Journal of Official Statistics
Vol. 10. No. 1, 1994, pp. 93–102

Miscellanea

Under the heading Miscellanea, essays will be published dealing with topics considered to be of general interest to the readers. All contributions will be refereed for their compatibility with this criterion.

Electronic Data Collection at the National Center for Education Statistics: Successes in the Collection of Library Data

Carrol Benner Kindel[1]

Abstract: The U.S. National Center for Education Statistics (NCES) collects and disseminates statistical information on public, academic (i.e., college and university), and elementary and secondary school libraries. Nationwide public and academic library statistics are collected by state data coordinators annually for public libraries and biennially for academic libraries using PC software packages. This software, called DECTOP for the public library data collection and IDEALS for the academic library data collection, uses Borland's (Ashton-Tate's) dBASE IV and Nantucket's Clipper data base application. In DECTOP, all data collection and processing are done through the software system. Entry of the nearly forty data elements is accomplished through two simple data entry screens. DECTOP also offers respondents several different ways to import and convert data from their state dBASE, Lotus 1-2-3, and ASCII files. Currently, IDEALS allows coordinators in each state to send data to NCES on hard copy, or on diskette. Starting with the 1992 survey, the academic library data will be collected by the U.S. Census Bureau as part of the larger data collection effort on the Integrated Postsecondary Education Data System (IPEDS).

Key words: Survey; processing; diskette; PC software; methods.

1. Introduction

Public library data were published by the U.S. Office of Education as long ago as 1876. However, the establishment of a regular, ongoing program of statistical data collection is a recent phenomenon, dating, essentially, from a 1988 law requiring a Federal-State Cooperative System (FSCS) for collecting public library data.

There was great interest in exploring the use of electronic technology, particularly floppy diskettes, for data collection purposes. Unfortunately, tested prototypes

[1] National Center for Education Statistics, 555 New Jersey Ave. NW., Washington, D.C. 20208, U.S.A.

were not available in the mid-1980s. For example, the U.S. Department of Energy's Petroleum Electronic Data Reporting System was also under development at that time. "Electronic technology" usually meant sending computer *tapes* through the mail from respondent to data collector.

State-level personnel were enthusiastic about the use of diskettes, which had advantages over computer tapes in being easily transferable and widely usable through personal computers. Building on this interest, the National Center for Education Statistics (NCES) initiated the development of an electronic diskette data collection system, and also supplied personal computer equipment to states for this effort.

NCES's first academic library survey was conducted in 1966. Initially, data were collected at irregular intervals, but, since the late 1980s, collection has been biennially.

Based on user acceptance of the new public library diskette reporting system, a similar system was introduced into the academic data program in 1990. The system was basically the same, but modifications were needed to reflect the differences in data needs in academic versus public library communities.

This paper will describe experience with both DECTOP, the public library system, and IDEALS, the academic library system. Its purpose is to share information about the operations of these systems to increase our common knowledge about electronic data collection methods. Sections 2 and 3 of this paper describe the systems.

A second purpose of this paper is to inform decisions about whether or not to adopt similar methods to other surveys. For this purpose, one would want to know how use of an electronic system affects overall data collection operations and the quality of the resulting data. Since the new public library data collection started fresh with DECTOP, there are no "before and after" data for comparison. However, there are indicators of the system performance. The situation for academic libraries was different. IDEALS overlaid a hardcopy data collection system already in place. Therefore, electronic-nonelectronic comparisons are possible. These performance data are provided in Section 4.

2. Public Library Data Collection

2.1. Introduction

The purpose of the public library data collection is to provide the national census of public libraries. Its users include offices of the federal government for planning and developing national programs, professional organizations for developing informed policy positions, and businesses and researchers to reach conclusions about the state of librarianship and to improve practice. In addition, as responsibility for library service is decentralized in the United States, state governments are important data users. States report that data are used for planning and evaluating the condition and progress of public libraries, determining the effect of federal and state revenues and legislation, and developing and justifying budget requests. States often combine library data with demographic or economic data to complete the statewide picture of library service.

For the public library census, 50 states and the District of Columbia report data to NCES on over 9,000 public libraries. Thirty-nine data items are collected annually on library staffing, revenues, expenditures, size of collection, circulation, hours open, transactions, interlibrary loans, population of the library's legal service area, and library service outlets.

These data items were identified by the pilot studies conducted in the mid-1980s, as being available from administrative records in every state. Provision was also made for amendments to data items over time, with the consent of the states. Determining the content of the public library census remains, therefore, a truly cooperative activity.

Two components contribute to the functioning of the public library census: the network of federal and state personnel who collect and process the data, and the electronic system that facilitates it. Both are described below.

2.2. Organization

The census is collected through a network of state library agency personnel and contractors, and it is processed and published by NCES staff and contractors. These individuals, together with staff from the National Commission on Libraries and Information Science (NCLIS), the American Library Association (ALA), and the Chief Officers of State Library Agencies (COSLA), comprise the Federal-State Cooperative System for public library data (FSCS).

FSCS meets once a year at a training workshop, but works through a steering committee on issues of data collection feasibility, survey content, and non-response follow-up. The steering committee and its technical subcommittee have also been instrumental in the development, revision, and ongoing testing of the DECTOP software.

