Journal of Official Statistics
Vol. 6. No. 1. 1990. pp. 75–87

Graduation Models for the Age-Specific Marital Fertility Rates: The Hungarian Example

Samarendranath Mitra[1], *Emil Valkovics*[2], *Diane Jones*,[1] *and Richard Clarke*[1]

Abstract: In this paper we have proposed two models that express the age-specific marital fertility rate $m(x)$ as a function of age x. Specifically, linear functions of $\ln x$ and $\ln (50 - x)$ have been suggested as possible estimators of $\ln m(x)$ and $\ln(-\ln m(x))$. These models have been fitted on Hungarian data covering a twenty year period from 1960. Both of these models have three parameters which have been estimated by the method of least squares and by solving a set of three simultaneous equations generated by three data points. We have compared these models and methods with another model based on a Pearsonian Type I distribution fitted earlier by the method of moments. Using the square of the correlation coefficient between the observed and the model values as a measure of goodness of fit, we have found that the model based on $\ln(-\ln m(x))$ fits the data very well by both methods.

Key words: Age-specific marital fertility rates; linear model; logarithmic transformation; method of least squares; solutions of simultaneous equations.

1. Introduction

Graduation of rates and probabilities specific for age and other variables by mathematical models has a relatively long tradition in demography. The graduation of detailed rates can be useful in several respects. First, the pattern of variation of the rates can be accurately described by a model that fits the data well. As a result, the few parameters that define such a model can replace the entire data set with little loss of information. That is to say, whenever necessary, any or all of those rates can be reproduced by the model with sufficient accuracy. Second, such models are often used to smooth the observed data when the fluctuations can be attributed to errors. Third, when data are available by age groups or for only a few ages at specified intervals, the model can be used to generate estimates for the entire range of variation. Fourth, the estimates of parameters at several points in time can be effectively used to generate estimates at other points in time by appropriate methods of interpolation and extrapolation.

Sometimes a model can be used as a part or a component of another model. Thus, a model of age-specific marital fertility rates (ASMFR) can be looked upon as a part of a model of age-specific fertility rates (ASFR) in which marital status is not con-

[1] Department of Sociology, Emory University, Atlanta, Georgia 30322, U.S.A.
[2] Central Statistical Office, Demographic Research Institute, Budapest, Hungary.

Acknowledgment: The authors would like to thank the editor and the anonymous reviewers for helpful comments and suggestions.

sidered. This is true because, in special cases, the latter model can be regarded as having two multiplicative components (Coale and Trussel (1974)): the ASMFR model and a model of the age-specific proportion ever married. Such a multiplicative model can describe in greater detail the pattern of variation of ASFR than a single model and that is one of the reasons for our attempting to develop models of ASMFR. As such, modeling of the age-specific proportion ever married is an interesting topic for further research. However, this type of modeling will not be applicable to populations with large extra-marital fertility rates.

It may also be noted that literature on ASMFR models is somewhat scanty compared with that on the models of ASFR. In contrast, the literature on modeling ASFR is vast and falls into two categories. In the first category, the ASFR values are modeled by a frequency function of a probability distribution. Since the pattern of the general ASFR can be expressed by a bell-shaped curve with the mode assuming a value in the age interval 20–29, attempts have been made in the past to fit the distributions by mathematical functions, such as Hadwiger, gamma, lognormal, Pearsonian Type I, Wald, Weibull, Gompertz and others. See, for instance, Brass (1960), (1974), and (1978); Brass, Coale, Demeny, Heisel, Lorimer, Romaniuc, and Van de Walle (1968), Duchene and Gillet de Stefano (1974), Gilje (1969), Gilje and Yntema (1970), Hoem and Berge (1975), Hoem, Madsen, Nielsen, Ohlsen, Hansen, and Rennermalm (1981), Hyrenius, Sundwall, and Nygren (1974), Mitra (1967) and (1970), Mitra and Romaniuc (1973), Romaniuc (1973), Talwar (1970), Tekse (1967), Valkovics (1983a), (1983b), and (1984); Yntema (1952) and (1969). In the second category, modeling efforts have concentrated on specific transformations, for example, cumulative (Murphy and Nagnur (1972); Wunsch (1966)), trigonometric, logarithmic, and other functions of ASFR (Valkovics (1983b)).

In contrast, attempts to model marital fertility rates have been made by Henry (1961), Coale and Trussel (1974), and Valkovics (1984). They argue that marital fertility is similar to natural fertility in populations in which there is little or no voluntary birth control. Furthermore, ASMFR in such populations decline monotonically with age and therefore can be successfully modeled by mathematical functions different from those most appropriate to ASFR. Even in populations practising family planning, the general pattern of variation of ASMFR does not change and can be modeled by similar functions. Recently, experiments with mathematical models (Valkovics (1984)) on relatively low-level marital fertility rates of Hungary have produced promising results. Three functions have been attempted on the data, namely, (1) the Beta function, (2) the Gompertz function, and (3) a fourth degree polynomial. Of these three, the first function fitted by the standard method of moments has been found to be the best in 80% of the cases.

