Journal of Official Statistics
Vol. 5. No. 3, 1989, pp. 241–251

Modeling Childhood Mortality

Michael Hartmann[1]

Abstract: A new parametric model of child mortality is introduced and it is shown that it gives close fits to observed childhood survival functions. The model can be used for a variety of tasks including graduation or representation of childhood mortality and as an aid in indirect estimation of child mortality. The model is illustrated partly in terms of the survival function partly in terms of the mortality intensity.

Key words: Laws of mortality; parametric representation of child mortality; graduation; indirect estimation of child mortality.

1. Introduction

Attempts to model mortality by means of parametric functions can be traced back to the beginning of the 18th century (see, e.g., Hooker and Longley-Cook (1953) p. 161). For historic reasons such functions have become known as "laws of mortality." While most laws are partial in the sense that they only apply to a limited age range, a few laws cover all ages (see, e.g., Hartmann (1987)). Most partial laws of mortality apply to adult ages and only a few to childhood ages (see, e.g., Hartmann (1980), and Krishnamoorthy (1982)). The present paper gives a new law of mortality which applies to infant and childhood ages and which, among other things, can be used in a Brass-type process of indirect estimation of childhood mortality.

[1] Population Research Office, Statistics Sweden, 115 81 Stockholm, Sweden.

Acknowledgements: The author wishes to extend his sincere thanks to the editor and two anonymous referees for their comments on an earlier version of this paper.

Here s denotes the survival function so that $s(x)$ is the probability for a child born alive surviving to age x. The intensity μ (also known as the force of mortality or the hazard rate) then becomes $\mu(x) = -s'(x)/s(x)$. The probability for a person aged x to die before age $x + 1$ (the life table mortality rate) is $q_x = 1 - s(x + 1)/s(x)$.

From a practical point of view, an important reason for modeling the survival function for young ages arose with the advent of indirect, or Brass-type, estimation of childhood mortality in developing countries. Without going into a detailed discussion of Brass-type estimation of infant and childhood mortality (see, e.g., Brass (1975) pp. 50–55, and Brass and Coale (1968) pp. 104–122), the reason for creating a parametric model of infant and childhood mortality is that, subject to fairly light assumptions, it enhances the estimation process from the point of view of precision as well as from the point of view of computational flexibility (Hartmann (1982)).

In Brass estimation of infant and childhood mortality, the observational plan is one of utilizing retrospective reports from women concerning their ever born children and surviving children. Based on such reports, the proportion of deceased children, tabulated by age of women, can be used to infer the underlying survival function for children. Assuming stationary mortality and fertility as well as a uniform age-distribution of women, and by letting s denote the survival function for both sexes, and f the fertility function for women, the proportion of deceased children theoretically reported by women aged x is

$$H_x = \frac{\int_0^{x-a_0} f(x-a)\{1-s(a)\}\,da}{\int_0^{x-a_0} f(x-a)\,da}, \quad (1)$$

where a_0 denotes the starting age of fertility (see, e.g., Brass (1975) pp. 50–51). The basic idea behind Brass estimation of childhood mortality is that if the fertility function f in (1) can be estimated, or specified in advance, then corresponding to the observed statistic $\hat{H}_x$ it is possible to estimate a survival function $\hat{s}$ so that (1) is satisfied with H_x replaced by $\hat{H}_x$. The solution $\hat{s}$ is then accepted as an estimate of child mortality. The paper, however, does not discuss issues relating to Brass-type estimation of child mortality but merely discusses the new model of the survival function for childhood ages.

2. Theoretical Issues

In traditional parametric modeling of mortality, the focus has been on the intensity μ or the mortality risk q_x (see, e.g., Harper (1936), Hartmann (1980), Hoem (1980) and (1983), McCutcheon (1976), Pressat (1972) pp. 90–100, Steffensen (1930), and Weibull (1939)). The reasons for this are both theoretically and empirically motivated. In fact, because $s(x) = \prod_{i=0}^{x-1} (1 - q_i)$, two different (observed) experiences $\{\hat{q}_x\}_A$ and $\{\hat{q}_x\}_B$, say, may produce very nearly the same survival function. It is clear, therefore, that an estimation process which focuses on the survival function does not necessarily lead to good estimates of the underlying intensity μ, or of the risk q_x. On the other hand, because (1) is used to arrive at an estimate of child mortality, there is obviously a practical need for modeling the survival function. The present paper shows how the new model fits a selection of child mortality patterns and the extent to which it captures the underlying mortality intensity.

