was 34.4% (36.6%) to M_1, 13.7% (13.9%) to M_2, 34.5% (36.8%) to M_3, and 13.8% (13.9%) to M_4. On average, each state should contribute about 2% to these M_i, which indicates that California's undercount has been estimated badly. Because only one sample was available, it cannot be ascertained whether California is a hard state to count or whether the sample taken was unrepresentative. (For the sample taken it should be noted that every stratum within California *was* sampled).

Table 1. Measures of improvement for eba (eq. (4.2)), syn (eq. (4.3)) and cen (no adjustment). Small values are preferred

	Estimators		
	eba	syn	cen
M_1	19,594	19,607	84,744
M_2	597	611	1,447
M_3	19,173	19,172	82,339
M_4	591	605	1,430

5. Discussion and Conclusions

5.1. Discussion

One of the advantages of using an artificial population is that various conjectures about the undercount mechanism can be tested. Since this artificial population was constructed at the enumeration-district (ED) level, questions of aggregation and disaggregation of estimators can be addressed.

Under the model (2.5) and (2.6), Cressie (1988b) proved that if the synthetic estimator has smaller risk than the census at any one-level, then it will always have smaller risk at a lower level, and that the risk-gap widens; here risk is calculated with respect to M_1. By analogy, a similar result should hold for M_3. For the artificial population, the risk-gap (i.e., M_3^{cen} minus M_3^{syn}) is 59, 743 at the national level, it is 63,167 at the state level, and it is 73,092 at the county level. However, as a percentage of M_3^{cen}, the risk-gap drops from 99% (national level) to 77% (state level) to 54% (county level). In other words, as disaggregation proceeds, although the synthetic estimator and the census counts are getting further apart in the absolute sense, it appears as if they are getting closer in a proportionate sense. This observation partly resolves conflicting claims about the performance of adjusted counts *vis à vis* census counts, as a function of the level of aggregation.

A consequence of making the model assumption (2.5) at a particular level (e.g., counties) is that it will hold at all aggregated levels (Cressie 1988a). But what is the level at which it is a reasonable assumption? To check (2.5) at the ED level, we chose stratum 1 and made a histogram of the values

$$\{C_{ji}^{1/2}(F_{ji} - F_{j\cdot}) : C_{ji} > 0; \quad i = 1, \ldots, I\} \tag{5.1}$$

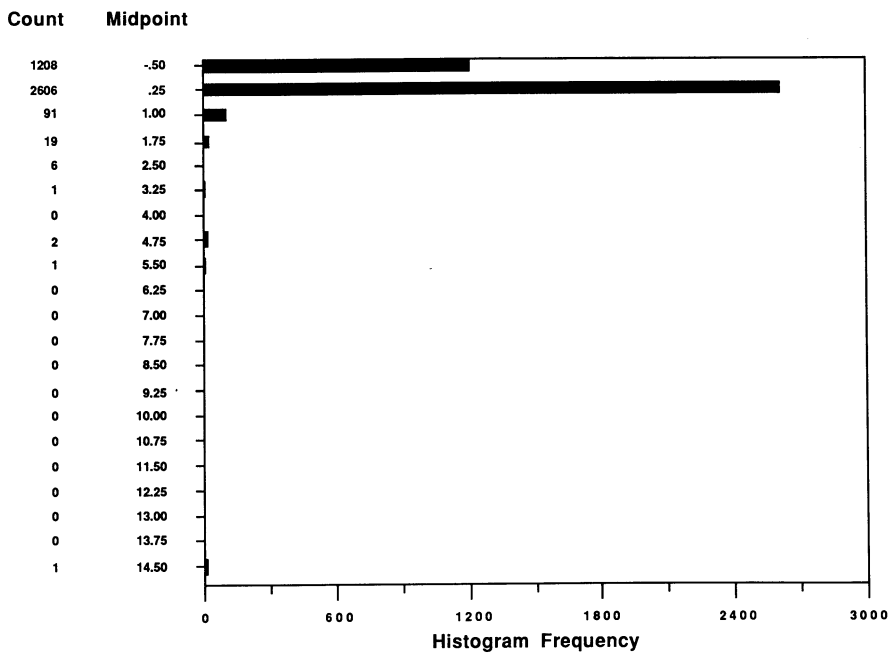

Fig. 2a. Histogram of 3,935 normalized adjustment factors (given by (5.1)) at the ED level, for stratum 1 (White population in New England Central cities of over 50,000 people); there are 96 possible strata.