These encouraging results have led to further experiments with alternative models for the graduation of ASMFR. In this paper, we provide a description of the three models mentioned above and their goodness of fit on the Hungarian data. Hungarian data were chosen because of their excellent quality, general availability, and also because the ASMFR values are tabulated by single year of age. Unfortunately, comparable ASMFR data are not available in most countries. However, as long as the general pattern of the distribution of the ASMFR remains monotonic and declining, it seems reasonable to assume that the models will work as well on other data sets. The following sections describe the models, the data and methods used to estimate the parameters, and finally, the findings when the models are applied on Hungarian data.

2. The Models

2.1. Model I

We begin by noting that unlike the distribution of ASFR which is generally unimodal and asymmetric, the distribution of ASMFR is monotonically declining. Furthermore, Pearson's Type I frequency function, i.e., the Beta frequency function which has been successfully fitted earlier to the ASFR is flexible enough to be used on either a monotonically increasing or a declining distribution. Consequently, we have started our experiment by attempting to model the distribution of ASMFR by the Type I function.

However, for the present investigation using the Hungarian ASMFR, we have chosen not to adopt the traditional method of moments to fit a Type I distribution for two reasons. First, the method of moments requires the full distribution and cannot be applied on a truncated distribution. In Section 4 we show why it is necessary to truncate the distribution of ASMFR by excluding the rates for ages 15, 16, and 17, since these ages do not follow the expected trend manifested in the following ages. Second, the graduation model provided by the Type I distribution can be written as

$$m(x) \;=\; Cx^{A_1}(\beta - x)^{A_2} \tag{1}$$

with origin at age $x = 0$. After a logarithmic transformation (1) is expressed as

$$\ln m(x) \;=\; A_0 + A_1 \ln x + A_2 \ln (\beta - x), \tag{2}$$

where

$$A_0 \;=\; \ln C, \tag{3}$$

with β as the upper limit of the reproductive interval. For this study, β is assumed to be 49 years at last birthday or a continuous variable beginning at the 50th year. In this form, the parameters A_0, A_1, and A_2 can be estimated by methods described in Section 4. The reader will see that the logic justifying the application of these methods is not affected by the nature of the distribution, truncated or otherwise.

2.2. Model II

In an earlier study of the modeling of the life table survivorship function $l(x)$, which is also a monotonically declining function, a good fit was obtained by a model (Mitra (1984)) that expressed $\ln(-\ln l(x))$ as a linear function of $\ln x$ and $\ln(\alpha - x)$. In this model, x and α represent the age and the length of the life span, respectively. A goodness-of-fit statistic of about 0.99 was achieved in that study, as measured by the squared multiple correlation coefficient between the observed and the expected values. Encouraged by this result, we felt that it would also be appropriate to attempt to fit the ASMFR by a similar model. Thus, denoting these rates by $m(x)$, we defined the model

$$\ln(-\ln m(x)) \;=\; B_0 + B_1 \ln x + B_2 \ln(\beta - x), \tag{4}$$

where the last term on the right-hand side contains the upper limit of the reproductive interval β, which, as before, is assumed to be 50 years.

2.3. About the two models

The similarity between models (2) and (4) is apparent. A linear function of $\ln x$ and

$\ln(\beta - x)$ has been taken as the estimate of $\ln m(x)$ in the former as against $\ln(-\ln m(x))$ in the latter. A formal comparison between the two models can be made by expressing (4) as

$$-\ln m(x) = \exp(B_0 + B_1 \ln x + B_2 \ln(\beta - x)). \quad (5)$$

Thus, we see that, according to (2), the relationships among $\ln m(x)$, $\ln x$, and $\ln(\beta - x)$ are linear whereas these relationships according to (5) are curvilinear. The magnitude of the departure from linearity in any given example will, however, depend on the values of the parameters.

Observe that both of these models remain undefined for both $x < 0$ and $x > \beta$. Also, in any given distribution of ASMFR a discontinuity will surely be found at some age, say 15, where the rate is the largest. However, the rates are assumed to be zero at all ages under 15. Not only do these models fail to match that pattern but when projected for ages under 15 they show increasingly larger values. However, these should not be of concern since the main quality of a model is its ability to reproduce the observed distribution in the modeled range of variation and not by its performance elsewhere.

