Estimating Marginal Cohort Working Life Expectancies from Sequential Cross-sectional Survey Data

Markku M. Nurminen¹, Christopher R. Heathcote², and Brett A. Davis³

This article applies recently developed health expectancy methodologies to estimate the average duration of future work life in different states of work ability. Data on working capacity obtained from sequential cross-sectional samples of the cohort population were available from Finnish surveys conducted among active municipal employees. We used these data to estimate cohort marginal probabilities and expected occupancy times in the work ability states. One finding is that the proportion of workers with excellent or good work ability decreased monotonically with advancing age for both genders, but men were prone to have worse work ability and a shorter work career than women. Transition from poor to good or excellent work ability state was estimated to increase working life expectancy of a 45-year-old person by four years for both genders. This study indicates that the work ability of aging Finnish workers deteriorates prematurely, leading to serious socio-economic consequences. Thus it is important to examine the development of work ability already at an early age when it is still possible to intervene in the process.

Key words: Biostatistics; epidemiological methods; health expectancy; work ability.

1. Introduction

Working life expectancies are indicators of the expected duration of working life. This article is concerned with estimating cohort working life expectancies of the three states (1) excellent or good work capacity, (2) fair work capacity, (3) poor work capacity, for aging Finnish municipal workers, using large sample logistic regression methods that seem not to have been used previously in studies of working life expectancy.

The estimation of working life expectancies generally requires data on transitions between states, such data being available from cohort studies. However, longitudinal studies are costly and typically suffer from loss of information due to recurring changes and incomplete follow-up of the population under study. The use of multiple cross-sectional samples of the population is an easier and cheaper alternative approach to measure trends in working expectancies. Sullivan (1971) first presented a popular method for the calculation of conceptually similar health expectancies; this method is predominantly based on period life tables. Using Sullivan’s method and indicators of

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disability and poor health based on three variates (limiting long-standing illness, functional disability or poor self-rated health) from the nationwide survey of living conditions, Valkonen et al. (1997) calculated the disability-free life expectancy and the life expectancy with disability in Finland. This provides estimates of life expectancies adjusted to the prevalence and severity of the health state at issue.

Davis et al. (2001) and Davis, Heathcote, and O’Neill (2002) recently proposed an alternative method to estimate cohort occupation times of disability-free and disabled states. This approach uses cross-sectional information from sequential independent surveys in an attempt to reconstruct relevant parts of the longitudinal stochastic process, using logistic parametrization of the marginal probabilities of the various health states. These are marginal expectancies in the sense that it is known only that initially the individuals are actively employed in the workforce, and their commencing state of health or work ability is not known. This technique was applied to the data from the Australian disability surveys previously discussed by Mathers (1996). Deaton (1985) developed a different approach which is based on measurement error models for a series of independent cross-sectional samples. Brunsdon and Smith (1998) developed time series models to estimate probabilities from a sequence of surveys. Although there is some similarity with our approach, the methods are essentially different.

In this article, we apply the methodology of Davis et al. (2001) and Davis, Heathcote, and O’Neill (2002) to estimate expectancies of the three states of work ability mentioned above for an active population of Finnish municipal workers. Cross-sectional survey data are used in the application. An estimating equations approach for a multinomial counting process was applied in the estimation of working life expectancies. The method differs from the frequency-based arguments usually used to estimate these expectancies, as for example that of Richards and Abele (1999). This estimation supplements the comprehensive research project on the health and work ability of aging workers (Tuomi 1997). Although we describe the Finnish experience in terms of an applied measure of working life expectancy, the discussion of some general methodological issues may offer a wider perspective on measuring population health or work capacity.

The article is organized as follows. Work capacity is described in the next section, Section 3 defines working life expectancies in probabilistic terms, Section 4 presents numerical results, and Section 5 comprises a discussion of alternatives to these expectancies, such as measures of disability-adjusted life years. The Appendix gives an outline of the weighted least squares estimation of probabilities and prevalences.

2. Finnish Work Ability Surveys

The data analyzed here originate from the sequential surveys on aging and work ability carried out by the Finnish Institute of Occupational Health (FIOH) in 1981, 1985, and 1992. We refer to Tuomi (1997) for a description and discussion of the cross-sectional surveys. For the purposes of this article, it suffices to note the following points.

The purpose of the survey was to collect information related to retirement age policies from different types of municipal occupations and to compare these occupational groups with respect to health status and work ability. At the start of the survey in 1980, the sampling frame comprised 335,232 employed workers within the municipal pension
system, which included about 2,500 professional or job titles. The member communities of the Municipal Pension Institute, which is to say the 879 municipal employers, were at that time usually city municipalities, rural communities (towns) or unions of towns. A total of 112 professional titles were selected fulfilling the following criteria for variation: the title comprises a sufficiently large number of employees; the rate of retirement from the profession on a disability pension is either high or low; professions that involve either heavy or light physical or mental stress; professions that are predominantly pursued by women or men or both. The sample comprised persons born during the period 1923–1936 who had worked in their present occupation for at least five years. The subjects were selected in co-operation with the municipal employers, using information on municipal workers provided by Statistics Finland. The study sample was selected so that it represented municipal workplaces of various sizes (mainly larger ones) from all the 11 provinces of Finland. The minimum size of the occupational groups was set at 200, which was fulfilled by 40 groups (obtained by combining occupational titles). As for the other occupational groups, the attainment of the target size was deemed too tedious or impossible, because there were not a sufficient number of workers in the relevant age groups. A sample was taken that was representative with regard to occupational grouping, and the regional and administrative representations were regarded as reasonably satisfactory. Thus the cohort was purposefully assembled from a national subpopulation. Documentation on details of the sampling scheme is no longer available. However, it is reasonable to assume that the sample design was such that, if not yielding unbiased estimates of population subtotals, these estimates are approximately unbiased in the sense of satisfying the theoretical property of consistency, i.e., asymptotic unbiasedness as the sample size increases indefinitely. More formally put, we assume:

(A1) The sample yields consistent estimates of the year- and age-specific subtotals in the population health-state groups of interest, (State 1) excellent or good work capacity, (State 2) fair work capacity, (State 3) poor work capacity.

