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Response Burden and Panel Attrition

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When maintaining a consumer panel one is confronted with the question of how large the response burden can be. It is our experience that the larger the response burden (i.e., the number and size of the respondent's tasks) the higher the panel attrition. Consequently, we have a trade-off problem. In this article we derive some rules for optimum response burden for change estimators under simple model assumptions. The consequences of these results for daily research design in frequently measuring panels are discussed.

Key words: Budget survey; consumer panels; estimators of change; survey designs

1. Introduction

In budget surveys and consumer panels the burden of the respondents is a major problem. The workload associated with filling out diaries for these surveys has several effects on the quality of the budget data. The large amount of work is considered to be an important reason for the high initial nonresponse rate for budget surveys compared to other household surveys (Lindström, Lindqvist, and Näsholm 1989; Lyberg 1991). A second effect can be expected in the phenomenon of underreporting. It is commonly found that the number of reported purchased products in the first week is higher than in the second week (Harrison 1991; Nevraumont 1991; Ribe 1991). In panel surveys the response burden also influences panel attrition (Silberstein and Jacobs 1989). Modern techniques like bar-scanning methods and electronic diaries (Saris, Prastacos, and Recober 1992) are applied to ease the respondent's task. But in spite of these techniques a respondent still has to do a considerable amount of work, especially in panels that aim at continuous measurement of expenditures. It may, however, be unnecessary to collect the budget data every week. If we take only a sample of weeks this may result in a relatively small loss of precision. On the other hand, this may result in a lower attrition rate as fewer respondents become fed up with their task.

In this article we propose an optimal survey design for a consumer panel. Such a design includes the method to sample respondents, the range of products that is reported (durable goods, fast moving goods, regular expenditures) and the time period covered. We can divide these aspects into two groups: the sampling of respondents works between

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individuals, while the range of products and the time period work within individuals. The last group determines the task for each respondent, and is responsible for the response burden. Many different designs are used to collect budget data. For example, in the Dutch Budget Survey, respondents do not report all expenditures all the time. They report large expenditures (expenditures over a certain amount) throughout the whole year, and they report all expenditures only for a short period of two weeks (see e.g., Statistics Netherlands 1992). We are interested in finding an optimal design for a consumer panel where the set of products is fixed. We can only vary the number of weeks that respondents report (every week or one week every two weeks, etc.). As an indicator of the adequacy of the design we will look at the precision of the estimators in terms of mean squared error. Let us assume that we are interested in a population parameter φ (e.g., the total expenditures on a specific product in a certain year), and that we use an estimator $\hat{\varphi}$ based on budget data that was gathered with a certain survey design. Then we like to minimise

$$\mathrm{MSE}(\hat{\varphi}) = var(\hat{\varphi}) + bias^2(\hat{\varphi})$$

with respect to the number of weeks that respondents report. In practice, cost plays an important role, which means that we want the minimal MSE for a given research budget. A reduction of the report frequency has several effects. The initial nonresponse rate may drop, since there is less deterrence by the workload. In turn this may result in a bias reduction if the (non-) response was selective. A lower workload will reduce under-reporting and the bias from it. We may also expect lower panel attrition. Again this will reduce the bias of the estimators, since households that have many purchases may remain in the panel. Besides a bias reduction, for estimators of change we find a positive effect to the variance from a larger panel overlap. Next to the mentioned positive effects we expect some negative effects from a reduction in report frequency. We will have an extra variance component from sampling in weeks. It may be that an irregular reporting scheme confuses respondents, leading to results that are less reliable.

In this article we will focus on the variance reduction caused by reduced panel attrition. Most literature on panel attrition deals with bias effects. Examples are the use of Markov-chain models for non-random nonresponse to estimate gross flows in categorical data (Stasny 1987), econometric regression analyses to correct for attrition bias (Hausman and Wise 1979), and bias corrections by weighting techniques (Van de Pol 1993). Special survey designs are also used to deal with the bias from attrition: split-panel designs and rotating panels are generally applied in budget and labour-force surveys (Van de Pol 1989).

For ease of exposure we will assume that households are sampled by simple random sampling, although the theory can easily be generalised to other sampling methods. For households that dropped out of the panel we assume that they are immediately replaced using a quota sampling method, keeping the panel size constant over time. This implies that attrition brings extra costs, since the replacement of a respondent will cost money. Although these costs should be incorporated in finding an optimal survey design, we will ignore it for the sake of simplicity. We will study two models that correspond to two panel designs. The first design deals with a panel that is dedicated to measure expenditures on consumption goods. This is the case for most consumer panels. We will call this a single-purpose panel. In the second design, however, the panel is also used for other purposes. An example of this is the Dutch Telepanel, where the burdensome questionnaires on expenditures are interchanged with questionnaires on a variety of other topics. We will call this the multi-purpose panel. The theory derived here is stated in general terms as to make it applicable to a more general situation.