3. The Data

The Hungarian marital fertility data used in this research come from the Hungarian *Demographic Yearbook* (as reproduced in Valkovics (1984)). The yearbook provides data by single year of age (at last birthday) over the age interval (15, 49) for the years 1960 to 1980. To save time and space we have selected the data sets for 1960, 1980, and the intervening five-year intervals; the data is summarized in the Appendix, Table 1. As noted earlier, the pattern of marital fertility rates is generally monotonically declining although a few exceptions have been observed in Hungary for a number of years, mostly in the 1960s. In those years, a high rate of age 15 was followed by a steep decrease at age 16, then a significant increase in the rates was recorded at age 17. One possible explanation is that, compared to other ages, many of the marriages among 15-year-olds take place following premarital conception, resulting in a higher than normal marital fertility rate at that age. In fact, the highest rate recorded in those years was found to be 0.84 in 1978; that is to say, 84% of 15-year-old married women became mothers in that year. Since most of the women who become mothers at age 15 could not give birth again in the following year, the rates at age 16 are expected to be considerably lower than those at the preceding age. However, they can become mothers again when they are 17 and because few marriages take place at these ages, the rate at age 17 can be higher than that at age 16, a phenomenon that has been observed in Hungary several times in the early 1960s. In general, the rates for these three ages show considerable instability during the period under investigation while the rates at the remaining ages show reasonably smooth patterns of variation (see Figures in Section 5).

4. Methods

The mathematical functions employed by the earlier researchers have captured the pattern of ASMFR at ages 18 and above quite well. Yet for the reasons mentioned above, these models could not reproduce the actual rates at the first few ages as accurately as they did at other ages. Indeed, the rates at ages 15, 16, and 17 may be considered outliers. Therefore, one may expect a better fit in the age interval 18 and above if the outliers are dropped when estimating the parameters. As noted earlier, graduation formulas that are frequency functions fitted by the method of moments require the full data set for generating the estimates of the parameters. The moments computed from a truncated distribution are thus inappropriate. But when such formulas are looked upon strictly as mathematical functions, the parameters can quite often be estimated by methods which are robust with respect to their data requirements. From a strictly theoretical point of view, an appropriate segment of a curve generated by a mathematical function is all that is needed for estimating the parameters.

For our models we have experimented with two such methods in order to derive our parametric estimates from data sets covering the age interval (18, 49). The specification of the upper limit of the age variable reduces both of the proposed models to a form such that the solution by the method of least squares suggests itself almost instantly. Accordingly, we applied this method on the ASMFR data sets given by single year of age in the age interval (18, 49).

Next, we also noted that since the equations are linear, using exactly three parameters, a set of only three data points should be sufficient for generating the parametric estimates if the models fit the data well. The simplicity of a procedure based on the exact fit of a carefully selected set of three data points makes it attractive. We have used this additional method of curve fitting with the ages 20, 30, and 40 as our data points. These ages are equispaced and cover the principal segment of the reproductive interval, which lends operational simplicity. In addition, we have also chosen to substitute the observed rates at those ages with the simple averages of the respective five-year intervals centered around them.

To summarize, we have experimented with two linear models which use $\ln x$ and $\ln(\beta - x)$ for $L1 = \ln m(x)$ and $L2 = \ln(-\ln(m(x)))$, respectively. The choice of the notations L1 and L2 reflects the fact that in L1 the logarithm is taken only once, whereas in L2 it is taken twice. The parameters of those models have been estimated by the method of least squares (LS) and from the solutions of a set of simultaneous equations (SE), respectively. The combinations of the models and the methods give rise to four types which we refer to by their abbreviations L1LS, L1SE, L2LS, and L2SE. Thus, L1LS denotes the least square solution of the parameters of the linear model for $\ln m(x)$. The abbreviations for the remaining three types can be similarly understood.

5. Results

For each of the four types, the sequence of the computational scheme begins with the estimation of the parameters (see Appendix, Table 3). Next is the calculation of the expected values of the single year ASMFR $m(x)$ by substituting into the model equations the parametric estimates obtained in the first step. The usefulness of the models and the methods of their application can be tested by devising a measure of goodness of fit based on a comparison of the observed and the expected values. Guided by Mitra (1984), we decided to measure the goodness of fit by the square of the correlation coefficient between the observed and the expected values (by single year of age). These coefficients are shown in Table 1 for each type and for each of the five years. It may be mentioned that for the LS method, the square of the multiple correlation coefficient is the same as the squared correlation coefficient between the observed and the expected values.