(A2) Estimates of these population subtotals are asymptotically normally distributed.

These are assumptions that are reasonable and will often hold irrespective of the details of the sample design. The first one is the most relevant for the application of our large sample-weighted least squares methodology. If the covariance matrix of the asymptotic normal distribution is unknown or cannot be conveniently estimated, or even if approximate normality cannot be assumed, then the covariance matrix used in our least squares method is that which would be obtained from simple random sampling. This is shown in the Appendix and leads to consistent but not necessarily efficient estimators. Standard errors are calculated using the technique of Liang and Zeger (1986). The Appendix presents a brief outline of the methods of Davis et al. (2001) and Davis, Heathcote, and O’Neill (2002).

The health status of the workers was investigated by means of a questionnaire mailed to 7,534 persons, using information obtained from the municipal employers. Of those who responded, 190 did not fulfil the inclusion admissibility criteria. These persons were excluded from the sample, whose size thus became 7,344. The nonresponse rate after two mailings was 14.3%. After omitting a few retired and deceased persons, the purposefully assembled cohort finally comprised 6,257 active municipal employees, aged 45 to 58 years.
in 1981 (Table 1). Since the follow-up commenced in 1981, data from the three surveys could be used in the estimation procedure. At the time of the second survey, the subjects were divided into groups of active workers, persons who did not reply to the questionnaire but who were not on pension (e.g., unemployed persons), disability pensioners (including persons with a permanently lowered work capacity who are on individual early disability pension), retired workers (persons on old-age, early old-age, unemployment or veterans’ pension), and deceased persons. In the third survey, when the subjects were 56–69 years old, altogether 18% of them were still actively working, 5% did not respond to the questionnaire (but were not on pension), 30% were on disability pension, 41% were retired, and 6% had died. The actual retirement age is on average 59 years in Finland. At present the general statutory retirement age for municipal workers varies within the age-range 63–65 years. The identification of work ability and its level relied on self-assessment by the workers who responded to the questionnaire.

The so-called work ability index was constructed from the following seven items (Tuomi et al. 1998): subjective estimation of present work ability compared with the lifetime best, perceived work ability in relation to both physical and mental demands of the work, number of diagnosed diseases, subjective estimation of work impairment due to disease, sickness absence during the past year, own prognosis of work ability after two years, and psychological resources. The index score (re-scaled to run from 0 to 1) was classified into four mutually exclusive and exhaustive categories: 1 = excellent (score ≥ 0.85), 2 = good (score 0.7–<0.85), 3 = fair (score 0.5–<0.7) and 4 = poor work ability (score < 0.5). Of course, the cut-off scores are arbitrary. These categories comprised 21%, 33%, 32%, and 14%, respectively, of those who responded to the inquiry in 1981 (Table 1). Because epidemiological questionnaire studies (Maddox and Douglass, 1973) and validation studies (Eskelinen et al. 1991) have shown a tendency to self-appraise health at a

<table>
<thead>
<tr>
<th>Employment or work ability status</th>
<th>1981 Age: 45–58</th>
<th>1985 Age: 49–62</th>
<th>1992 Age: 56–69</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Men</td>
<td>Women</td>
<td>Men</td>
</tr>
<tr>
<td>Active employee</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excellent</td>
<td>640</td>
<td>464</td>
<td>268</td>
</tr>
<tr>
<td>Good</td>
<td>976</td>
<td>690</td>
<td>701</td>
</tr>
<tr>
<td>Fair</td>
<td>972</td>
<td>778</td>
<td>1,048</td>
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<tr>
<td>Poor</td>
<td>369</td>
<td>362</td>
<td>358</td>
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<tr>
<td>No information on work ability</td>
<td>503</td>
<td>503</td>
<td>310</td>
</tr>
<tr>
<td>No response; no information on pension</td>
<td>—</td>
<td>—</td>
<td>215</td>
</tr>
<tr>
<td>Disability pensioner</td>
<td>—</td>
<td>—</td>
<td>246</td>
</tr>
<tr>
<td>Retired</td>
<td>—</td>
<td>—</td>
<td>289</td>
</tr>
<tr>
<td>Deceased</td>
<td>—</td>
<td>—</td>
<td>25</td>
</tr>
<tr>
<td>Total</td>
<td>3,460</td>
<td>2,797</td>
<td>3,460</td>
</tr>
</tbody>
</table>

Table 1. The original Finnish cohort of 6,257 municipal employees by sex, employment, or work ability status in the survey years 1981, 1985, and 1992.
higher level than found in a clinical examination, the categories of excellent and good work ability were combined in the analyses. The data from the inquiry were complemented with information from disability and mortality registers. The analysis of transitions between the different employment and vital statuses will be reported elsewhere.