3. A New Model of the Survival Function for Children

A study of the shape of many different survival curves during the first 15–20 years of life reveals that these behave in a very similar manner (Hartmann (1980) pp. 20–22). More specifically, when plotting the logits of survival probabilities as a function of age, it is seen that almost regardless of the shape and level of the survival function, the transformed probabilities follow a logarithmic curve (Fig. 1). In other words, letting

$$\text{logit}\, p = 0.5 \ln \frac{1-p}{p}, \quad 0 < p < 1,$$

it is noted that

$$\text{logit}\, s(x) = a + b \ln x, \quad (2)$$

where a and b are parameters, gives an accurate representation of child mortality; the corresponding model survival function is

$$s(x; \theta) = \frac{1}{1 + e^{2a} x^{2b}}, \quad (3)$$

with parameter vector $\theta = (a, b)$. The

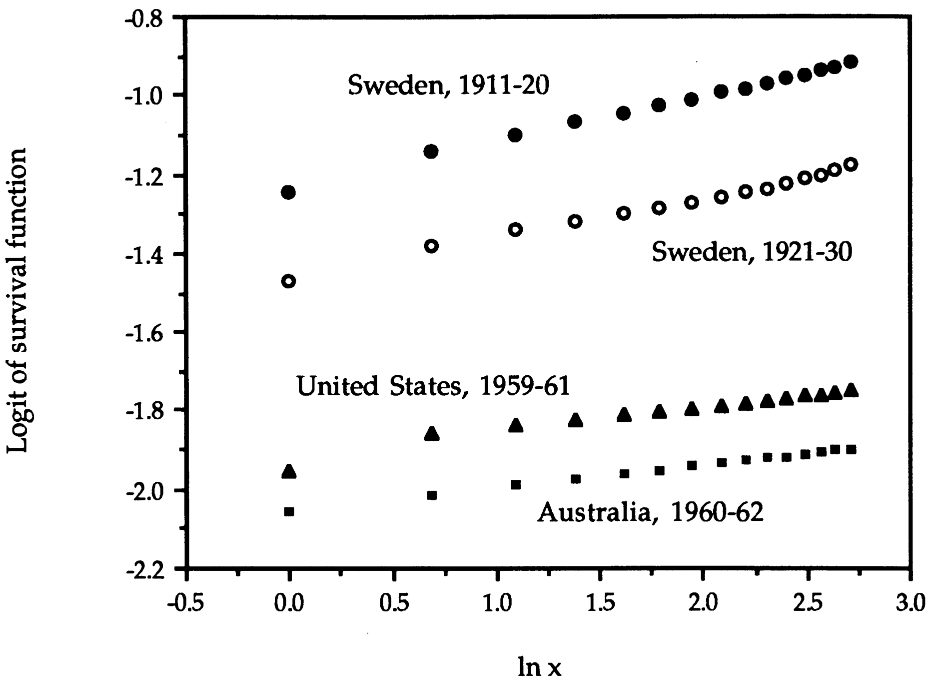

Fig. 1. Logits of survival probabilities for the United States, both sexes, 1959–61, Australian females 1960–62, Swedish males 1911–20, and Swedish females 1921–30 plotted against ln x

model intensity is

$$\mu(x;\,\theta) \;=\; \frac{2b\,\exp\,(2a)x^{2b-1}}{1+\exp\,(2a)x^{2b}}. \tag{4}$$

In order to illustrate that logit $s(x)$ very nearly is a logarithmic function of age, Fig. 1 shows logit $s(x)$ plotted against ln x for a choice of four different survival functions (see, Gross and Clark (1975) pp. 27–30, Pollard, Yusuf, and Pollard (1981) pp. 172–173, and Swedish Life Tables for the Decades 1911–20 and 1921–30). These plots are nearly straight lines. Extensive studies of survival functions from widely different time periods and societies reveal similar graphs (Hartmann (1980) and (1981)), that is, regardless of the level and shape of the survival function for childhood, the logits of survival probabilities very nearly follow a logarithmic curve. This is also true of the survival functions in the Coale-Demeny model life tables (Coale and Demeny (1966), Hartmann (1980) and (1981)).