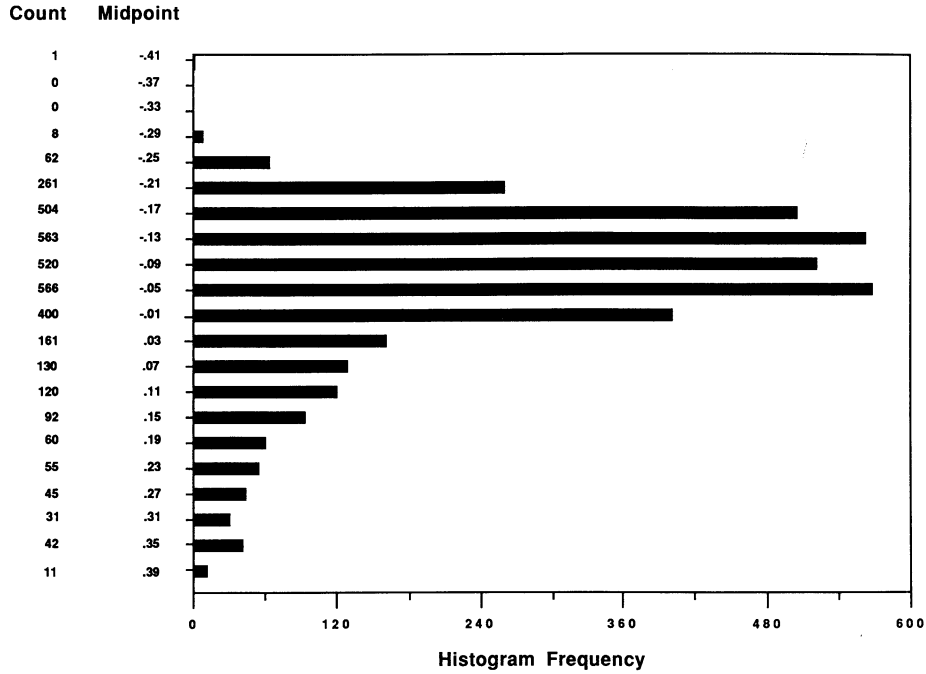

Fig. 2b. Central portion of the histogram presented in Figure 4a, showing 3,631 of the 3,935 normalized adjustment factors shown in Figure 2b.

Count	Stem	Leaf
2	-4	3 1
	-3	
5	-2	9 4 1 1 1
13	-1	9 8 8 8 8 5 4 3 3 3 2 1 0
23	-0	9 9 9 8 8 8 7 7 7 6 6 5 4 3 3 3 2 1 1 0 0 0 0
16	0	1 2 2 2 3 4 5 5 5 6 6 6 7 7 7 8
8	1	0 0 1 1 2 3 3 4
4	2	0 1 3 3
1	3	2
1	4	6
	5	
1	6	8

Fig. 3. Histogram (stem-leaf plot) of 73 normalized adjustment factors (given by (5.7)) at the state-strata level, for the $k = 1$ group (Central City White) of strata; there are 4 possible groups.

where $j = 1$, and $I \simeq 300{,}000$. Figure 2a shows the histogram of all the values and Figure 2b shows the central portion of it. Clearly, some of the F_{ji} are badly biased and the distribution is skewed towards high values. Some of this could be cured by modeling *transformed* adjustment factors (Cressie 1986); indeed, Figure 1 suggests a relationship between mean and variance that might be accounted for by transformation.

To see whether these features disappear at higher levels of aggregation, we made a histogram, at the state-stratum level, of the values

$$\{C_{ji}^{1/2}(F_{ji} - F_{j.}) : C_{ji} > 0; \quad j \in A_k;\ i = 1, \ldots, I\} \tag{5.2}$$

where $k = 1$, and $I = 51$. Figure 3 shows that the outliers have been reduced and the central portion of the distribution is Gaussian in shape, as we had hoped.

5.2. *Conclusions*

In view of the consistency of results over a wide variety of measures of improvement (Cressie and Dajani 1988), some recommendations can be made. Estimation of census coverage can be made at the state level using statistical procedures that yield counts superior to the census counts. Although empirical Bayes estimators hold a slight edge over synthetic estimators, the latter are much easier to explain and implement (Schirm and Preston 1987). Apart from comparing the two estimators, this article has set about to gain a deeper understanding of the undercount mechanism. Figures 1, 2, and 3 illustrate the limitations of recent model-based approaches to estimating undercount.

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