Table 1. Squared correlation coefficients between the observed and the expected values of L1 = ln m(x) and between the same of L2 = ln(− ln m(x)) by single years of age by the two methods. (Age-interval: 18–49 years), Hungary: 1960–80

Year	Squared correlation coefficients for type			
	L1LS	L1SE	L2LS	L2SE
1960	0.983	0.989	0.995	0.996
1965	0.983	0.990	0.997	0.995
1970	0.973	0.979	0.995	0.997
1975	0.964	0.966	0.995	0.991
1980	0.970	0.967	0.997	0.998

All the squared correlation coefficients shown in Table 1 are large and the values are uniformly higher for L2, though by small amounts. Thus both the models are quite good, with L2 having a slight edge over L1. Interestingly enough, the difference between LS and SE is not only very slight, but in a few instances SE turned out to be better than LS. Although the occasional advantage of SE may be due to rounding errors associated with small numbers, it provides a clear demonstration that both methods are comparable and work very well in these examples.

It should be noted here that these measures of goodness of fit compare the values of a function of $m(x)$ like L1 or L2 with those of its expected values E(L1) or E(L2) and as such do not provide a direct comparison between $m(x)$ and $E(m(x))$. But the high correlations between L1 and E(L1) and between L2 and E(L2) lead us to expect similarly high correlations between $m(x)$ and $E(m(x))$ where the $E(m(x))$ values are obtained from E(L1) and E(L2) by reversing the procedure used to compute L1 and L2 from $m(x)$. This is clear from the squared correlation coefficients between $m(x)$ and $E(m(x))$ shown in Table 2.

The last column of Table 2 shows the squared correlation coefficients for the Type I model fitted by the method of moments (Valkovics (1984)) which like the others are also very large. Comparing the values in Table 2 with their counterparts in Table 1, we find that L1LS has lower correlation coefficients for the untransformed data, but the opposite is true for the other three. In fact, the coefficients for L1SE are so high in Table 2 that L1SE, L2LS, and L2SE can hardly be distinguished from one another. The smallest value of the coefficient for these three is found to be 0.990 while the others are 0.995 and higher with several as high as 0.999.

Next, we turn our attention to the fact that the least squares method has the inherent quality of equalizing the sums of the observed and the expected values. This is

Table 2. Squared correlation coefficients between the observed and the expected values of m(x) by the four types and the method of moments. (Age-interval: 18–49 years), Hungary: 1960–80

Year	Squared correlation coefficients for type				
	L1LS	L1SE	L2LS	L2SE	Moments
1960	0.980	0.990	0.997	0.996	0.989
1965	0.968	0.995	0.997	0.997	0.996
1970	0.959	0.997	0.997	0.997	0.975
1975	0.940	0.999	0.998	0.999	0.997

Calculated from the observed and the expected values shown in Valkovics (1984).

Table 3. Total of the observed and expected marital fertility rates over the age interval 18–49 years by types. Hungary: 1960–80

Year	Observed	Total marital fertility rates (18–49)				
		L1LS	L1SE	L2LS	L2SE	Moments
1960	2.867	2.978	2.881	2.868	2.909	2.809
1965	2.729	2.900	2.779	2.730	2.788	2.690
1970	2.923	3.185	2.978	2.922	2.984	2.892
1975	3.451	3.879	3.499	3.440	3.500	3.466
1980	2.776	3.192	2.796	2.779	2.805	2.777

a highly desirable feature and is found in our least squares examples dealing with the transformations L1 and L2 of the $m(x)$ functions. However, even though it is highly desirable, there is no guarantee that the graduated values of $m(x)$ obtained by inverting the expected values of L1 or L2 will add up to the sum of the corresponding observed values. Nevertheless, we have reason to expect that these two sums should be quite close since the quality of the model in the transformed plane seems to have very little scope for improvement. For the same reason we can also expect that SE should produce similarly matching values in spite of its not having the control mechanism of equalizing the observed and the expected sums. In Table 3, we show these sums of ASMFR values or the total marital fertility rates (TMFR) obtained from these procedures.

The discrepancy between the observed and the expected model values of TMFR is somewhat large for L1LS but quite small for the other three. The difference ranges from 0.014 to 0.055 children for L1SE, from 0.029 to 0.061 for L2SE and from −0.011 to 0.003 for L2LS. For these three, the discrepancies are small and ignorable for all practical purposes.

The last column of Table 3 shows the TMFR values obtained by the method of moments (Valkovics (1984)). Because this model was based on the entire age-range (15–49), the discrepancies between the observed and the expected values are likely to be higher for the 18–49 age interval and indeed they are.