Here it is not amiss to mention that although log-linear models appear frequently in statistical bioassay and that $s(x;\,\theta)$ to some extent could be interpreted as a dose-response model where the dose is time and the response is survival (see, e.g., Finney (1952), and Brass (1971)), the basic motivation for the model has its roots in several empirical studies of survival curves (Hartmann (1980) and (1981)).

3.1. Estimation of parameters

If $s(x;\,\mathbf{a})$ with parameter vector $\mathbf{a}$ is a model of the survival function, the most common method of estimating $\mathbf{a}$ is by means of least squares, that is, by minimizing the sum of squares

$$Q = \sum_x \omega(x)[\hat{s}(x) - s(x; \mathbf{a})]^2 \qquad (5)$$

with respect to **a** where $\hat{s}(x)$ is the observed survival function and $\omega(x)$ a weight which should preferably be proportional to the inverse of the variance of $\hat{s}(x)$. Because (5) leads to non-linear normal equations in the case of $s(x; \theta)$, estimation of θ requires an electronic computer and an algorithm for non-linear least squares minimization. In the present paper, the non-linear module in Macintosh SYSTAT is used for minimization of (5). With respect to the choice of weights, it should be noted that in most cases it suffices to work with the constant weight $\omega(x) = 1$. In what follows it is assumed then that $\omega(x) = 1$.

In passing it is proper to mention, of course, that other criteria could be used for fitting a parametric model to observed mortality. Because it is the principal aim of the paper to show that $s(x; \theta)$ is a reliable demographic model of the survival function for young ages, is easy to fit to data, and can be used in (1) as an aid in indirect estimation, I abstain from discussing, e.g., a maximum-likelihood approach (see, e.g., Gross and Clark (1975), Lee (1980), and Miller (1981)). For alternative approaches to estimating the parameters in a parametric model of child mortality see Choe (1981) and Luther (1983).

3.2. *Interpretation of parameters*

With respect to the role of the parameters, it will be seen that parameter a determines the level of infant mortality since $a = 0.5 \ln [(1 - s(1; \theta))/s(1; \theta)]$ and that parameter b determines the shape or age-pattern of child mortality. It has been shown elsewhere (Hartmann (1980) and (1981)), that the estimated parameters can be used to classify age-patterns of child mortality in terms of the four families in the Coale-Demeny model life tables (Coale and Demeny (1966)).

3.3. *Judging the goodness of fit*

There is considerable difference between fitting $s(x; \theta)$ to a survival curve calculated for a large respectively a small population. In the case of a large population, the empirical survival curve is accurately determined, and even if the fit provided by the model is excellent the small (almost zero) variances in the observed survival function automatically lead to a rejection of a statistical test of whether the model curve and the empirical curve are in statistical agreement. In such a situation it is reasonable to refer to the fitted curve as a representation of the survival function. In this situation, the absence of a generally acknowledged standard for judging the fit necessitates an inspection of a tabulation or diagram of observed and fitted values. Consequently, the evaluation depends on the given circumstances, and is, of course, more or less subjective. Indeed, it is in this spirit that Brass (1971) validates his logit life table system.

When the survival curve is calculated for a small population, a test of whether observed mortality is in statistical agreement with that of the fitted model is best determined by studying the intensity functions (or, alternatively, the probabilities q_x). More specifically, when there is visible random variation in the observed mortality intensities $\hat{\mu}_x$, one may make use of the result that if the intensities are considered piecewise constant, then the $\hat{\mu}_x$ are asymptotically independent and $\hat{\mu}_x$ is asymptotically normally distributed with estimated mean $\hat{\mu}_x$ and estimated variance $\hat{\mu}_x / R_x$ where R_x is the observed number of person-years lived by those exposed to risk while at age x (see, e.g., Brownlee (1965) pp. 217–218).