2.3. Data collection and processing system

The electronic data collection system for public libraries is known as DECTOP, Data Entry Conversion Table Output Program. DECTOP is a PC software package which uses Borland's (Ashton-Tate's) dBASE IV and Nantucket's Clipper data base applications. It is menu driven and there are numerous pop-up windows within the various functions.

Early in the calendar year, NCES mails to each state's FSCS data coordinator two floppy diskettes, a user's guide, and a cover letter. One diskette contains the state's data files, including prior year data, and a "template" file for current data, containing records for each of the state's libraries which are blank except for identifiers. The other diskette holds the program software.

Data for DECTOP, and other state purposes, are collected by states from their local public libraries. Data thus reported by local libraries are maintained by states in hardcopy and electronic files. When the DECTOP request arrives each year, state data coordinators provide the requested data from those files.

Data may be entered into DECTOP in two ways: via keyboard, or imported from another electronic file. States enter the data, edit them on the basis of both consistency and previous values, correct individual records as needed, and send diskettes and copies of error listings/files to NCES.

As each state's diskettes are received in NCES, the software edits are rerun. Any additional errors are followed up on a flow basis. States are asked to indicate corrections in hardcopy for input by NCES in order to facilitate response and to avoid inadvertent changes to correct data already on the file. States requiring no edit follow-up are simply mailed a thank you letter.

After all data are received and the edit follow-up completed, state files are merged and national summary tables created for publication purposes. DECTOP software is also used for these purposes. Prior to publication, tables run by NCES are shared with each state, and any errors surfacing at that time are corrected. As another data quality step, unit and item nonresponse are analyzed annually by the FSCS.

Primary data dissemination is handled by NCES. There are three basic formats: a hardcopy publication containing a brief narrative and two-way tables; the data file of microrecords on floppy diskette; and a version of the hardcopy publication released via electronic bulletin board. These releases are provided free to FSCS state data coordinators. In addition, some 11,000 public libraries, the states, and professional organizations receive free copies of the publication. A few thousand additional users receive announcements of data availability, with copies of the publication and data files available from the federal government for a nominal fee. Secondary dissemination in the form of other publications and articles is encouraged by NCES.

3. Academic Library Data Collection

3.1. Introduction

The purpose of the academic library data collection is to provide current and trend data on the status of academic libraries. In addition to federal and state users, academic institutions and accreditation panels use these data for evaluating postsecondary education institutions and establishing standards, and for institution-level planning, budgeting, and benchmarking among similar institutions. The popularity of comparisons called "peer analysis," has spawned development of several software packages that automatically select peer group institutions from the academic library data base.

Libraries in all accredited higher education institutions, including four-year and less than four-year institutions, are generally in scope. Representatives in the 50 states and the District of Columbia are usually the collection agents for the approximately 3,500–4,000 academic libraries within their jurisdictions, though there is some variation in this arrangement, as explained below.

Data are provided on operating expenditures, staffing, service outlets, total volumes, circulation, interlibrary loans, public service hours, attendance and reference transactions. The need for these data items goes back many years. They have been used traditionally to document the condition of academic libraries, and are available in institutional records. New data items reflecting changes in library functions, for example, as the result of technology, are reviewed as part of the ongoing academic library data improvement project, discussed below.

The data are collected as a biennial supplement to the annual census of postsecondary institutions, the Integrated Postsecondary Education Data System (IPEDS). As part of the IPEDS, academic library data had been collected on hardcopy forms. However, in 1990, an electronic system was introduced, solely for academic library data. At that time, also, development of a network of library representatives, also described below, was initiated. As with public libraries, both the network and the electronic system are integral to academic library data collection operations.

3.2. Organization

In 1990 the American Library Association proposed a plan for the improvement of quality and timeliness of the academic library data. The proposal was for a multiyear effort, and included forming a working committee to improve communications with the community, provide input to data content decisions, review survey results, and facilitate data collection and response. An outgrowth of the plan was development of a network of library representatives. Although the network is not yet fully operational, the idea is that a representative in each state would coordinate within-state data collection from individual academic libraries, provide edited data to NCES, and be responsible for nonresponse follow-up.

In this early stage, network participants are still being recruited by the library community. Where there is no state representative or only partial representation, data collection is direct, through the postsecondary institutions and academic libraries. Library representatives, themselves, take varying amounts of responsibility from simply turning over individual institution data to NCES, to editing data and actively following up nonresponse.

3.3. Data collection and processing system

The electronic data collection system for academic libraries is called IDEALS, Input and Data Editing for Academic Library Statistics, and, like DECTOP, was developed by MTL.

As mentioned above, IDEALS was first introduced in 1990. States were accustomed to reporting on hard copy through IPEDS, so a hardcopy alternative was offered. NECS mailed hardcopy forms designated for each library to the library representative or direct to the postsecondary institution as appropriate. If data collection was coordinated at the state level, library representatives received diskettes for entry of individual library data, much like FSCS, but mailed hardcopy forms to the institutions. Institutions reported back to the representatives (or to NCES where there was no representative) using the forms. Representatives had the option of entering those data on diskettes, or returning the hardcopy forms to NCES.

Once at NCES, data from hardcopy forms were entered into IDEALS. Diskettes were checked for valid entries. State files were then merged and the records matched against the IPEDS file universe of postsecondary institutions. The universe of academic libraries was then determined, duplicate records and out-of-scopes identified. Unit and item nonresponse were followed up. The merged data were edited and imputed based both on prior year information and characteristics from the IPEDS data for the academic institution.