As before, L2 seems to be the better and, of the two methods used to generate the estimates, LS seems to have some advantage

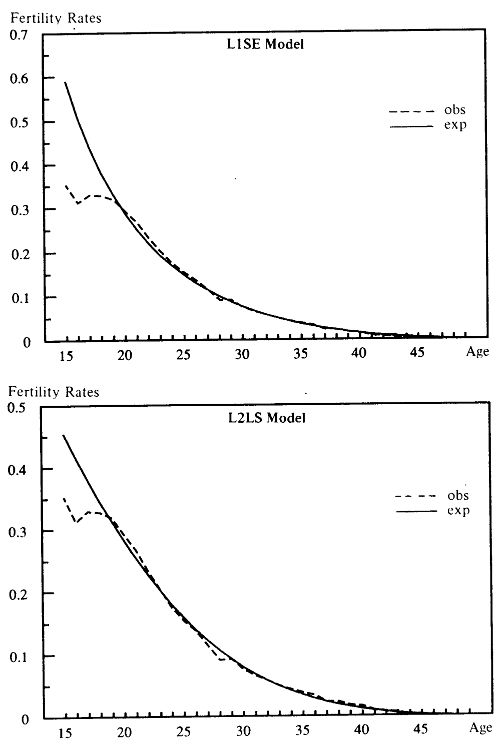

Fig. 1. Graduation of age-specific marital fertility rates, Hungary, 1960

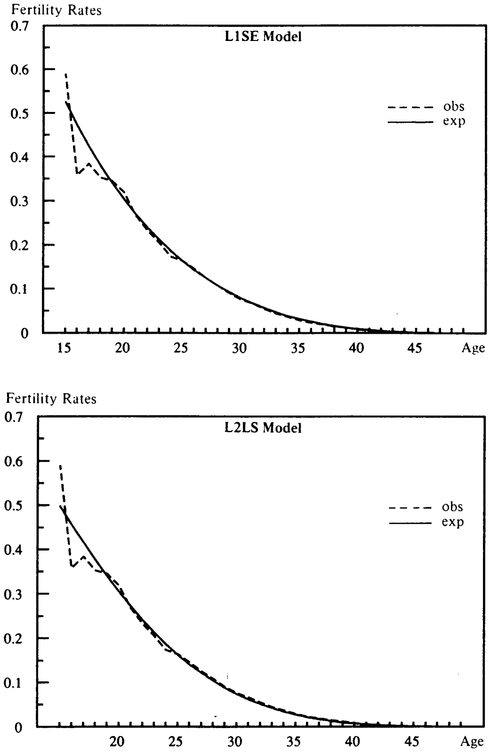

Fig. 2. Graduation of age-specific marital fertility rates, Hungary, 1970

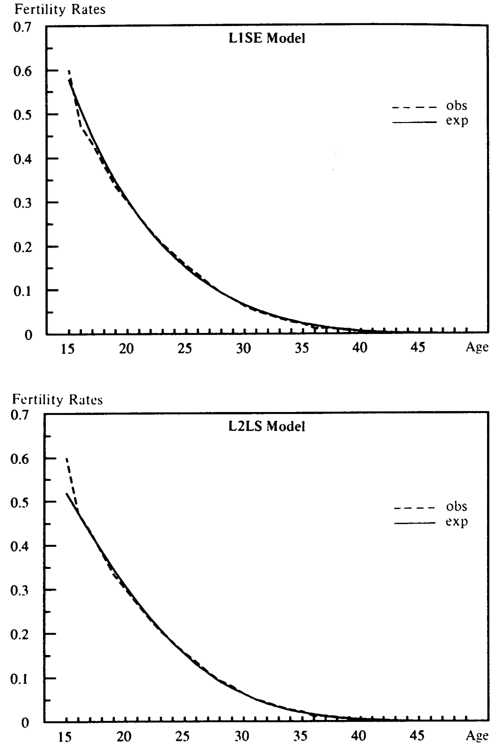

Fig. 3. Graduation of age-specific marital fertility rates, Hungary, 1980

over SE. The results are shown in greater detail in the Appendix, Table 2, which shows the ASMFR values averaged over five-year age intervals. In that table we have also shown the TMFR separately for the age intervals 18–49 and 15–49. The latter has been obtained by adding to the former the rates for the ages 15, 16, and 17 estimated from the model equations. As anticipated, the discrepancies between the actual and the graduated values are considerably larger for this latter comparison. Other than that, the goodness of fit of the models follows the same pattern noted earlier. Figures 1–3 provide a visual comparison of the observed and the model values by single year of age for L1SE and L2LS for the years 1960, 1970, and 1980.

The TMFR values remained fairly stable over these years except for 1975 (the Appendix, Tables 1 and 2), which apparently reflects the consequences of a short-lived pronatalist policy instituted in 1973. Accordingly, we would expect similar stability in the parametric estimates if all other sources of variation remain the same. However, their values (see the Appendix, Table 3) show considerable fluctuations for L1 which do not seem to be attributable to the other sources of variation. In contrast, the L2 parameters are relatively stable. It is possible that the variation in the L2 parameters may have been caused more by the differentials in the distribution of the rates by age than by any other factor of major consequence. These findings provide strong support for L2 for purposes of graduation of ASMFR leaving the choice between LS and SE to the researcher.