3. Working Life Expectancies

Consider a health surveillance program for active employees conducted over the period of active working years until retirement (due to disability or old age) or until death, whichever comes first. Let \( p(x, y) \) denote the conditional probability that an individual is working at age \( y \), given that he or she was working at an earlier age \( x \). For fixed \( x \), the average duration of the future work life or working life expectancy is the area under \( p(x, y) \) as \( y \) varies between \( x \) and the maximum age \( w \) of mandatory retirement, here taken as 63. A trapezoidal approximation to this area (see Chiang 1968, Section 10.5) with \( p(x, x) = 1 \) and \( p(x, w+1) = 0 \) is

\[
e(x) = 1/2 \sum_{y=x}^{w} \{ p(x, y) + p(x, y+1) \}
= 1/2 + \sum_{y=x+1}^{w} p(x, y) \tag{3.1}
\]

which is taken to define working life expectancy. Thus \( e(x) \) is the expected time that a person will be occupied in active employment between age \( x \) and withdrawal from working life due to retirement or death.

Suppose that a worker’s ability to work is partitioned into \( k-1 \) disjoint categories or states determined by the work ability index. These are nonabsorbing states in the sense that a worker may move between them with the passage of time. State \( k \) will denote an absorbing state, indicating departure from the work force by retirement or death. If \( p_i(x, y) \) denotes the probability that an individual is in State \( i \) at age \( y \) conditional on having worked at \( x \), then the future work life in \( i \) at age \( x \) or working life expectancy of State \( i \) at \( x \) is the area under \( p_i(x, y) \) as \( y \) varies from \( x \) to \( w \). Just as (3.1) was obtained by a discrete time approximation to an area, this working life expectancy is defined by

\[
e_i(x) = \frac{\pi_i(x)}{2} + \sum_{y=x+1}^{w} p_i(x, y), i = 1, 2, \cdots, k - 1 \tag{3.2}
\]

where \( \pi_i(x) \) is the prevalence at initial age \( x \) of work ability State \( i \). We note a minor error in the formula used by Davis et al. (2001) in which the factor 1/2 was omitted.

The epidemiological concept of prevalence is the proportion of individuals in the population at issue at a given point in time, and thus relates to a cross-sectional sample. From a statistical point of view, it is natural to interpret prevalence as probability and to relate observable quantities to this parameter via a probabilistic model (see Haberman 1978). Note that

\[
\sum_{i=1}^{k-1} p_i(x, y) = p(x, y), \sum_{i=1}^{k-1} \pi_i(x) = 1, \sum_{i=1}^{k-1} e_i(x) = e(x)
\]
The problem is to estimate the $e_i(x)$, or equivalently the $p_i(x, y)$, from cross-sections of a cohort at a sequence of time points. Our case has $k = 4$ with State 1 denoting excellent or good work ability, State 2 fair work ability, State 3 poor work ability, and State 4 no longer working due to disability, retirement, or death.

The method of Davis et al. (2001) models and estimates a logistic transform of the probabilities

$$p_1(x, y) = \left[ 1 + \sum_{i=2}^{4} \exp\{ \theta_i(x, y) \} \right]^{-1}$$

$$p_i(x, y) = p_1(x, y) \exp\{ \theta_i(x, y) \}, i = 2, 3, 4$$

where the $\theta_i(x, y)$ are polynomial functions of $x$ and $y$. We model and estimate the log odds

$$\theta_i(x, y) = \log\{ p_i(x, y)/p_1(x, y) \}, i = 2, 3, 4$$

Note that their form need not be fixed a priori, but can be determined from the data and can incorporate covariates of a temporal and socio-economic nature. In a similar way the prevalences are estimated from

$$\theta_i(x) = \log\{ \pi_i(x)/\pi_1(x) \}, i = 2, 3$$

It is assumed that a worker of the group retires no later than his or her 63rd birthday, and that the earliest commencing date is the middle of the age interval (45, 46). Thus the maximum duration of work life for the cohort members is 17.5 years. We estimated $e_i(x)$, $i = 1, 2, 3$ by the methods described in the Appendix and estimated $e_4(x)$ by

$$\hat{e}_4(x) = (63 - x) - 1/2 - \sum_{i=1}^{3} \hat{e}_i(x)$$

The data used comes from seven birth cohorts aged 45 to 51 in 1981, who were subsequently aged 49 to 55 in 1985 and 56 to 62 in 1992. Age groups 44 and 45 years were combined because there were only five persons in the former. The older birth cohorts aged 52 to 58 years in 1981 were not included in the analyses because of incomplete 11-year follow-up. After inspection of the plots of the observed values, the log partial odds, $\theta_i(x, y)$, were parameterized as linear or quadratic polynomials in $y$. The log partial odds $\theta_2(x)$ and $\theta_3(x)$, which correspond to the prevalences $\pi_1(x)$, $\pi_2(x)$ and $\pi_3(x)$, were estimated using data from all three surveys. That is, there were three data vectors for each of the seven-birth cohorts aged 45–51 in 1981. However, since all sampled individuals were actively working at the time of the first survey, then observation of the vector $\{ \tilde{\theta}_2(x, y), \tilde{\theta}_3(x, y), \tilde{\theta}_4(x, y) \}$ was possible only in the second and third surveys. Hence, under the assumption that the same parameters apply to all seven birth cohorts there were 21 data vectors from which to estimate the prevalences $\{ \pi_1(x), \pi_2(x), \pi_3(x) \}$.

For the probabilities $\{ p_1(x, y), p_2(x, y), p_3(x, y), p_4(x, y) \}$ there were 14 vectors on which to base inference. No predictions were made beyond the observed age-range. Nonresponse was ignored in the estimation, as it is believed to make little difference. The parameter estimates (and their standard errors), computed in S-Plus, are given in Table 2.
Table 1 shows changes in the marginal work ability distribution of the female and male cohort members at entry, and during the successive 4-year and 7-year follow-up periods, respectively. In the period 1981–1985, the marginal distributions shifted markedly towards worse capacity for work. In 1981, for example, 21% of the respondents were in excellent health, whereas in 1985 the corresponding figure was only 12%. During the latter period, 1985–1992, the shift in the distribution was slightly more lopsided towards poor work ability because of the longer follow-up. In 1985, for example, 15% of the women were in the poor work ability class, whereas in 1992 the corresponding figure had risen to 21%. The trends were similar for both genders. Weighted least squares estimation led to the fitted values of log partial odds for prevalences and probabilities shown in Figures 1 and 2.