Beginning with the 1992/93 data collection, the U.S. Census Bureau became NCES's data collection agent for the IPEDS system, including the biennial academic library survey. Mail out, check in, follow-up and processing are now conducted by the Census Bureau through the library representatives as described above.

Primary data dissemination is, again, handled by NCES in the same basic formats as those for the public library data collection. Characteristics of the parent institution, collected by IPEDS, are included on the academic library diskette for context. In addition to targetted free copies, the publication and diskettes are available from the federal government for a nominal fee.

4. System Performance

The foregoing sections describe the functioning of these data collection systems. Electronic methodology contributes to the overall data collection efforts in that software should be user friendly and facilitate data collection operations. It should also help identify data quality problems, such as nonresponse and edit errors, so that the user can make corrections before submitting the data.

IDEALS has been operational only for one year, yet 38 of 50 states and the District of Columbia used the software on the first implementation. These 38 states and the District of Columbia accounted for 55% of the libraries, while the 12 hardcopy states accounted for 45% of the libraries on the file. Initial DECTOP use was somewhat higher, 44 states in 1989, the first year of implementation. Since 1990, all 50 states and the District of Columbia have used DECTOP.

IDEALS acceptability seems good for a first attempt, but lags behind DECTOP. This might be expected for three reasons. IDEALS supplanted an existing hardcopy system, so acceptability may be slower. IDEALS users are a mixed group of academics, state personnel, and other interested persons. DECTOP users, through FSCS, have become an established network whose members rely on each other for support and assistance. Lastly, IDEALS users' computer arrangements vary, whereas, equipment was supplied for DECTOP.

4.1. Unit nonresponse

An important part of a state coordinator's work is identifying and tracking nonresponse so that it can be followed up and, converted. Table 1 below shows unit nonresponse for three surveys. Data for the 1990 Academic Library Survey are preliminary pending final universe coverage analysis. For academic libraries, there was little change in nonresponse from the 1988 (all hard copy) survey to the 1990 (partial electronic) survey. Within the 1990 survey, however, there is a considerable difference between hardcopy and diskette states. The public library (all electronic) nonresponse rate is very low. Unfortunately, unit nonresponse is not reported for the 1990 public library survey.

One factor accounting for the above differences might be the size of the state in terms of the number of libraries, and therefore the number of cases the state data coordinator must handle. The largest states might, therefore, have the highest nonresponse. On the other hand, if the largest states are automated, and report electronically, they may be more aware of response problems and take action to reduce them. This possibility is best investigated with 1990 academic library data.

The largest academic library states are defined here to be those with more than 100 libraries. There are eight such states: four report on hardcopy and four on diskette. Interestingly, hardcopy states are generally larger than diskette states, with

Table 1. Unit nonresponse for academic and public library surveys

Survey	Nonresponse rate (libraries)	
	Percentage	Number
1988 Academic	14.5	499
1990 Academic Total	14.2	463
Hard copy States (45% of the libraries)	9.2	301
Diskette States (55% of the libraries)	5.0	162
1991 Public	0.8	68

Table 2. Unit nonresponse for eight largest versus smaller states on the 1990 academic libraries survey

Survey component	Nonresponse rate (libraries)	
	Percentage	Number
Eight Largest States (46% of the libraries)	8.1	264
Smaller States (54% of the libraries)	6.1	199

241, 194, 162, and 146, for a total of 743 libraries. The diskette states had 153, 116, 109, and 104, for a total of 482 libraries.

These issues are illustrated in Tables 2 and 3. Table 2 shows differences by size of state only for the 1990 survey of academic libraries. The eight largest states account for more nonresponse than the remaining (smaller) states.

Looking at the eight largest states, Table 3 shows nonresponse by methodology. Large states that use hardcopy had considerably higher unit nonresponse than large states that use diskettes.

4.2. *Item nonresponse*

Table 4 shows item nonresponse for four critical data items which appear in both academic and public library surveys.

The 1988 academic survey had the lowest or nearly the lowest item nonresponse rates of all surveys. In 1990, the year of transition to electronic collection, response for circulation improved slightly; response for volumes was about the same as the 1988 survey. For public libraries, item response is better for some items and worse for others in the first year of electronic implementation (1989), than in the second year (1990). By 1991, item response is considerably improved, approaching the level for academic, a mixture of hardcopy and electronic.

The 1990 academic library survey provides the better indication of any hardcopy versus electronic differences, see Table 5 below.

Diskette item nonresponse was uniformly higher than hardcopy item nonresponse, although item nonresponse on these four items was overall very low.

For the 1991 public library census, data on item nonresponse may be further analyzed by large versus small states. The ratio of public libraries to academic libraries is about 3 to 1. So, for purposes of this discussion, states having more than 300 public libraries were considered large public library states. There were 10 of these states; two of them were high outliers (760 and 603 libraries). These 10 states account for 51% of the libraries.

Table 6 shows that though nonresponse for these items was low overall, it was uniformly higher for small states than for large ones. The stability of the numbers for all four items may also indicate that smaller states did, in fact, have more problems providing data.