6. Summary and Concluding Remarks

In this paper, we have fitted graduated models of age-specific marital fertility rates. The monotonically declining nature of the distribution of age-specific marital fertility rates (ASMFR) led us to search for appropriate mathematical functions with similar properties. We began by noting that the Pearsonian Type I distribution was successfully used by earlier researchers to model age-specific fertility rates (ASFR). Although these distributions are unimodal in nature, we were encouraged by the flexibility of the Type I distribution to accommodate monotonically declining functions. We operationalized the model by applying logarithmic transformations to a linear form involving the logarithms of ASMFR $m(x)$, age x, and the remainder of the fertile interval $\beta - x$. Next, we recalled the results of another successful modeling experiment involving the life table survivorship probability $l(x)$, x, and $\alpha - x$ where α is the life span. Thereafter, we decided to experiment with a parallel model, also linear, in terms of $\ln(-\ln m(x))$, $\ln x$, and $\ln(\beta - x)$.

Two methods were used to estimate the model parameters. The first was the traditional method of least squares. The second used parameters obtained by solving three simultaneous equations developed by equating the observed and the model ASMFR values at ages 20, 30, and 40. These ages were chosen because they were equispaced and cover a wide range of the reproductive interval. Our experiment with Hungarian data for the years 1960 through 1980 demonstrated excellent fit, especially by the second model, as measured by the correlation coefficients of 0.990 and above between the observed and model rates.

Further research with these and other models should be conducted with other data although we note that single-year data of ASMFR are not readily available. Valuable insights can be obtained about reproductive behavior when such data are available. We anticipate, for example, that the distribution of ASMFR is influenced by the pattern of marriage. Early marriages will tend to result in lower ASMFR at older ages and vice versa. Thus, the marriage pattern will be a partial determinant of the slope of the distribution of ASMFR. It is quite possible that some of the variations in the parameters of our model may have resulted from temporal changes in that pattern.

7. References

Brass, W. (1960): The Graduation of Fertility Distributions by Polynominal Functions. Population Studies, 14, pp. 148–162.

Brass, W., Coale, A., Demeny, P., Heisel, D.F., Lorimer, F., Romaniuc, A., and Van de Walle, E. (1968): The Demography of Tropical Africa. Princeton University Press, Princeton.

Brass, W. (1974): Perspectives in Population Prediction: Illustrated by the Statistics of England and Wales (with discussion). Journal of the Royal Statistical Society, ser. A, 137, pp. 532–583.

Brass, W. (1978): Population Projections for Planning and Policy. Papers of the East-West Population Institute, 55, Honolulu, Hawaii.

Coale, A.J. and Trussel, T.J. (1974): Model Fertility: Variations in the Age Structure of Childbearing in Human Populations. Population Index, 40, pp. 185–258.

Duchene, F. and Gillet de Stefano, S. (1974): Ajustement analytique des courbes de fecondite generale. Population et Famille, 32, pp. 53–93.

Gilje, E. (1969): Fitting Curves to Age-Specific Fertility Rates: Some Examples. Statistisk tidskrift, 7, pp. 118–139.

Gilje, E. and Yntema, L. (1970): Shifted Hadwiger Fertility Function. Arbeitsnotader, Statistisk Sentralbyra, 16 pp.

Henry, L. (1961): Some Data on Natural Fertility. Eugenics Quarterly, 8, pp. 81–91.

Hoem, J.M. and Berge, E. (1975): Some Problems in Hadwiger Fertility Graduation. Scandinavian Acturial Journal, 58, pp. 129–144.

Hoem, J.M., Madsen, D., Nielsen, J.L., Ohlsen, E.M., Hansen, H.O., and Rennermalm, B. (1981): Experiments in Modelling Recent Danish Fertility Curves. Demography, 18, pp. 231–244.

Hyrenius, H., Sundwall, A., and Nygren, O. (1974): Methods of Fitting a Pearson Type I Function to Age-Specific Fertility Rates. Statistisk tidskrift, 12, pp. 133–142.

Mitra, S. (1967): The Pattern of Age-Specific Fertility Rates. Demography, 2, pp. 894–906.

Mitra, S. (1970): Graduation of Net Maternity Function. Sankhya, ser. B, 32, pp. 63–68.

Mitra, S. (1984): On the Characteristics of the Parameters of a Life Table Function. Genus, 40, pp. 47–56.

Mitra, S. and Romaniuc, A. (1973): Pearsonian Type I Curve and Its Fertility Projection Potentials. Demography, 10, pp. 351–365.