Table 3 shows the estimated marginal probabilities of being at a given age either in one of the three states of self-rated work ability (with the states of excellent and good work ability combined) or outside the work force (disability pensioners, retired, or deceased). Of course, the probabilities add up to 100%. The standard errors of the probability estimates are shown in parentheses. The proportion of workers with excellent or good work ability decreased monotonically with advancing age for both genders (Figures 3a and 3b). However, a notable gender gap emerged: at most ages, women were estimated to have better work ability than men, but the difference was not always statistically significant. The standard errors in Table 3 enabled us to test the statistical significance of the difference in estimated probabilities between women and men for a particular age and state. For example, at age 45 the estimated marginal probability for State 1 (excellent or good health) was 56.8% for women and 59.4% for men. Since the standard errors of these estimates were 3.1% and 2.3%, we then calculated the standard error of the difference as $[(3.1\%)^2 + (2.3\%)^2]^{0.5} = 3.9\%$. Hence, the 95% confidence interval for this difference was $(-10.4\%, 5.2\%)$. We therefore inferred that the gender difference for State 1 at age 45 was not statistically significant. In contrast, at age 55 the corresponding 95% confidence interval was $(0.2\%, 7.4\%)$, indicating a significant difference. Applying the same
argument to other comparisons we found that for State 1 there was a significant gender difference only for ages 55–59 years; each of these significant differences favored women.

The estimated proportion of female workers with fair work ability first increased until age 48 years, after which it decreased. Similarly, for male workers the proportion increased to age 50 years and then decreased. Also, the proportion for women was higher than that for men at all ages from 45 to 62 years, but the only statistically significant difference (at the 5% level) occurred at age 58 years.

At ages 49 to 57 years, women were more prone to have poorer work ability than men, and less prone in the other age groups. For this state the gender difference was significant at the following ages: 45, 46, 51, 52, 53, 54, 60, 61, and 62 years. The gender gap among the proportion of workers remaining actively employed was in favor of women for ages 45–60 years, after which it favored men. The gender difference was significant at the following ages: 45, 51, 52, 53, 54, 55, 56, 57, and 58 years.
Finally, Table 4 gives the expected duration of remaining work life in the various states. Under the discrete approximation, at the age of 45 years a municipal worker has up to 17.5 years of working time left. Note that the expectations add up to the duration of maximum remaining work life at age \( x \) (i.e., \( 62 - x + 1/2 \) years). Of this time, on average, a female worker was actively employed for 12.2 years and a male worker for 11.5 years. While employed, women were expected to have excellent or good work ability for 5.6 years, fair work ability for 5.0 years and poor work ability for 1.6 years (Figure 4a). The average duration for men in the respective states of work ability was estimated to be 5.5, 4.5, and 1.5 years (Figure 4b). Women were expected to be working longer than men at the ages of 45–58 years, after which the difference was in favor of men. Statistically significant differences occurred for ages 45–52 years. Transition from poor to good or excellent work ability was estimated to increase working life expectancy of a 45-year person by four years for both genders.

Fig. 2. Log partial odds for probabilities: (a), (c), (e) \( \theta_i(45,y) \), \( i = 2,3,4 \) for men; (b), (d), (f) \( \theta_i(45,y) \), \( i = 2,3,4 \) for women. Fitted value = solid line, observed value from 1985 and 1992 surveys = *. The vertical axes of graphs (a) and (b) have the same scale. Similarly, the pairs (c), (d) and (e), (f) have the same scale on the vertical axis.
Table 3. Estimated probabilities (%), at age $x$, of being in a state of excellent/good, $p_1(x)$, fair, $p_2(x)$, or poor, $p_3(x)$, work ability, or not actively employed, $p_4(x)$. Standard errors are shown in parentheses.

<table>
<thead>
<tr>
<th>Age, years $x$</th>
<th>Men</th>
<th></th>
<th></th>
<th></th>
<th>Women</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_1(x)$</td>
<td>$p_2(x)$</td>
<td>$p_3(x)$</td>
<td>$p_4(x)$</td>
<td>$p_1(x)$</td>
<td>$p_2(x)$</td>
<td>$p_3(x)$</td>
<td>$p_4(x)$</td>
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<tr>
<td>45</td>
<td>59.4 (2.3)</td>
<td>34.1 (2.2)</td>
<td>5.8 (0.5)</td>
<td>0.7 (0.2)</td>
<td>56.8 (3.1)</td>
<td>39.5 (2.8)</td>
<td>3.5 (0.9)</td>
<td>0.2 (0.09)</td>
</tr>
<tr>
<td>46</td>
<td>57.4 (2.0)</td>
<td>35.0 (2.1)</td>
<td>6.6 (0.5)</td>
<td>1.0 (0.3)</td>
<td>55.1 (2.8)</td>
<td>39.9 (2.5)</td>
<td>4.6 (0.8)</td>
<td>0.4 (0.1)</td>
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<tr>
<td>47</td>
<td>55.2 (1.7)</td>
<td>35.8 (1.9)</td>
<td>7.4 (0.4)</td>
<td>1.6 (0.5)</td>
<td>53.2 (2.4)</td>
<td>40.1 (2.2)</td>
<td>6.0 (0.7)</td>
<td>0.7 (0.2)</td>
</tr>
<tr>
<td>48</td>
<td>52.9 (1.5)</td>
<td>36.5 (1.7)</td>
<td>8.2 (0.4)</td>
<td>2.4 (0.6)</td>
<td>51.2 (2.1)</td>
<td>40.2 (1.9)</td>
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<td>1.1 (0.3)</td>
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<td>49</td>
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<td>19.6 (1.6)</td>
<td>10.0 (0.7)</td>
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<td>20.7 (1.5)</td>
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<td>10.7 (0.9)</td>
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<tr>
<td>58</td>
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<td>15.3 (1.3)</td>
<td>8.6 (0.6)</td>
<td>63.9 (2.2)</td>
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Fig. 3a, b. Probability distribution of work ability states for (a) male/(b) female workers at different ages.
Table 4. Expected number of years of remaining work life at age x, spent with excellent/good, $e_1(x)$, fair, $e_2(x)$, or poor, $e_3(x)$, work capacity, or not actively employed, $e_4(x)$. Standard errors are shown in parentheses.