Table 3. Unit nonresponse for largest states within each methodology on the 1990 academic library survey

Methodology	Nonresponse Rate (libraries)	
	Percentage	Number
Hard copy	22.6	217
Diskette	8.9	47

Table 4. Item nonresponse for four critical data items by survey

Item	Survey				
	88 Acad.	90 Acad.	89 Public	90 Public	91 Public
Staff	—	1.1% (32)	6.2% (487)	4.6% (411)	2.0% (174)
Operating Expense	—	0.5% (15)	3.8% (299)	4.2% (381)	1.9% (165)
Volumes	1.1% (31)	1.3% (37)	5.8% (460)	4.7% (425)	2.2% (199)
Circulation	2.1% (62)	1.3% (36)	4.2% (333)	5.0% (447)	2.4% (214)

4.3. System operations and quality control

While there are no additional performance data on the academic library survey, there are additional data on the 1991 public library census which provide insights in evaluating system operations and quality control.

One measure might be timeliness of data submissions. Table 7 shows timing for various data collection and processing operations for the 1991 public library census.

The quick response to the edit follow-up and table reviews was encouraging as states had not been asked to participate in these operations in earlier years. Though states were not asked to respond electronically in either operation, certainly NCES data processing turnaround time and the format of the edit follow-up were facilitated by the electronic system.

Response to the initial mailing was slower than expected, however, with data collection exceeding the July deadline by four months. This remains somewhat unexplained, as the initial 1991 mail out occurred even earlier than in 1990. Speculation is that reductions in State Library Agency personnel may have delayed response in some states.

Another way to assess the effect of the electronic system on data collection quality control is by comparing item nonresponse before and after the final edits,

Table 5. Item nonresponse for hardcopy versus diskette states in the 1990 academic library survey

Item	Methodology	
	Hard copy	Diskette
Staff	0.04% (1)	1.1% (31)
Operating Expense	0.21% (6)	0.32% (9)
Volumes	0.32% (9)	1.0% (28)
Circulation	0.53% (15)	0.75% (21)

Table 6. Item nonresponse for large versus small states on the 1991 public library census

Item	Large states	Small states
Staff	0.7% (57)	1.3% (117)
Operating Expense	0.6% (51)	1.3% (114)
Volumes	0.6% (55)	1.6% (144)
Circulation	0.7% (62)	1.7% (155)

Table 7. Receipt control and edit status summary for the 1991 public library census

Timeliness	Initial data	Number of states received: Edit followup	Number of states received: Table review
July 31 deadline (or within one month of mailout)	26	27 (August mailout)	27 (Dec. mailout)
One month late	16	9	3
More than one month late	9	4	0
Total states*	51	40	30

*Eleven states required no edit follow-up. States repond to the table review only when they have changes.

again for the four critical data items. The "before" data have been through the initial round of software edits by both state and NCES personnel (Table 7). Resulting corrections have been incorporated so that, theoretically, all errors have been caught. The final edit consisted of reviewing each state's data and aggregate U.S. data, both in tabular form, for outliers and inconsistencies. This indicator is shown for the total file in Table 8 and for large and small states in Table 9.

The item nonresponse for these four critical items was low overall in 1991. But the tables show that there was some change in the item nonresponse before and after the final edit and review of the data, Table 8, and that the change occurred entirely in the smaller states, Table 9. There was more item nonresponse after final edit review than before it.

Table 8. Item nonresponse prior to and after final data review

Item	Prior to final review	After final review
Staff	1.8% (154)	2.0% (174)
Operating Expense	1.5% (133)	1.9% (165)
Volumes	1.9% (164)	2.2% (199)
Circulation	2.1% (182)	2.4% (217)

This finding may be due to a number of causes. First, "0's" are treated as responses in the census. NA's are indicated by "−1's." Several states confused the use of these two characters, so that substituting −1 for 0 raised the item nonresponse rate. Another possibility is that unit nonresponse was reduced from 86 cases originally to 68. Only four of the final nonresponding libraries were in large states, and the 18 converted cases were all in small states. Some of these converted nonrespondents may have contained proportionally higher item nonresponse.

5. Summary

We found that for academic libraries, large states and hardcopy states have higher unit nonresponse than diskette states and small states, as illustrated by Tables 1–3. This suggests that unit nonresponse may be easier for state coordinators to manage in small states and in diskette states than in hardcopy and large states. This also highlights the contribution software can make to

Table 9. Item nonresponse prior to and after final data review for large versus small states

Item	Prior to final review		After final review	
	Large states	Small states	Large states	Small states
Staff	0.7% (57)	1.1% (97)	0.7% (57)	1.3% (117)
Operating Expense	0.6% (51)	0.9% (82)	0.6% (51)	1.3% (114)
Volumes	0.6% (55)	1.3% (109)	0.6% (55)	1.6% (144)
Circulation	0.7% (62)	1.4% (120)	0.7% (62)	1.7% (155)

controlling nonresponse in large states (Table 3).

The findings on item nonresponse, Tables 4–6, are contradictory. Tables 4 and 5 show public library item nonresponse (entirely diskette) is higher than academic library nonresponse. In 1990, the mixed year for academic library methodology, diskette states also had higher item nonresponse than hardcopy states. Additionally, item nonresponse seems to be declining for public libraries, as the electronic methodology continues in use. Table 6 shows that, in the fully-automated public library census, item nonresponse is somewhat higher for small than for large states.

Concerning timeliness, Table 7 shows that the initial data submissions of nine of the states were more than one month late. As mentioned above, this may be at least partially due to reductions in state agency staff.