Murphy, E.M. and Nagnur, D.N. (1972): A Gompertz Fit That Fits: Application to Canadian Fertility Patterns. Demography, 9, pp. 35–50.

Romaniuc, A. (1973): A Three-Parameter Model for Birth Projections. Population Studies, 27, pp. 467–478.

Talwar, P.P. (1970): Age Patterns of Fertility. Institute of Statistics Mimeo Series, No. 656, University of North Carolina, Chapel Hill.

Tekse, K. (1967): On Demographic Models of Age-Specific Fertility Rates. Statistisk tidskrift, 5, pp. 189–207.

Valkovics, E. (1983a): On Some Directly Fittable Models of General Age-Specific Fertility Rates. Demografia, 26, pp. 522–567. (In Hungarian.)

Valkovics, E. (1983b): Une methode indirecte de modelisation des taux de fecondite generale par age. European Demographic Information Bulletin, 14(1), pp. 11–27.

Valkovics, E. (1984): An Attempt to Modeling the Age-Specific Marital Fertility Rates. Published by Central Statistical Office, Demographic Research Institute, Hungary. (In Hungarian.)

Wunsch, G. (1966): Courbes de Gompertz et perspectives de fecondite. Recherchas Economiques de Louvain, 6, pp. 457–468.

Yntema, L. (1952): Mathematical Models of Demographic Analysis. F.F. Groen and Zoon, N.V., Leiden, 78 pp.

Yntema, L. (1969): On Hadwiger's Fertility Function. Statistisk tidskrift, 7, pp. 113–117.

Received June 1987
Revised October 1989

Appendix Table 1. Hungarian age-specific marital fertility rates by single years of age[1]

Age	Calendar years 1960	1965	1970	1975	1980
15	0.3514	0.5600	0.5883	0.8325	0.5982
16	0.3119	0.3219	0.3573	0.4912	0.4718
17	0.3292	0.3488	0.3828	0.4692	0.4321
18	0.3275	0.3229	0.3524	0.4237	0.3826
19	0.3188	0.3151	0.3444	0.3891	0.3336
20	0.2923	0.2920	0.3183	0.3487	0.3000
21	0.2672	0.2625	0.2672	0.3025	0.2654
22	0.2331	0.2211	0.2321	0.2758	0.2333
23	0.2034	0.1951	0.2046	0.2470	0.2034
24	0.1751	0.1662	0.1728	0.2217	0.1792
25	0.1538	0.1478	0.1631	0.2023	0.1547
26	0.1370	0.1271	0.1457	0.1757	0.1354
27	0.1138	0.1139	0.1244	0.1516	0.1119
28	0.0902	0.0996	0.1077	0.1323	0.0931
29	0.0911	0.0829	0.0906	0.1099	0.0795
30	0.0743	0.0730	0.0756	0.0960	0.0630
31	0.0644	0.0599	0.0663	0.0827	0.0509
32	0.0568	0.0488	0.0544	0.0662	0.0427
33	0.0490	0.0410	0.0441	0.0554	0.0346
34	0.0428	0.0358	0.0369	0.0416	0.0272
35	0.0377	0.0283	0.0290	0.0337	0.0224
36	0.0331	0.0229	0.0228	0.0264	0.0177
37	0.0225	0.0180	0.0189	0.0200	0.0128
38	0.0226	0.0157	0.0147	0.0157	0.0103
39	0.0176	0.0121	0.0121	0.0111	0.0074
40	0.0154	0.0095	0.0081	0.0089	0.0055
41	0.0089	0.0068	0.0055	0.0058	0.0039
42	0.0073	0.0044	0.0043	0.0041	0.0023
43	0.0055	0.0029	0.0028	0.0019	0.0015
44	0.0025	0.0019	0.0015	0.0013	0.0009
45	0.0015	0.0011	0.0008	0.0007	0.0004
46	0.0008	0.0004	0.0003	0.0002	0.0002
47	0.0004	0.0000	0.0001	0.0000	0.0001
48	0.0002	0.0001	0.0001	0.0001	0.0001
49	0.0001	0.0000	0.0000	0.0000	0.0000

[1] *Source*: Demographic Yearbook, 1960–1980, Central Statistical Office, Budapest, Hungary

Appendix Table 2. Observed and the expected age-specific marital fertility rates by types. Hungary: 1960–80