<table>
<thead>
<tr>
<th>Age, years</th>
<th>Men</th>
<th></th>
<th></th>
<th></th>
<th>Women</th>
<th></th>
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<tr>
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<td>$e_1(x)$</td>
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<td>$e_4(x)$</td>
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<td>5.99 (0.26)</td>
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<td>52</td>
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<tr>
<td>57</td>
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<td>0.67 (0.04)</td>
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<tr>
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<td>2.56 (0.02)</td>
<td>0.33 (0.02)</td>
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<td>0.14 (0.01)</td>
<td>0.00 (0.00)</td>
<td>0.14 (0.01)</td>
<td>0.26 (0.02)</td>
<td>0.10 (0.02)</td>
<td>0.00 (0.00)</td>
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</table>
The standard errors given in Table 4 can be applied to test the statistical significance of difference in working life expectancy for a given state at a specific age. For example, in State 1 at age 45 years the expectancy of women was 0.19 years more than that of men. From the standard errors on these estimates, 0.12 years for women and 0.10 years for men,
we calculated the standard error of the difference as \(((0.12)^2 + (0.10)^2)^{0.5} = 0.16\) years. Hence, the 95% confidence interval for this difference was \((-0.13, 0.51)\) years. The 95% confidence intervals (in years) for the other states at this initial age were: \((-0.06, 1.06)\) for State 2, \((-0.16, 0.20)\) for State 3 and \((-1.37, -0.05)\) for State 4. We inferred that the only statistically significant difference between the genders at age 45 years occurred in State 4. In contrast, at age 51 years the 95% confidence intervals (in years) for the gender differences were: \((0.02, 0.54)\) for State 1, \((-0.05, 0.59)\) for State 2, \((-0.14, 0.22)\) for State 3 and \((-1.15, -0.03)\) for State 4. Thus, there was a significant difference in State 1 as well as in State 4.

5. Discussion

5.1. Remarks on results

The aim of this article was to propose an applicable regression methodology for estimating working life expectancies, illustrated by the particular experience of aging Finnish municipal workers. Working life expectancy gives the number of years one expects to continue in work life. This concept takes into account the fact that a person may choose to leave the work force, or may be unable to continue to work during his or her lifetime. The contingencies to be considered include disability, retirement and death. For example, Gamboa et al. (1989) pointed out the importance of considering disability when addressing working life expectancy.

The World Health Organization (WHO) defines disability as “any restriction or inability (resulting from an impairment) to perform an activity in the manner or within the range considered normal for a human being” (WHO 1980). Although the prevalence of disability differs across populations, two findings are generally consistent. The first is, naturally, that disability increases with age, and the second is that a larger percentage of women are disabled than men (see Newman and Brach 2001, and references therein). The latter finding has to do with the fact that women live longer than men, so also the period of disabled life expectancy is longer for women than for men, as discussed for example by Guralnik et al. (1993). However, largely because the status after age 63 was not considered in the Finnish municipal worker population, the percentage of men who were on work disability pension was larger than that of women (Table 1).

These statistics may be explained by the fact that men run a greater risk of suffering from work-related injuries and occupational diseases than women do (Karjalainen et al. 2001). Men are more often engaged in physically heavier outdoor work (e.g., construction) than women. Men’s work in industrial settings exposes them to chemical and physical hazards (e.g., noise can cause work-related stress). More men than women hold demanding managerial positions that can involve psychological stress. On the other hand, women’s professions (e.g., a nurse doing shift work) also involve psychological hazards that can cause work-related stress (Kauppinen and Kandolin 1998).

The law stipulates that work disability pension always be based on a medically diagnosed disease, whereas health interview surveys on disability often define disability as the person’s inability to cope with the everyday routines of normal life. Individual early pension is granted on less strict medical grounds than the proper disability pension, but it is
based on permanently lowered work capacity. In the 1980s, women’s occupational positions were at a lower level than those of men. Therefore, the work ability index based on self-rated assessment may be distorted by gender differences in the working conditions (Sköld 2000). There is also evidence that women tend to complain more often than men about their symptoms (Nolen-Hoeksema et al. 1999). Be that as it may, a useful way to illustrate differences in disability is to examine working life expectancies. A disability-free working life expectancy may be defined as the number of years spent in excellent or good capacity for work. With this definition, the disability-free expected duration of remaining working life was shorter for men than for women at all ages (Table 4).