The finding in Table 9 complements the size-of-state finding on item nonresponse. Small states required more "last minute" changes than large ones and these changes had the effect of increasing the item nonresponse.

Further analysis of the relationships between types of respondents, methodologies, and size of state may provide additional insights.

Received June 1992
Revised September 1993

Journal of Official Statistics
Vol. 10. No. 1, 1994, pp. 103–108

Letter to the Editor

Letters to the Editor will be confined to discussion of papers which have appeared in the Journal of Official Statistics and of important issues facing the statistical community.

A New Functional Form for Price Indexes' Elementary Aggregates

Jörgen Dalén's paper on "Computing Elementary Aggregates in the Swedish Consumer Price Index" (Journal of Official Statistics, Vol. 8, no. 2, 1992, pp. 129–147) provides important theoretical and empirical results on the construction of the basic components of consumer price indexes. Two especially noteworthy contributions are a new index number test and a new index number functional form that is now used by Statistics Sweden for aggregating outlets' prices to form the basic components of its Consumer Price Index (CPI). The purpose of this note is to propose a modification of Dalén's new price index that incorporates a logarithmic mean in lieu of an arithmetic mean. This modification offers two advantages: (1) Dalén's new index number test is satisfied exactly rather than just approximately; and (2) the geometric mean index is approximated more closely than by the index introduced by Dalén. This second advantage is especially noteworthy because Dalén's index itself approximates the geometric mean index better than other alternatives, including one resembling a Fisher's ideal index.

Dalén's new index number test, called the permutation test, is motivated by the observed tendency of outlets' prices for a good to oscillate. The second version of this test, which Dalén (p. 138) suggests is "enough for practical purposes," concerns the behavior of the functional form for the basic component price index when every price receives the same weight. It requires the index to equal 1 whenever the outlets in the index collectively offer the same set of prices, even if the identity of the outlet offering each particular price changes. This test, as well as other tests discussed by Dalén, is satisfied by the geometric mean index

$$G_{0t} = \exp\left\{\sum w_i \log(r_i)\right\}$$

where the w_i, which sum to 1, are outlets' weights in the basic component index and $r_i \equiv p_{it}/p_{i0}$.

The disadvantage of the geometric mean index is its dissimilarity to the Edgeworth index that Statistics Sweden uses to form higher level CPI aggregates. Dalén's new index provides for a unified approach at all levels of aggregation yet behaves quite similarly to a geometric mean index. It is

$$RA_{0t} = \frac{\sum w_i p_{it}/a(p_{i0}, p_{it})}{\sum w_i p_{i0}/a(p_{i0}, p_{it})}$$

where $a(p_{it}, p_{i0})$ denotes the arithmetic mean $(p_{it} + p_{i0})/2$. Ideally, the w_i represent outlets' shares of consumers' expenditure on the good in question over the period from time 0 to time t. If so, for any arbitrary outlet i, $w_i/a(p_{i0}, p_{it})$ is proportional to i's estimated quantity during the period from 0 to t calculated as the ratio of i's sales to consumers during that period to an estimate of i's average price during that period.

Assuming that prices have constant growth rates between times 0 and t implies a different estimator for the average price over the interval $[0, t]$. Let $p_{is} = p_{i0}e^{\rho_i s}$ for any $s \in [0, t]$. Then (assuming that $\rho_i \neq 0$), the average price during the interval $[0, t]$ is $\int_0^t (1/t) p_{i0} e^{\rho_i s} ds = (p_{it} - p_{i0})/[\ln(p_{it}) - \ln(p_{i0})]$, which is the logarithmic or "Vartia" mean, denoted $v(p_{i0}, p_{it})$. This mean may be thought of as lying two-thirds of the way from an arithmetic mean to a geometric mean because $v(p_{i0}, p_{it}) \approx [a({p_{i0}}^{1/3}, {p_{it}}^{1/3})]^3$, while the geometric mean $g(p_{i0}, p_{it}) = \lim_{\epsilon \to 0}[a({p_{i0}}^{\epsilon}, {p_{it}}^{\epsilon})]^{1/\epsilon}$, or $\exp\{a[\log(p_{i0}), \log(p_{it})]\}$. In fact, before the role of the logarithmic mean in ideal log-change index numbers was discovered, both $\frac{2}{3}g(p_{i0}, p_{it}) + \frac{1}{3}a(p_{i0}, p_{it})$ and $[g(p_{i0}, p_{it})]^{2/3} \times [a(p_{i0}, p_{it})]^{1/3}$ were suggested as approximations to it.

Replacing $a(p_t, p_0)$ in the formula for RA_{0t} by the average price during $[0, t]$ under the constant growth rate assumption gives

$$RV_{0t} = \frac{\sum w_i p_{it}/v(p_{i0}, p_{it})}{\sum w_i p_{i0}/v(p_{i0}, p_{it})} = 1 + \frac{\sum w_i \log(r_i)}{\sum w_i/v(r_i, 1)}.$$

Since RV_{0t} equals 1 whenever G_{0t} equals 1, it satisfies the permutation test exactly. Furthermore, Dalén notes that under the common condition of right skew in the distribution of the r_i, RA_{0t} has a slight downward bias compared with G_{0t}. In contrast, for the range of price changes that generally enter price indexes, RV_{0t} will be numerically indistinguishable from G_{0t}. For example, suppose that ten outlets furnish price quotes for a good, nine of which report no price change. If the tenth outlet reports an outlier of $r_{10} = 2$, then, rounding to the nearest thousandth, $G_{0t} = RV_{0t} = 1.072$, while $RA_{0t} = 1.069$. If $r_{10} = 10$ – an extreme outlier that would certainly be rejected for use in the U.S. CPI – then $G_{0t} = 1.259$, $RV_{0t} = 1.249$ and $RA_{0t} = 1.178$.