If $\mu(x; \mathbf{b})$, with parameter vector $\mathbf{b} = (b_1, \ldots, b_n)$, is a model of the underlying piecewise constant mortality intensity, it may be tested if the fitted intensities $\mu(x; \hat{\mathbf{b}})$ are in statistical agreement with the observed ones by studying the variable

$$\chi^2(N - n) = \sum_x \frac{R_x}{\hat{\mu}_x} [\hat{\mu}_x - \mu(x; \hat{\mathbf{b}})]^2 \quad (6)$$

which, for a least squares estimate $\mu(x; \hat{\mathbf{b}})$, is approximately Chi-square distributed with $N - n$ degrees of freedom where N is the number of age groups. It should be noted that since the intensity is considered piecewise constant over single-year intervals, x is replaced by $x + 0.5$ for computational purposes in (6). The exception is for age zero where the intensity changes rapidly with age and where, approximately speaking, it is appropriate to let 0.1 represent the pivotal age (see, e.g., Chiang (1968) pp. 194–202).

As already emphasized, an important question is whether the fitted intensities $\mu(x; \hat{\theta})$ given by (4), where $\hat{\theta}$ has been estimated by means of (5), lead to acceptable results in terms of (6). In other words, in the context of an experience with visible random variation in the observed intensities, it is important that the intensities captured from the estimated model survival function $s(x; \hat{\theta})$ are reasonably close to the observed ones.

We now turn to numerical illustrations where the focus is on the ability of $s(x; \hat{\theta})$ to, on the one hand, represent the survival function in the case of large populations where there is no visible random variation in the observed intensities and, on the other, model both the survival and the intensity function in the case of populations where there is visible random variation in the observed intensities.

4. Numerical Illustrations

4.1. Applications to observed survival curves for large populations

As an illustration of the goodness of fit given by $s(x; \theta)$ in the case of experiences based on large numbers of person-years, with the parameters estimated by means of (5), it has been fitted to the survival functions for the United States, 1959–61 (both sexes), Australia, 1960–62 (females) Sweden, 1921–30 (females) and Sweden, 1911–20 (males) (see Gross and Clark (1975) pp. 27–30, Pollard, Yusuf, and Pollard (1981) pp. 172–173, and Swedish Life Tables for the Decades 1911–20 and 1921–30). These survival functions are different both in level and shape (Fig. 1). It will be seen (Table 1) that the fitted functions are very close to the observed ones. Indeed, if one were to show a diagram of fitted and observed survival curves these would more or less overlap. Hence, at least in an impressionistic sense it can be concluded that the model gives a reliable representation of the survival curves in Fig. 1. The estimated parameters are given in Table 2.

4.2. Estimation of the underlying intensity: the case of a moderately large population

To demonstrate the ability of $s(x; \theta)$ to reproduce the underlying intensity in the case of a moderately large population, Table 3 shows observed and fitted survival probabilities and corresponding intensities along with the risk times for the total population of children in California in 1960 (see, Chiang (1968) p. 196). It will be noted that the fitted survival function virtually coincides with the observed one (Table 3).

It will also be noted that, except for age 0, there is fairly close agreement between

Table 1. Observed and fitted survival functions

Age	$0_1(x)$	$s_1(x;\hat{\theta})$	$0_2(x)$	$s_2(x;\hat{\theta})$	$0_3(x)$	$s_3(x;\hat{\theta})$	$0_4(x)$	$s_4(x;\hat{\theta})$
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.9782	0.9765	0.9824	0.9824	0.9495	0.9488	0.9236	0.9215
2	0.9732	0.9741	0.9807	0.9808	0.9403	0.9416	0.9076	0.9092
3	0.9719	0.9726	0.9797	0.9798	0.9361	0.9370	0.9001	0.9012
4	0.9710	0.9715	0.9791	0.9791	0.9332	0.9335	0.8945	0.8952
5	0.9703	0.9706	0.9785	0.9785	0.9308	0.9307	0.8900	0.8903
6	0.9697	0.9699	0.9781	0.9780	0.9287	0.9283	0.8862	0.8861
7	0.9692	0.9692	0.9776	0.9776	0.9269	0.9262	0.8827	0.8825
8	0.9687	0.9687	0.9773	0.9772	0.9251	0.9243	0.8796	0.8793
9	0.9682	0.9681	0.9769	0.9769	0.9235	0.9227	0.8768	0.8764
10	0.9678	0.9677	0.9766	0.9766	0.9219	0.9211	0.8743	0.8738
11	0.9675	0.9673	0.9764	0.9763	0.9204	0.9197	0.8718	0.8713
12	0.9671	0.9669	0.9761	0.9761	0.9188	0.9185	0.8694	0.8691
13	0.9667	0.9665	0.9758	0.9758	0.9172	0.9172	0.8672	0.8670
14	0.9663	0.9662	0.9756	0.9756	0.9154	0.9161	0.8649	0.8650
15	0.9658	0.9658	0.9753	0.9754	0.9133	0.9150	0.8623	0.8632