Age-group	Observed	Age-specific marital fertility rates L1LS	L1SE	L2LS	L2SE	Moments
				1960		
18–19	0.323	0.400	0.355	0.329	0.340	0.299
20–24	0.234	0.239	0.224	0.230	0.230	0.220
25–29	0.117	0.114	0.115	0.124	0.119	0.127
30–34	0.058	0.053	0.058	0.059	0.057	0.063
35–39	0.027	0.022	0.026	0.023	0.025	0.025
40–44	0.008	0.007	0.009	0.007	0.011	0.006
45–49	0.000	0.000	0.002	0.001	0.005	0.000
18–49	2.867	2.978	2.881	2.868	2.909	2.809
15–49	3.859	4.794	4.413	4.116	4.218	3.871
				1965		
18–19	0.319	0.428	0.341	0.331	0.332	0.317
20–24	0.227	0.239	0.223	0.225	0.227	0.217
25–29	0.114	0.104	0.115	0.115	0.118	0.116
30–34	0.052	0.044	0.054	0.051	0.053	0.054
35–39	0.019	0.017	0.021	0.018	0.020	0.020
40–44	0.005	0.005	0.006	0.005	0.006	0.004
45–49	0.000	0.001	0.001	0.001	0.001	0.000
18–49	2.729	2.900	2.779	2.730	2.788	2.690
15–49	3.960	4.957	4.172	3.999	4.061	3.948
				1970		
18–19	0.348	0.495	0.359	0.358	0.354	0.345
20–24	0.239	0.266	0.242	0.243	0.244	0.236
25–29	0.126	0.109	0.126	0.123	0.128	0.124
30–34	0.056	0.044	0.058	0.052	0.057	0.056
35–39	0.020	0.016	0.021	0.018	0.021	0.020
40–44	0.004	0.004	0.005	0.004	0.005	0.004
45–49	0.000	0.001	0.000	0.000	0.001	0.000
18–49	2.923	3.185	2.978	2.922	2.984	2.892
15–49	4.250	5.660	4.400	4.290	4.325	4.242
				1975		
18–19	0.406	0.632	0.400	0.418	0.400	0.424
20–24	0.279	0.326	0.285	0.288	0.285	0.283
25–29	0.154	0.126	0.155	0.147	0.156	0.147
30–34	0.068	0.049	0.070	0.061	0.070	0.066
35–39	0.021	0.017	0.024	0.020	0.024	0.023
40–44	0.004	0.004	0.005	0.004	0.005	0.005
45–49	0.000	0.001	0.000	0.000	0.000	0.000
18–49	3.451	3.879	3.499	3.440	3.500	3.466
15–49	5.244	7.163	4.978	5.010	4.979	5.220

Appendix Table 2. Continued.

Age-group	Observed	Age-specific marital fertility rates L1LS	L1SE	L2LS	L2SE	Moments
				1980		
18–19	0.358	0.574	0.370	0.366	0.360	0.363
20–24	0.236	0.269	0.235	0.240	0.239	0.235
25–29	0.115	0.093	0.112	0.112	0.115	0.113
30–34	0.044	0.033	0.047	0.043	0.046	0.046
35–39	0.014	0.011	0.015	0.013	0.015	0.014
40–44	0.003	0.003	0.003	0.003	0.003	0.003
45–49	0.000	0.000	0.000	0.000	0.000	0.000
18–49	2.776	3.192	2.796	2.779	2.805	2.777
15–49	4.278	6.493	4.329	4.199	4.195	4.267

Appendix Table 3. Estimates of the parameters of the L1 and L2 models by the LS and the SE methods as well as the observed values of the total age-specific marital fertility rates (TMFR) over the age-interval 18–49. Hungary: 1960–80

Year	TMFR	Parameter	Estimates by type L1LS	L1SE	L2LS	L2SE
1960	2.867	Constant	−1.491	−2.324	−3.961	−5.748
		Coefficient of				
		(1) ln x	−1.898	−1.600	1.579	1.904
		(2) ln $(50 - x)$	1.793	1.737	−0.169	0.064
1965	2.729	Constant	−0.970	−7.785	−4.319	−4.503
		Coefficient of				
		(1) ln x	−2.217	−0.764	1.691	1.708
		(2) ln $(50 - x)$	1.935	2.596	−0.163	−0.125
1970	2.922	Constant	−0.256	−10.529	−4.766	−4.256
		Coefficient of				
		(1) ln x	−2.456	−0.306	1.813	1.694
		(2) ln $(50 - x)$	1.974	3.017	−0.158	−0.201
1975	3.451	Constant	0.497	−14.882	−5.567	−3.761
		Coefficient of				
		(1) ln x	−2.683	0.486	2.023	1.645
		(2) ln $(50 - x)$	2.021	3.616	−0.153	−0.339
1980	2.770	Constant	3.698	−10.886	−5.652	−5.316
		Coefficient of				
		(1) ln x	−3.504	−0.490	2.041	1.952
		(2) ln $(50 - x)$	1.764	3.285	−0.102	−0.118