Letting $\mathbf{r}$ denote the vector of the r_i, $G(\mathbf{r})$, $RV(\mathbf{r})$, and $RA(\mathbf{r})$ have identical first and second derivatives at the point $\mathbf{r} = \mathbf{1}$. Dalén reports the third derivatives for $G(\mathbf{1})$ and $RA(\mathbf{1})$, though a minus sign appears to have been inadvertently inserted in front of his expression for $RA'''_{iij}(\mathbf{1})$. The third derivatives of $RV(\mathbf{1})$ are derived in the mathematical appendix. Table 1 displays all three sets of third derivatives.

Because $RV'''_{iii}(\mathbf{1})$ differs less from $G'''_{iii}(\mathbf{1})$ than does $RA'''_{iii}(\mathbf{1})$, right skew in $\mathbf{r}$ causes less discrepancy between $RV(\mathbf{r})$ and $G(\mathbf{r})$ than between $RA(\mathbf{r})$ and $G(\mathbf{r})$. Furthermore, unless $w_i \leq \frac{1}{6}$ (in which case both derivatives are near zero), $RV'''_{iij}(\mathbf{1})$ is closer to $G'''_{iij}(\mathbf{1})$ than is $RA'''_{iij}(\mathbf{1})$. The equality or near-equality of the first three derivatives of $RV(\cdot)$ and $G(\cdot)$ at $\mathbf{r} = \mathbf{1}$ accounts for the tendency for $G(\mathbf{r})$ and $RV(\mathbf{r})$ to agree precisely when expressed to three decimal places if the price changes in $\mathbf{r}$ are moderate.

A final noteworthy consequence of the similarity between $RV(\mathbf{r})$ and $G(\mathbf{r})$ is an expression in price-quantity form for

Table 1. Third derivatives of $G(\cdot)$, $RA(\cdot)$ and $RV(\cdot)$

	$G'''_{ijk}(\mathbf{1})$	$RA'''_{ijk}(\mathbf{1})$	$RV'''_{ijk}(\mathbf{1})$
$k = j = i$	$w_i(1 - w_i)(2 - w_i)$	$1.5w_i(1 - w_i)^2$	$w_i(1 - w_i)(2 - 1.5w_i)$
$k = i; j \neq i$	$-w_iw_j(1 - w_i)$	$-w_iw_j(1 - 1.5w_i)$	$-w_iw_j(\frac{7}{6} - 1.5w_i)$
$k \neq j \neq i$	$w_iw_jw_k$	$1.5w_iw_jw_k$	$1.5w_iw_jw_k.$

the Törnqvist price index $T(\mathbf{w}_0, \mathbf{w}_t, \mathbf{r}) \equiv \exp[\sum a(w_{i0}, w_{it}) \log(r_i)]$, where $\mathbf{w}_0$ and $\mathbf{w}_t$ are vectors of *good's* expenditures shares at times 0 and t. Let $\bar{q}_i = a(w_{i0}, w_{it})/v(p_{i0}, p_{it})$. Although the $\bar{q}_i$ are deflated expenditure *shares,* rescaling them by a measure of average expenditures would not change the value of the approximation. It is

$$T(\mathbf{w}_0, \mathbf{w}_t, \mathbf{r}) = \frac{\sum p_{it}\bar{q}_i}{\sum p_{i0}\bar{q}_i}.$$

If consumers' expenditures on goods are constant, as would occur with constant budgets and Cobb-Douglas preferences, then $\bar{q}_i$ is proportional to $[v(1/q_{i0}, 1/q_{it})]^{-1}$. Assuming that prices (and, therefore, quantities) change at a constant rate in the interval $[0, t]$, this, in turn, equals a harmonic mean of the quantities of good i that consumers buy at various times between time 0 and time t.

Marshall Reinsdorf
U.S. Bureau of Labor Statistics
Washington, D.C. 20212
U.S.A.

Appendix

In order to find the third derivatives at $\mathbf{r} = \mathbf{1}$ of $RV(\mathbf{r}) = 1 + [\Sigma w_i \log r_i]/[\Sigma w_i (\log r_i)/(r_i - 1)]$, it is useful to know the derivatives of $(\log r_i)/(r_i - 1) \equiv u(r_i)$ at $r_i = 1$. To find these derivatives, define $x(r_i)$ as $(r_i - 1)/(r_i + 1)$. In evaluating the derivatives of $u(\cdot)$ it is convenient to suppress the i subscripts.