Note:
$0_i(x)$ denotes the observed and $s_i(x;\hat{\theta})$ the fitted function.
$0_1(x)$: United States, both sexes, 1959–61
$0_2(x)$: Australia, 1960–62, females
$0_3(x)$: Sweden, 1921–30, females
$0_4(x)$: Sweden, 1911–20, males

observed and fitted intensities (Table 3). The individual squared standardized residuals in (6) are also given in Table 3. Whereas, as noted, there is a considerable deviation between the observed and fitted intensity at age 0, the remaining observed and fitted intensities agree reasonably well. In fact, for the 11 single year age groups ($1 \leqslant x < 12$), $\chi^2(9) = 13.2$ which is below the 5% limit. Hence, for these ages the fitted and observed intensities are not significantly different on the 5% level. With respect to age 0 it is always difficult to compare observed and fitted intensities because the assumption of a constant intensity does not hold on account of the rapid changes in mortality during infancy. And, as already noted, the assumption that the estimated model intensity corresponds to age 0.1 is, of course, only an approximation. It will be seen, however, that the observed and fitted q_0, for all practical purposes, are the same.

Table 2. Estimated parameters

Survival function	Estimated parameters		Infant mortality
	$\hat{a}$	$\hat{b}$	q_0
Australia, 1960–62, females	−2.0108	0.0631	0.018
United States, 1959–61, both sexes	−1.8636	0.0712	0.022
Sweden, 1921–30, females	−1.4595	0.1001	0.051
Sweden, 1911–20, males	−1.2312	0.1146	0.076

Table 3. Observed and fitted survival probabilities and intensities for total California population, 1960

Age	Observed Survival	Fitted Survival	Person-Years	Observed Intensity	Fitted Intensity	Goodness of Fit
0	1.0000	1.0000	356,435	0.02430	0.01883	438.85
1	0.9762	0.9764	351,611	0.00150	0.00164	4.31
2	0.9748	0.9747	351,828	0.00097	0.00103	1.37
3	0.9738	0.9737	344,966	0.00079	0.00076	0.36
4	0.9730	0.9730	341,712	0.00060	0.00061	0.03
5	0.9725	0.9724	335,837	0.00052	0.00051	0.12
6	0.9720	0.9719	331,427	0.00043	0.00044	0.02
7	0.9715	0.9715	322,881	0.00044	0.00038	2.43
8	0.9711	0.9711	307,669	0.00041	0.00034	3.50
9	0.9707	0.9708	297,664	0.00034	0.00031	0.84
10	0.9704	0.9705	299,407	0.00029	0.00028	0.06
11	0.9701	0.9702	297,343	0.00025	0.00026	0.12

Note: The goodness of fit has been calculated by means of (6) for each of the 12 single-year age groups. The intensities correspond to the middle of each age interval except for age 0 where the pivotal age is 0.1. The estimated parameters, obtained by minimization of (5), are $\hat{a} = -1.86096$ and $\hat{b} = 0.04993$.