The present analyses add new quantitative information not included in the original study of aging workers by Tuomi (1997). The value of statistical modeling of health state probabilities lies in that it facilitates their joint estimation. By contrast, Tuomi et al. (1997) estimated, for example, the relative frequencies leading to the state of disability directly by first excluding persons who had retired or died. Because the exclusions were of different magnitude for women and men at different ages, and because disability is associated with mortality, the obtained estimates of the sex-specific proportions are not comparable and do not characterize the underlying counting process. In addition, the relative frequency approach treats, for example, all women aged 54–58 years alike with regard to their working status. The four-state labor-force status model adopted in this article assumed that the probabilities of being in the states depend on age and calendar year. The advantage of the modeling approach is that we can estimate more precisely a person’s occupancy in these states at each age. This also enables us to circumvent problems of small sample sizes.

The new analyses of the data from the surveys suggest that the work ability of Finnish aging workers deteriorates prematurely (that is, before the statutory retirement age, which varied between 55 and 63 years) leading to far-reaching socio-economic consequences. This calls for greater labor flexibility and concerted effort to encourage higher labor force participation of older workers, particularly skilled people. Especially aging male workers would need more flexible working time arrangements than the present ones. Free time is clearly appreciated more when one ages, and particularly men tend to retire early in order to gain time for their own activities. Mature-age workers have the potential to make a continuing social and economic contribution to Finland. The extent of this contribution will depend upon the ability of these workers to continue to work, and their interest in doing so. It will also be influenced by the changing nature of work and the attitudes of employers towards the value of mature-age workers. If the employment rate declines, this scenario entails risks arising from the effects of population aging. The present working-age population (15–64 years), especially the large age cohorts born in 1945–1949 (“baby-boomers”) will be reaching retirement age in the years 2005–2014, as younger and smaller cohorts are joining the labor force. The inevitable consequence is a steady shrinking of the working-age population starting in 2005. Hence Finland will face a future of dwindling numbers of employed persons who will have to pay for the increased costs of health care and social security of the expanding retired population. The young may resent the tax burden imposed on them. Priority should be given to curbing the burden of taxation on the working population. On the other hand, there is a looming intergenerational conflict if baby-boomers must prepare themselves to give way economically to the succeeding
generations. It could be reasoned that the generations now approaching retirement age do not have an automatic right to expensive social welfare subsidized by younger workers.

When preparing for this aging demographic progression in order to alleviate the pressure for growing health care expenditure, the Parliament of Finland passed new legislation in 2003 for the private sector to postpone pensioning off from work. The legislative package includes several measures. The age at which it is first possible to receive early old-age pension will go up from 60 to 62 years, and the upper limit for work life will be extended from 65 to 68 years. The transfer to a disability pension will be made easier, but the threshold will be clearly higher than that of the present early retirement of individual early retirement pension. The latter type of retirement is intended for persons whose work ability has diminished but who are not entitled to disability pension. This individual old-age pension will no longer, as of 2004, be granted to persons born after 1943. Unemployment pension will be discontinued. The age limit for part-time pension will be elevated from 56 to 58 years for persons born in 1947 or thereafter (with a lowering of the amount of subsequent old-age pension). Most importantly, pension entitlements will be linked to lifetime earnings, and increasing life expectancy will be allowed to affect the retirement age. All these measures, which will come into effect in 2005, are geared to providing more incentives in the future for people to remain in the labor force over a longer period.

Thus it is vital to examine the development of the working ability of the population at an early age when it is still possible to intervene in the process. Figure 5, modified from Williams (1999), illustrates the effect of intervention, such effects being measured by changes in probabilities and expectations. In this example, for a 45-year-old worker the effect would be three additional years of working life.

5.2. Notes on statistical assumptions and methods

The study conducted by the FIOH was designed for research purposes other than those of the present study. That is, the survey aimed at comparing municipal occupations with

![Fig. 5. Probability of working and expected length of future work life gained for an aging worker via an occupational health intervention. (Modified from Figure 4 in Williams (1999).)](image-url)
regard to their incumbents’ retirement plans, self-rated work ability, etc. However, the different aims of the two studies do not by any means render the population sample unusable for the estimation and comparison of working life expectancies. Admittedly, the design would have been more efficient statistically if a simple random sample had been drawn from the base population. But we wanted to utilize the existing data containing valuable information on work ability. Nevertheless, we maintain that the readily available sample suited the requirements of our analysis reasonably well, particularly since only fairly general assumptions are required for the least squares technique used, and maximum efficiency is not claimed.

The objective of the present study was not to focus on the particular population of municipal workers, but to gain a general understanding of the work ability of an aging Finnish employed population from the point of view of the age trend. Moreover, the original interest was in the differences in the working life expectancies of employees in the work ability classes and between the genders, rather than in the absolute expected duration of future working life of an average municipal worker. Thus, the statistical inference was not concerned with the particularistic experience of the municipal workers per se. Of course, the study could be extended to concern the relative performance of different occupational groups regarding work force participation. Finally, it is believed that the survey design did not involve any complex sampling scheme that would have had to be taken into account in the analysis, as was the case in the multi-purpose Mini-Finland Health Survey that used a nationally representative stratified sample (Helio¨vaara et al. 1993). Therefore, in our judgement, the sampled data were adequately pertinent to our study.

It is also important to note that individuals within the same cohort who are in the same initial health state evolve with age independently and identically, as far as the transitions between health states are concerned. Then the distribution of the number of people in work state \( i \) at age \( y \) is multinomial. As shown in Davis et al. (2001), for large total numbers an asymptotic normal distribution can be derived for the log partial odds, \( \theta(x, y) \). If simple random sampling can be taken as an adequate large-sample approximation to the Finnish sampling scheme, then this will hold for the data discussed here.