Since $r = (1 + x)/(1 - x)$

$$\log r = \log \frac{1 + x}{1 - x}.$$

A Taylor series expansion for log $[(1 + x)/ (1 - x)$ is $2(x + x^3/3 + x^5/5 + \cdots)]$ where $x < 1$ since $r > 0$. Furthermore, $r - 1 = 2x/(1 - x)$. Hence

$$u(r) = f(x(r))$$

where

$$f(x) \equiv (1 - x)\left(1 + \frac{x^2}{3} + \frac{x^4}{5} + \cdots\right).$$

We then have

$$\begin{aligned} u'(r) &= f'(x(r))x'(r) \\ u''(r) &= f''(x(r))[x'(r)]^2 + f'(x(r))x''(r) \\ u'''(r) &= f'''(x(r))[x'(r)]^3 \\ &\quad + 3f''(x(r))x''(r)x'(r) \\ &\quad + f'(x(r))x'''(r). \end{aligned}$$

Evaluating the derivatives of $f(x)$ at $x = 0$ gives

$$\begin{aligned} f'(x) &= -\left(1 + \frac{x^2}{3} + \frac{x^4}{5} + \cdots\right) \\ &\quad + (1 - x)(\tfrac{2}{3}x + \tfrac{4}{5}x^3 + \cdots) = -1 \\ f''(x) &= -2(\tfrac{2}{3}x + \tfrac{4}{5}x^3 + \cdots) \\ &\quad + (1 - x)(\tfrac{2}{3} + \tfrac{12}{5}x^2 + \cdots) = \tfrac{2}{3} \\ f'''(x) &= -3(\tfrac{2}{3} + \tfrac{12}{5}x^2 + \cdots) \\ &\quad + (1 - x)(\tfrac{24}{5}x + \tfrac{120}{7}x^3 + \cdots) = -2. \end{aligned}$$

Evaluating the derivatives of $x(r)$ at $r = 1$ gives

$$x'(r) = 2(r+1)^{-2} = \tfrac{1}{2}$$
$$x''(r) = 4(r+1)^{-3} = -\tfrac{1}{2}$$
$$x'''(r) = 12(r+1)^{-4} = \tfrac{3}{4}.$$

Substituting these values into the expressions for the derivatives of $u(\cdot)$ gives

$$u'(1) = (-1)(\tfrac{1}{2}) = -\tfrac{1}{2}$$
$$u''(1) = (\tfrac{2}{3})(\tfrac{1}{2})^2 + (\tfrac{1}{2}) = \tfrac{2}{3}$$
$$u'''(1) = (-2)(\tfrac{1}{2})^3 + 3(\tfrac{2}{3})(-\tfrac{1}{2})(\tfrac{1}{2}) - (\tfrac{3}{4}) = -\tfrac{3}{2}.$$

With these results in hand, we now turn to the derivatives of $RV(\cdot)$. We have

$$RV(\mathbf{r}) = 1 + \frac{\Sigma w_i \log r_i}{\Sigma w_i u(r_i)}$$

$$\frac{\partial RV(\cdot)}{\partial r_i} = \frac{w_i/r_i}{\Sigma w_h u(r_h)} - \frac{[\Sigma w_h \log r_h] w_i u'(r_i)}{[\Sigma w_h u(r_h)]^2}$$

$$\frac{\partial^2 RV(\cdot)}{(\partial r_i)^2} = \frac{-w_i/r_i^2}{\Sigma w_h u(r_h)} - \frac{2(w_i^2/r_i)u'(r_i)}{[\Sigma w_h u(r_h)]^2} - \frac{[\Sigma w_h \log r_h] w_i u''(r_i)}{[\Sigma w_h u(r_h)]^2} + \frac{2[\Sigma w_h \log r_h][w_i u'(r_i)]^2}{[\Sigma w_h u(r_h)]^3}$$

$$\frac{\partial^2 RV(\cdot)}{\partial r_i \partial r_j} = -\frac{w_i w_j u'(r_j)/r_i}{[\Sigma w_h u(r_h)]^2} - \frac{w_i w_j u'(r_i)/r_j}{[\Sigma w_h u(r_h)]^2} + 2\frac{[\Sigma w_h \log r_h][w_i u'(r_i)][w_j u'(r_j)]}{[\Sigma w_h u(r_h)]^3}$$

$$\frac{\partial^3 RV(\cdot)}{(\partial r_i)^3} = \frac{2w_i/r_i^3}{\Sigma w_h u(r_h)} + \frac{(w_i/r_i)^2 u'(r_i)}{[\Sigma w_h u(r_h)]^2} + 2\frac{(w_i/r_i)^2 u'(r_i) - (w_i^2/r_i)u''(r_i)}{[\Sigma w_h u(r_h)]^2} + 4\frac{[w_i^3/r_i][u'(r_i)]^2}{[\Sigma w_h u(r_h)]^3} - \frac{w_i\{(w_i/r_i)u''(r_i) + [\Sigma w_h \log r_h]u'''(r_i)\}}{[\Sigma w_h u(r_h)]^2} + 2\frac{[\Sigma w_h \log r_h] w_i^2 u''(r_i) u'(r_i)}{[\sum w_h u(r_h)]^3} + 2\frac{w_i^3[u'(r_i)]^2/r_i + 2[\Sigma w_h \log r_h] w_i^2 u'(r_i) u''(r_i)]}{[\Sigma w_h u(r_h)]^3} - 6\frac{[\Sigma w_h \log r_h][w_i u'(r_i)]^3}{[\Sigma w_h u(r_h)]^4}.$$