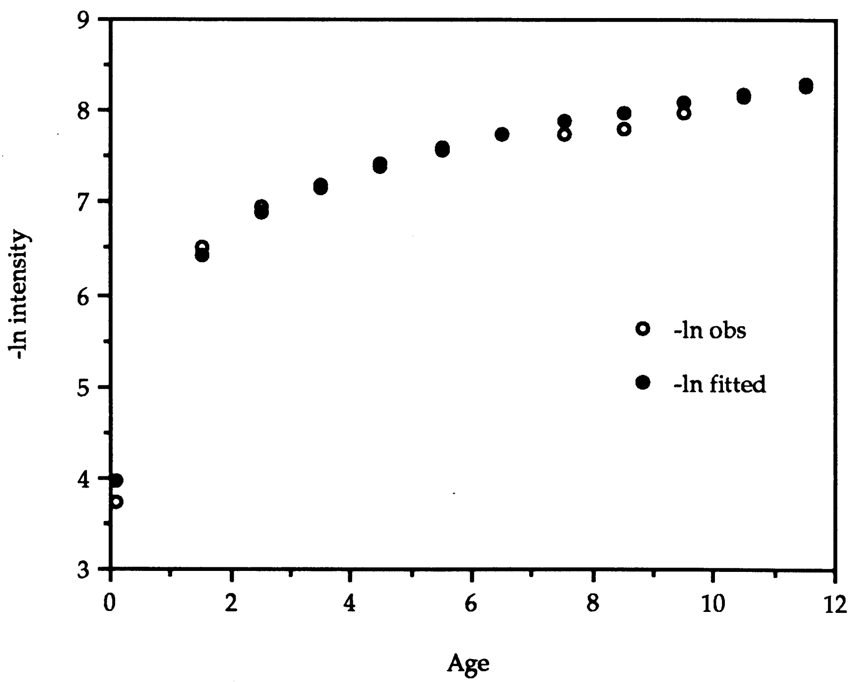

Fig. 2. Observed and fitted intensities for total California population in 1960

Table 4. Observed and fitted survival probabilities and intensities for total Swedish male population, 1980

Age	Observed Survival	Fitted Survival	Person-Years	Observed Intensity	Fitted Intensity	Goodness of Fit
0	1.0000	1.0000	49,860	0.00806	0.00785	0.28
1	0.9919	0.9922	48,676	0.00055	0.00076	3.81
2	0.9914	0.9914	48,836	0.00029	0.00049	6.55
3	0.9911	0.9909	50,372	0.00034	0.00036	0.09
4	0.9907	0.9906	52,400	0.00032	0.00029	0.12
5	0.9904	0.9903	55,471	0.00025	0.00025	0.00
6	0.9902	0.9900	56,952	0.00030	0.00021	1.42
7	0.9899	0.9898	57,214	0.00021	0.00019	0.12
8	0.9897	0.9896	58,149	0.00024	0.00017	1.21
9	0.9894	0.9895	57,255	0.00026	0.00015	2.48
10	0.9892	0.9893	55,848	0.00016	0.00014	0.13
11	0.9890	0.9892	57,003	0.00011	0.00013	0.22

Note: The estimated parameters, obtained by minimization of (5), are $\hat{a} = -2.42211$ and $\hat{b} = 0.06880$.

This result is encouraging and speaks in favor of $s(x; \theta)$ as a sound model of child· mortality, especially if one considers the large number of person-years lived by those exposed to risk (Table 3). In order to enhance visible inspection of observed and fitted intensities, Fig. 2 shows the transformed intensities $-\ln \hat{\mu}_x$ and $-\ln \mu(x; \hat{\theta})$.

4.3. *Estimation of the underlying intensity: the case of a relatively small population*

The final illustration concerns survival data for a relatively small population, namely Swedish males in 1980 (Table 4). The person-years of this experience (see Swedish Life Tables for the Decade 1971–80) are substantially fewer (and hence the variances substantially larger) than in the case of Table 3. First, it will be seen that the fitted survival function gives a close fit to the observed one; it is, among other things, for this reason that it is sufficient to make use of (5) for estimation of the parameters. Second, because the variances in the observed intensities are much larger than in Table 3, the observed and fitted intensities agree quite well in terms of (6). For the ages $0 \leqslant x < 12$, χ^2 (10) = 16.4 which means that observed and fitted intensities agree on the 5% level. The transformed intensities $-\ln \hat{\mu}_x$ and $-\ln \mu(x; \hat{\theta})$ are shown in Fig. 3.