It may be thought that assuming an underlying Markov process would simplify the statistical methodology. However, in the present case, where it is known only that initially the individuals were in the work force. Of importance is the “working” assumption that repeated observations on an individual are independent and that age cohorts are independent. The first of these assumptions is the more restrictive. Violation will not result in inconsistent estimates, but standard methods of calculating standard errors would be invalid. For this reason the method of Liang and Zeger (1986) was used to calculate the standard errors of regression coefficients. Curiously enough, the situation is different and much simpler when the initial health state is known (see Davis, Heathcote, and O’Neill 2002). Nevertheless, there is need for methodologic studies in which survey data sets are simulated from an underlying process that accounts for dependence between repeated observations on the same individual and age cohorts.

For a cohort on which survival data are available, we can perform repeated cross-sectional surveys among the surviving persons to obtain assessments of their work ability at different ages. These data can be combined with work disability and mortality data to estimate the expected duration of the various states of work ability as well as the active
working life expectancy. To avoid bias, the surveys should draw representative samples from the study base population and the causes of nonresponse should be investigated for possible lack of representativeness. If there is any reason to suspect that the workers who responded were more likely to have a higher or lower rate of work disability than those who did not respond, then this issue should be discussed when making inferences. For example, the nonresponse and disability rates of the men were higher than those of the women (Table 1). But, fortunately, only 5% of the subjects who were examined in 1981 did not respond to the third questionnaire in 1992.

Regarding the interpretation of the results, an essential issue is the cross-sectional design. The health state of a population affects the rates of labor force participation. Owing to the lack of information about health-based selection of the municipal workers, it is not known how much the demands of work have affected the rates of work ability and disability before the commencement of the study. At any rate, in the first cross-sectional survey, in 1981, all the subjects were still active employees.

We applied the method of Davis et al. (2001) and Davis, Heathcote, and O’Neill (2002) to estimate working life expectancies, at the same time correcting a minor error in that article in which the factor of 1/2 was omitted from the defining equation of $e_i(x)$. The effect of this factor reduces the total working life expectancy by six months. For example, if the initial age of the person is 45 years, his or her working life expectancy amounts to 17.5 years (rather than 18 years). Note that this is consistent with the assumption of uniformly distributed entry times during the first year. For the Finnish data, the coincidence of the origin year and the first year of the survey is the reason for separately modeling the prevalences of the active working states, States 1, 2, and 3, at initial age $x$, that is, modeling $\pi_i(x)$ as a function of $x$. Our aim was to ensure that the working life expectancies estimated from the Finnish data have the same interpretation as those estimated from the Australian disability data (see Mathers 1996). Hence, for $i = 1, 2, 3$ we have used $\pi_i(x)$ as an estimate of the conditional probability of an individual being in state $i$ at age $x$, given that they were actively employed at this age. Note that from cross-sectional data we can only estimate marginal expectancies for an individual known to be in the work force at a given initial age.

5.3. Direct versus adjusted life year measures

Working life expectancies, like health expectancies, are expected future occupation times of well-defined states. As such they are direct summary measures of population health. Most health expectancies are linked to a particular health status measurement, focussing on an individual’s perceived health and long-term illness (chronic disease, defect or injury). For example, the cost-effectiveness of health-care interventions has been assessed in terms of a gain in quality-adjusted life expectancy (QALE) (see Bowling 1993; Gold et al. 1994; Wilkins and Adams 1992). For the estimation, experts need to assign values for the years of life lived with disability, and to calculate a weighted average over the years survived.

The work ability index developed by Tuomi et al. (1998) is used for the self-assessment of physical and psychological work strain as well as an individual’s work ability. The index, which was developed for measuring health in relation to quality of work life in Finland, has been tested in many other countries. The self-assessment has been validated.
using clinical methods (Eskelinen et al. 1991). The results indicate that the questionnaire responses on health and work ability have related well with the clinically assessed factors at the group level. However, there remain methodological issues pertaining to reliability and item response scaling (Bowling 1993) that need to be examined more closely.

As noted by Mathers (1991), self-reported measures of health status are based on perceptions and expectations of health that vary with culture and community, and are also likely to vary with time – for example, as societies undergo epidemiological transition (Johansson 1991) or as public health campaigns and legislative measures alter the community’s awareness of health problems and attitudes to health hazards (Heloma 2003). Murray and Lopez (1996) reviewed studies which pointed to significant cross-cultural differences between self-report and observation of disability and poor health.

These considerations appear to have motivated an alternative method, described in the World Development Report (1993) and elsewhere, which measures health gaps. This measure is the disability-adjusted life years (DALY), that is, a combination of the time lost because of premature mortality and the time lived with a disability, adjusted for the severity of the disability. This is a concept that has been used, among other applications, to measure the “burden of disease” in populations, and as a tool for setting priorities in health policy, especially to guide allocation of resources for health care interventions. Obviously, the effect of an intervention can be calculated either as an increase (gain) in the QALE or as a reduction (gap) in the DALY. The procedure analogous to DALY in the context of work capacity requires the estimation, probably with the inclusion of subjective considerations, of factors affecting future work. This may be unavoidable in the field of forensic economics, but is perhaps best avoided in other contexts and, generally, when direct measures are more readily interpretable.