Evaluating this expression at $\mathbf{r} = \mathbf{1}$ gives

$$2w_i + (-1/2)w_i^2 + 2w_i^2[(-1/2) - (2/3)] + 4w_i^3(1/4) - w_i^2(2/3) + 2w_i^3(1/4)$$
$$= (3/2)w_i^3 - (7/2)w_i^2 + 2w_i$$
$$= w_i(1 - w_i)(2 - \tfrac{3}{2}w_i).$$

We next evaluate $RV_{iij}(\mathbf{1})$. It is

$$\frac{\partial^3 RV(\cdot)}{(\partial r_i)^2 \partial r_j} = \frac{w_i w_j u'(r_j)/r_i^2}{[\Sigma w_h u(r_h)]^2} + 4\frac{(w_i^2/r_i) w_j u'(r_i) u'(r_j)}{[\Sigma w_h u(r_h)]^3} - \frac{w_i w_j u''(r_i)/r_j}{[\Sigma w_h u(r_h)]^2} + 2\frac{[\Sigma w_h \log r_h] w_i w_j u''(r_i) w_j u'(r_j)}{[\Sigma w_h u(r_h)]^3} - 6\frac{[\Sigma w_h \log r_h][w_i u'(r_i)]^2 w_j u'(r_j)}{[\Sigma w_h u(r_h)]^4} + 2\frac{[w_i u'(r_i)]^2 (w_j/r_j)}{[\Sigma w_h u(r_h)]^3}.$$

Evaluating this expression at $\mathbf{r} = \mathbf{1}$ gives

$$-\tfrac{1}{2}w_iw_j + w_i^2w_j - \tfrac{2}{3}w_iw_j + \tfrac{1}{2}w_i^2w_j$$
$$= \tfrac{3}{2}w_i^2w_j - \tfrac{7}{6}w_iw_j = w_iw_j(\tfrac{3}{2}w_i - \tfrac{7}{6}).$$

Finally, we evaluate the last derivative in Table 1. It is

$$\frac{\partial^3 RV(\cdot)}{\partial r_i \partial r_j \partial r_k} = 2\frac{w_iw_jw_ku'(r_j)u'(r_k)/r_i}{[\Sigma w_h u(r_h)]^3}$$
$$+ 2\frac{w_iw_jw_ku'(r_i)u'(r_k)/r_j}{[\Sigma w_h u(r_h)]^3}$$
$$+ 2\frac{[w_iu'(r_i)][w_ju'(r_j)][w_k/r_k]}{[\Sigma w_h u(r_h)]^3}$$
$$- 6\frac{[\Sigma w_h \log r_h][w_iu'(r_i)][w_ju'(r_j)][w_ku'(r_k)]}{[\Sigma w_h u(r_h)]^4}.$$

Evaluating this expression at $\mathbf{r} = \mathbf{1}$ gives $\tfrac{3}{2}w_iw_jw_k$.

References

Dalén, J. (1992). Computing Elementary Aggregates in the Swedish Consumer Price Index. Journal of Official Statistics, 8, 129–147.

Diewert, W.E. (1978). Superlative Index Numbers and Consistency in Aggregation. Econometrica, 46, 883–900.

Lorenzen, G. (1990). A Unified Approach to the Calculation of Growth Rates. The American Statistician, 44, 148–150.

Sato, K. (1974). Ideal Index Numbers that Almost Satisfy the Factor Reversal Test. Review of Economics and Statistics, 56, 549–552.

Sato, K. (1976). The Ideal Log-Change Index Number. Review of Economics and Statistics, 58, 223–228.

Theil, H. (1973). A New Index Number Formula. Review of Economics and Statistics, 55, 498–502.

Törnqvist, L., Vartia, P., and Vartia, Y.O. (1985). How Should Relative Changes Be Measured? The American Statistician, 39, 43–46.

Vartia, Y.O. (1976). Ideal Log-Change Index Numbers. Scandinavian Journal of Statistics, 3, 121–126.

Reply

I thank Marshall Reinsdorf for his many insightful and enlightening comments on my article. His letter is an indication of the increasing attention given by statistical agencies to the effects of different methods for computing elementary aggregates in consumer price indices.

I have a few detailed comments to make:

1. Reinsdorf's *RV* index satisfies the weak (VI-B in my article) but not the strong (VI) version of the permutation test.

2. Reinsdorf correctly points out an erroneous minus sign in one of my formulas. This error does not, however, influence my main result 10.2.3, where I have used the correct expression as given by Reinsdorf.

3. In 10.2.3 in my article I give the approximate expression

$$G - RA = \gamma/12 + (\mu - 1)\sigma^2/4.$$

A similar expression could be given for *RV*: $G - RV \approx (\mu - 1)\sigma^2/12$. Although this approximation is only valid for price ratios $\mathbf{r}$ sufficiently close to one, it confirms Reinsdorf's statement that *RV* is a closer approximation to *G* than is *RA*.

4. The *G* and *RA* formulas can be derived from well-known superlative index formulas (Törnqvist and Edgeworth, resp.) if weights are set equal. This seems not to be possible with the *RV* formula.

5. Reinsdorf's "fixed basket" approximation to the Törnqvist index is very

interesting, since the principal argument against using geometric means in price index computations seems to be that they could not be given a fixed basket interpretation. Reinsdorf's approximation shows the contrary. My own numerical experimentation indicates that this approximation is very good, but it still would be interesting to see a more elaborate mathematical explanation.

Jörgen Dalén
Statistics Sweden, Price Statistics
S-115 81 Stockholm
Sweden