Journal of Official Statistics
Vol. 5. No. 3, 1989, pp. 293–295

Letters to the Editor

Letters to the Editor will be confined to discussion of papers which have appeared in the Journal of Official Statistics and of important issues facing the statistical community.

Dear Editor:

In Andersson, Forsman and Wretman (1987), there is a Monte Carlo investigation of several variance estimators which appears to be biased against the jackknife. Although some of the exact formulae are missing from the article, it appears that they used an ordinary jackknife on a stratified design. The other variance estimators reflected the stratification. Since the stratification was effective in reducing variance, any variance estimator that fails to take the stratification into account will, of course, be positively biased. Thus, if my deduction of which variance estimator they used is correct, then their finding of positive bias in the jackknife is a fault of their application, not an inherent fault of the jackknife.

The problem in the paper by Andersson, Forsman and Wretman was to estimate the variance of the ratio, R, of a consumer price index measured in terms of retail prices to a similar index measured in terms of list prices. Each consumer price index was, in turn, a weighted average of price indices for specific commodities. To get the price index for a specific commodity, another ratio was formed of the total value of the commodity sold at time 1 to the total value sold at time 0. The total value of the commodity sold at a particular time was estimated as

$$\hat{Y}_i = \frac{\Sigma_h \Sigma_k \, y_{hki}}{\pi_{hk}},$$

where y_{hki} is the total value of the commodity sold at sample store k in stratum h at the particular time, π_{hk} is the probability of selection for the kth store, the summation on k is over all sample stores within stratum h, and the summation on h is over all strata.

As the authors state, to estimate the variance of R using the jackknife, it is necessary to drop one store at a time from the sample. "Each time an estimate $\hat{R}(j)$ of R is calculated, analogous to $\hat{R}$, but based only on data from the remaining $n - 1$ stores ($j = 1, \ldots, n$)." However, they did not state how they calculated the analogous estimates. The question is seen more easily in terms of forming analogous estimates, $\hat{Y}_i^{(j)}$, of $\hat{Y}_i$ rather than $\hat{R}$ itself, given the complexity of $\hat{R}$. It appears that they used a formula something like

$$\hat{Y}_i^{(j)} = \hat{Y}_i^{(hk)} = \left(\hat{Y}_i - \frac{y_{hki}}{\pi_{hk}}\right) \frac{n}{(n-1)},$$

rather than the correct

$$\hat{Y}_i^{(j)} = \hat{Y}_i^{(hk)} = \sum_{h' \neq h} \sum_{k'} \left(\frac{y_{h'k'i}}{\pi_{h'k'}}\right) + \sum_{k' \neq k} \left(\frac{y_{hk'i}}{\pi_{hk'}}\right) \frac{n_h}{(n_h - 1)}.$$

After computing $\hat{Y}_i^{(j)}$ and the corresponding totals for other times and price sources, they computed $\hat{R}^{(j)}$ from its jackknifed components. They then used the variance formula

$$\hat{V}_{JK} = \frac{(n-1)\,\Sigma_j(\hat{R}^{(j)} - \hat{R}^{(\cdot)})^2}{n}.$$

The best formula (Wolter (1985) formula 4.5.3) for the stratified jackknife is

$$\hat{V}_{JK1} = \sum_h \left(\frac{(n_h - 1)}{n_h}\right) \sum_k (\hat{R}^{(hk)} - \hat{R}^{(h.)})^2.$$

Also possible and used frequently at Westat is

$$\hat{V}_{JK4} = \sum_h \left(\frac{(n_h - 1)}{n_h}\right) \sum_k (\hat{R}^{(hk)} - \hat{R}^{(\cdot)})^2.$$

Given the mistaken use of $\hat{V}_{JK}$ instead of $\hat{V}_{JK1}$ or $\hat{V}_{JK4}$, it would be consistent for them to have erred in the suggested manner in the calculation of $\hat{R}^{(j)}$. A further error was made in the jackknife estimate of the ratio (used to remove bias). It was given as

$$\hat{R}_{JK} = n\hat{R} - (n-1)\hat{R}^{(\cdot)},$$

where

$$\hat{R}^{(\cdot)} = \frac{\Sigma_j \hat{R}^{(j)}}{n}.$$

That formula is correct for a simple random sample (with replacement) but not for a stratified sample. The formula (Wolter (1985) formula 4.5.2) for a stratified sample (with replacement) is

$$\hat{R}_{JK} = (n + 1 - H)\hat{R} - \sum_h (n_h - 1)\hat{R}^{(h.)}$$

where

$$\hat{R}^{(h.)} = \frac{\Sigma_k \hat{R}^{(hk)}}{n_h}.$$

Andersson and his co-authors showed the "jackknife" to be consistently positively biased with the worst mean square error of any of the methods studied. This is, of course, what would be expected if stratification is ignored for variance estimation. Their results would have been much different if they had used the correct version of the jackknife. Theoretical work has shown that for sufficiently smooth statistics (of which the ratio is one), $\hat{V}_{JK1}$ is unbiased to third order moments (Jones (1974) and Dippo (1981)). Furthermore, a number of other Monte Carlo studies have established that the jackknife behaves very similarly to linearization estimators for variances of ratios (most recently Kovar, Rao, and Wu (1988)).

References

Andersson, C., Forsman, G., and Wretman, J. (1987): Estimating the Variance of a Complex Statistic: A Monte Carlo Study of Some Approximate Techniques. Journal of Official Statistics, 3, pp. 251–265.

Dippo, C.S. (1981): Variance for Nonlinear Estimators Based upon Stratified Samples from Finite Populations. Unpublished Ph.D. dissertation, George Washington University (Washington, DC).

Jones, H.L. (1974): Jackknife Estimation of Functions of Stratum Means. Biometrika, 61, pp. 343–348.

Kovar, J.G., Rao, J.N.K., and Wu, C.F.J. (1988): Bootstrap and Other Methods to Measure Errors in Survey Estimates. The Canadian Journal of Statistics 16, (Supplement) pp. 25–46.

Wolter, K.M. (1985): Introduction to Variance Estimation. New York: Springer-Verlag.

Sincerely,

David Judkins
Westat, Inc.
1650 Research Blvd.
Rockville, MD 20850-3129
U.S.A.

Reply

The jackknife estimator of variance that we used was of the type given by formula (4.3.8) in Wolter (1985) for the general case of unequal probability sampling without replacement.

Later in his book, Wolter says (pp. 174–175) that "the jackknife runs into some difficulty in the context of stratified sampling" and that one "should be especially careful not to apply the classical jackknife estimators . . . to stratified sampling problems." He then presents special jackknife estimators of variance that perform better than the classical one under stratified sampling.

In light of Wolter's book (which was published after we had conducted our study) and the interesting comments by David Judkins, it is clear that under stratified sampling there are special jackknife estimators of variance that are better than the one we used.

It is an interesting fact that the *general* jackknife variance estimator, given for a varying probability sampling design, fails for the special case of stratified sampling.

Reference

Wolter, K.M. (1985): Introduction to Variance Estimation. Springer-Verlag, New York.

Claes Andersson
Gösta Forsman
Jan Wretman