The basic requirements for the fitted intensities to be a graduation of the observed intensities is that (i) they are smooth, (ii) do not deviate significantly from one another in the sense of χ^2, and (iii) the sign changes between fitted and observed values are sufficiently many for a sign-test not to lead to a rejection on the 5% level, say. These requirements are all met in the case of Table 4. In the case of Table 3 the small variance in infant mortality, as well as the choice of a pivotal age of 0.1, cause a highly significant difference between the observed and estimated infant mortality intensity. However, as already noted, the observed and estimated q_0 are in agreement.

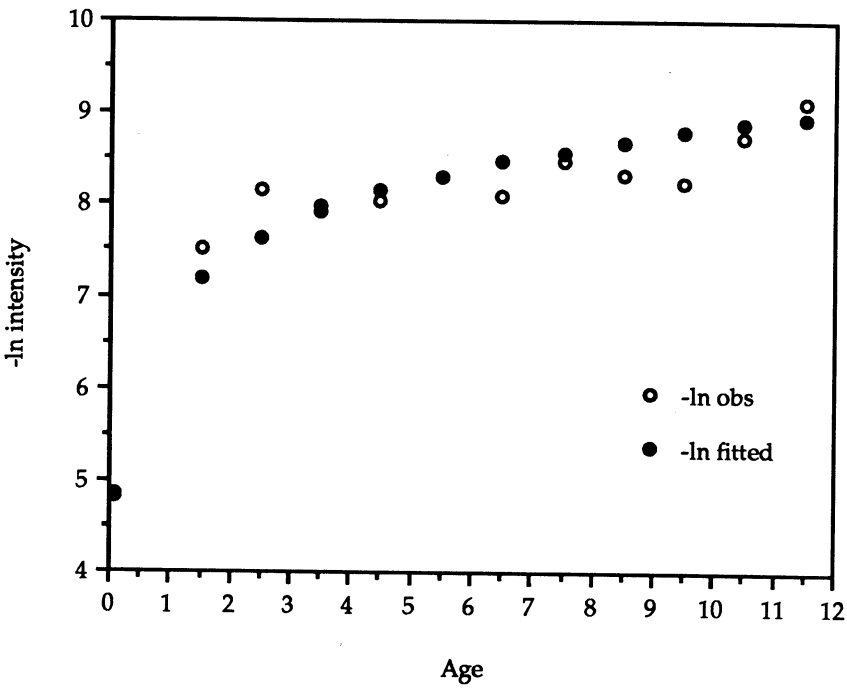

Fig. 3. Observed and fitted intensities for Swedish males, 1980

5. Conclusions

In the case of large populations, $s(x;\ \theta)$ appears to give "close fits" to observed survival functions. However, because the estimated variances in the observed intensities are almost zero, it has little or no meaning to test if the estimated model representations agree with the observed ones. Instead, one has to rely on common sense and the prevailing circumstances when deciding if the fit is adequate. Here it should be added that the fits to survival curves for large populations shown in this paper are quite similar to those shown elsewhere.

In the case of populations where there is visible random variation in the observed intensity (or in q_x), $s(x;\ \theta)$ can also be seen as function which graduates or represents both the survival function and the observed intensities. Previous findings (Hartmann (1980) and (1981)) as well as the results given in this paper suggest, therefore, that $s(x;\ \theta)$ is a highly reliable parametric model of infant and child mortality.

Perhaps the use of $s(x;\ \theta)$ with greatest potential is in (1). Here $s(x;\ \theta)$ can be chosen so as to represent a first guess of the underlying survival function. Because the intensities derived from $s(x;\ \theta)$ appear to give a realistic picture of the true underlying intensities, it is an ideal choice of parametric function for fine-tuning a Brass-type process of child mortality estimation. It will be realized, however, that $s(x;\ \theta)$ also may serve as a useful model for graduating child survival data obtained in a survey where the person-years are few and, hence, there is clearly visible random variation in the observed intensity function.

6. References

Brass, W. and Coale, A.J. (1968): Methods of Analysis and Estimation. In the Demography of Tropical Africa, edited by William Brass, pp. 88–150. Princeton University Press, Princeton.