5.4. Concluding remarks

In summary, we restate briefly the major findings and conclusions of the study. The formulated working life expectancy measure shows the number of years one expects to continue to participate in the work force. This concept takes into account the fact that a person may choose to leave employment, or may be unable for health reasons to continue to work any longer. We found that for a Finnish male worker aged 45 years in 1981, the working life expectancy was 11.5 years; the corresponding figure for women was 12.2 years. The probability of future working life to age 63 when not actively employed was more than 50% from the age of 57 for men and 58 years for women. Transition from poor to good or excellent work ability state was estimated to increase working life expectancy of a 45-year person by four years for both genders. The study indicates that the work ability of aging Finnish workers starts to deteriorate long before the general statutory retirement age of 63 years. In view of the labor force shortage due to the high unemployment rate and the approaching retirement of the so-called large age cohorts (born in 1945–1949), it would be extremely valuable to maintain and promote the work ability of aging workers, starting at the relatively young age of 45 years.
Appendix

Estimation of Probabilities and Prevalences

We give a brief exposition of the method developed by Davis et al. (2001) and Davis, Heathcote, and O’Neill (2002) to estimate the probabilities $p_i(x, y), y = x + 1, x + 2, \ldots, w,$ and the prevalences $\pi_i(x)$ for the Finnish data. There is a total of $k$ states, of which the first $k-1$ are nonabsorbing and denote work ability levels, in contrast to state $k$ which is absorbing. A first assumption is that for a cohort of $n(x)$ independent individuals known to be working at age $x$ the frequencies $N_i(x, y), i = 1, 2, \ldots, k$, of states at the subsequent age $y$ are multinomially distributed. This assumption is not essential although it will frequently be a good approximation for large sample sizes. Its failure leads to inefficient but still consistent estimators. Secondly, assume that the $N_i(x, y)$ satisfy a law of large numbers in the sense that $n(x)^{-1}N_i(x, y)$ converges in probability to $p_i(x, y)$ as $n(x)$ tends to infinity. Note also that the $N_i(x, y)$ and $N_i(x, u), y < u,$ are correlated, but this affects only the calculation of standard errors and not point estimates. We ignore this autocorrelation in the estimation procedure and use the method of Liang and Zeger (1986) to find standard errors.

For the case of interest with $k = 4$ states (State 4 absorbing) and reference State 1, the probabilities are given by (3.3) in terms of log partial odds, $\theta_i(x, y) = \log \{p_i(x, y)/p_1(x, y)\} = \log \{n_i(x, y)/n_1(x, y)\}, i = 2, 3, 4$. The prevalences of the non-absorbing states at the initial age $x$ are $\pi_i(x) = n_i(x)/n(x), i = 1, 2, 3$, where $n_i(x)$ is the expected number of individuals in State $i$ $(i = 1, 2, 3)$ at age $x$, and the relevant log partial odds are $\theta_i(x) = \log \{n_i(x)/n_1(x)\}, i = 1, 2, 3$. Note that in contrast to (3.3)

\[
\pi_1(x) = \left[1 + \sum_{i=2}^{3} \exp \{ \theta_i(x) \} \right]^{-1} \tag{A.1}
\]

\[\pi_i(x) = \pi_1(x) \exp \{ \theta_i(x) \}, i = 2, 3\]

Natural estimates for the log partial odds are $\tilde{\theta}_i(x, y) = \log \{N_i(x, y)/N_1(x, y)\}$ and for the log partial odds derived from prevalences, $\hat{\theta}_i(x) = \log \{N_i(x)/N_1(x)\}$, where $N_i(x)$ is the frequency of State $i$ at age $x$. Substitution into (3.3) and (A.1) gives estimates of the probabilities and prevalences.

A basic result established in the Appendix of Davis et al. (2001), and more simply and in greater generality in Section 4 of Davis, Heathcote et al. (2002), is that for a large cohort the random vector $\hat{\theta}(x, y) = \{\hat{\theta}_2(x, y), \hat{\theta}_3(x, y), \hat{\theta}_4(x, y)\}^T$ is approximately normally distributed with mean $\theta(x, y) = \{\theta_2(x, y), \theta_3(x, y), \theta_4(x, y)\}^T$. If simple random sampling holds, the covariance matrix $V(x, y)$ can be estimated from the data and for the case of interest, with $k = 4$, this $V(x, y)$ is equal to

\[
\begin{pmatrix}
  n_2(x, y)^{-1} + n_1(x, y)^{-1} & n_1(x, y)^{-1} & n_1(x, y)^{-1} \\
  n_1(x, y)^{-1} & n_3(x, y)^{-1} + n_1(x, y)^{-1} & n_1(x, y)^{-1} \\
  n_1(x, y)^{-1} & n_1(x, y)^{-1} & n_4(x, y)^{-1} + n_1(x, y)^{-1}
\end{pmatrix}
\]

In practice, the expectations $n_i(x, y)$ are replaced by observed $N_i(x, y)$. 


A plot of the observed \( \theta(x, y) \) suggests a suitable parametrization, for example a linear or quadratic fit. If the matrix of parametrized log partial odds is \( \theta(\beta x, y) = Z(x, y)\beta \), then parameter vector \( \beta \) is estimated by minimizing

\[
L(\beta) = \sum_y [\hat{\theta}(x, y) - Z(x, y)\beta]^T V(x, y)^{-1} [\hat{\theta}(x, y) - Z(x, y)\beta]
\]

Here the design matrix \( Z(x, y) \) is determined by the nature of the parametrization and we note that indicators for socio-economic variates could be included when appropriate data are available. Figures 1 and 2 illustrate the results. Estimates of the probabilities follow from (3.3). A similar argument yields estimates of the prevalences.

For a given cohort, observed values of the log partial odds in consecutive surveys cannot be assumed to be asymptotically independent. Although weighted least squares gives a consistent estimate \( \hat{\beta} \) of the regression coefficients, the corresponding estimate of their covariance matrix is invalid because it does not take into account the autocorrelation in the observed values of the \( \theta(x, y) \). For this reason the Liang and Zeger (1986) “sandwich” form estimator was used for the covariance matrix \( \text{Var}(\hat{\beta}) \). We stress that this is a method of calculating standard errors, and it does not affect the value of the point estimate \( \hat{\beta} \). Standard errors of the estimated probabilities and expectancies were calculated by the delta method.

6. References


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