Brass, W. (1971): On the Scale of Mortality. In Biological Aspects of Demography, edited by William Brass, pp. 69–110. Taylor and Francis, London.

Brass, W. (1975): Methods for Estimating Fertility and Mortality from Limited and Defective Data. The University of North Carolina at Chapel Hill, Chapel Hill.

Brownlee, K.A. (1965): Statistical Theory and Methodology (In Science and Engineering). Second Edition, Wiley, New York.

Chiang, C.L. (1968): Introduction to Stochastic Processes in Biostatistics. Wiley, New York.

Choe, M.K. (1981): Fitting the Age Pattern of Infant and Child Mortality with the Weibull Survival Distribution. Asian and Pacific Census Forum, 7, (4), pp. 10–13.

Coale, A.J. and Demeny, P.D. (1966): Regional Model Life Tables and Stable Populations. Princeton University Press, Princeton.

Finney, D.J. (1952): Statistical Method in Biological Assay. Charles Griffin, Glasgow.

Gross, A.J. and Clark, V.A. (1975): Survival Distributions: Reliability Applications in the Biomedical Sciences. Wiley, New York.

Harper, F.S. (1936): An Actuarial Study of Infant Mortality. Skandinavisk Aktuarietidskrift, pp. 234–270.

Hartmann, M. (1980): Infant and Childhood Mortality. University of Lund. Department of Statistics. CODEN: LUSADG/SAST-3074/1-65. (Working paper.)

Hartmann, M. (1981): Laws of Mortality on the Logit Scale. University of Lund. Department of Statistics. CODEN: LUSADG/SAST-3079/1-84. (Working paper.)

Hartmann, M. (1982): Some Reflections on the Use of Child Survivorship Data to Estimate Childhood Mortality. University of Lund, Department of Statistics. CODEN: LUSADG/SAST-3092/1-97. (Working paper.)

Hartmann, M. (1987): Past and Recent Attempts to Model Mortality at All Ages. Journal of Official Statistics, 3, (1), pp. 19–36.

Hoem, J.M. (1980): Who First Fitted a Mortality Formula by Least Squares? Blätter, Deutsche Gesellschaft für Versicherungsmatematik, 14, (3), pp. 459–460.

Hoem, J.M. (1983): The Reticent Trio: Some Little-known Discoveries in Life Insurance Mathematics by L.H.F. Oppermann, T.N. Thiele and J.P. Gram. International Statistical Review, 51, pp. 213–221.

Hooker, P.F. and Longley-Cook, L.H. (1953): Life and Other Contingencies. Vol. 1. Cambridge University Press, Cambridge.

Krishnamoorthy, S. (1982): On Graduating Infant and Childhood Mortality with Mathematical Curves. Asian and Pacific Census Forum, 8, (3), pp. 14–15.

Lee, T.E. (1980): Statistical Methods for Survival Data Analysis. Life Time Learning Publications, Belmont, CA.

Luther, N.Y. (1983): A Simple Linearization Procedure for Local Fit of the Weibull Survival Distribution: A Note on an Article by Choe. Asian and Pacific Census Forum, 9, (2), pp. 5–8.

McCutcheon, J.J. (1976): On the Measurement of Infant Mortality. Transactions of the Faculty of Actuaries, 34, pp. 433–442.

Miller, R.G. (1981): Survival Analysis. Wiley, New York.

Pollard, A.H., Yusuf, F., and Pollard, G.N. (1981): Demographic Techniques. Second Edition. Pergamon Press, Sydney.

Pressat, R. (1972): Demographic Analysis: Methods, Results, Applications. Edward Arnold, London.

Statistics Sweden (1980): Swedish Life Tables for the Decades 1911–20, 1921–30 and 1971–80. (Sveriges Officiella Statistik), Stockholm.

Steffensen, J.F. (1930): Infantile Mortality from an Actuarial Point of View. Skandinavisk Aktuarietidskrift, pp. 272–286.

Weibull, W. (1939): A Statistical Theory of the Strength of Material, Ingenjörsvetenskapsakademiens Handlingar, 151.

Received December 1988
Revised October 1989