2. Preliminary Notation and Relations

We are interested in estimates (e.g., of consumption or purchases of fast moving consumer goods) of a certain period of M weeks. Usually M is equal to 13, a three month period, but we may also consider $M = 4$, $M = 26$ or $M = 52$. Furthermore we are interested in estimates of change between successive periods t and $t + 1$. We assume that in every wave (or week) we have n households from a much larger population of N households. It is assumed further that there is a constant attrition q. For each week every panel member has a probability q of dropping out of the panel independently of the other panel members and independently of what happens in other weeks. Hence, if we denote the probability to stay in the panel by $p\,(p = 1 - q)$, a respondent who is in the panel during week j has probability p^{k-j} to still be a panel member in week $k > j$. Let

$X_{ij}^{(t)}$ be the amount of purchases by household i in week j of period t

$$X_j^{(t)} = Nn^{-1} \sum_{i=1}^{n} X_{ij}^{(t)}, \text{ the estimated population total for week } j \text{ of } t$$

$$X^{(t)} = \sum_{j=1}^{M} X_j^{(t)} \text{ the estimated population total for period } t.$$

It is assumed that for given j and t the $X_{ij}^{(t)}$ are i.i.d. (independent, identically distributed) with variance σ^2 for the households i. For different values of j and t we assume the $X_{ij}^{(t)}$ to be homoscedastic (possessing equal variances). We focus our attention to more or less daily shopping routines, and assume that this is a stable process, which is in equilibrium, although the process may be different for each household. When there is a regular weekly pattern in such purchases it is reasonable to assume that for a combination $(j, u) \neq (k, v)$ we have

$$\text{cov}(X_{ij}^{(u)}, X_{ik}^{(v)}) = \rho\sigma^2 \tag{1}$$

(a covariance matrix with compound symmetry). Of course, such an assumption is violated for a product that does not follow such a weekly pattern, e.g., when it is purchased on a two-weekly basis. Such types of variables are outside the scope of this chapter. This may suggest that the assumptions are rather restrictive. When, however, broad categories are used like meat, green vegetables, fruit or candies, the assumptions apply, at least in the Dutch society, to the most important results that have to come out of a budget survey.

Our main interest is to measure the changes from one period to another. The absolute consumption level of a product in itself is sometimes not a very useful figure. In terms of the variables defined above this means that we are interested in the precision of $X^{(t+1)} - X^{(t)}$ (and, as a by-product, of $X^{(t)}$). Consequently, we require the covariances of the terms of which these quantities consist. Let n_{jk} be the number of panel members that are in both wave j and in wave $k > j$ in a period t. Then n_{jk} has a binomial distribution with parameters n and p^{k-j}. Assume that the panel members are numbered such that the first n_{jk} are included in both waves. Then we can compute

$$\begin{aligned}
\mathrm{cov}(X_j^{(t)}, X_k^{(t)}) &= E_{n_{jk}} \mathrm{cov}(X_j^{(t)}, X_k^{(t)} | n_{jk}) + \mathrm{cov}_{n_{jk}}(EX_j^{(t)}, EX_k^{(t)} | n_{jk}) \\
&= E \frac{N^2}{n^2} \sum_{i=1}^{n_{jk}} \mathrm{cov}(X_{ij}^{(t)}, X_{ik}^{(t)}) \\
&= N^2 p^{k-j} \rho \sigma^2 / n
\end{aligned} \tag{2}$$

This yields the following equality for the variance of $X^{(t)}$:

$$\begin{aligned}
\mathrm{var}(X^{(t)}) &= \mathrm{var}\left(\sum_{j=1}^{M} X_j^{(t)} \right) \\
&= \sum_{j=1}^{M} \sum_{k=1}^{M} \mathrm{cov}(X_j^{(t)}, X_k^{(t)}) \\
&= \frac{N^2 \sigma^2}{n} \left(M + \frac{2\rho(M-1)p}{1-p} - \frac{2\rho p^2 (1-p^{M-1})}{(1-p)^2} \right)
\end{aligned} \tag{3}$$

Additionally, for the variance of $X^{(t+1)} - X^{(t)}$ we have:

$$\begin{aligned}
\mathrm{var}(X^{(t+1)} - X^{(t)}) &= \mathrm{var}\left(\sum_{j=1}^{M} X_j^{(t+1)} - \sum_{j=1}^{M} X_j^{(t)} \right) \\
&= \mathrm{var}(X^{(t+1)}) + \mathrm{var}(X^{(t)}) - 2 \sum_{j=1}^{M} \sum_{k=1}^{M} \mathrm{cov}(X_j^{(t+1)}, X_k^{(t)}) \\
&= \frac{2N^2 \sigma^2}{n} \left(M + \frac{2\rho(M-1)p}{1-p} - \frac{2\rho p^2 (1-p^{M-1})}{(1-p)^2} - \frac{\rho p (1-p^M)^2}{(1-p)^2} \right)
\end{aligned} \tag{4}$$

These (rather complex) equations were derived under the assumption that the correlations between purchases among different weeks are constant. This assumption is unrealistic: one can at least expect to find some sampling fluctuation, seasonal fluctuation and fluctuation for special occasions like Christmas. Before we continue our analyses based on the assumption of a constant correlation we will consider the results of a theoretical exercise and show some practical examples where this assumption is not exactly met in our data. In the case of non-constant correlations we have

$$\mathrm{cov}(X_{ij}^{(u)}, X_{ik}^{(v)}) = \rho_{jk}^{(u,v)} \tag{5}$$

As a consequence the variances of our estimators cannot be simplified into Equations (3) and (4), since they will not depend on a whole matrix of correlations.

We have studied the effect of varying single correlations from the average on a theoretical basis. In the case where the value $\rho_{jk}^{(u,v)}$ differs from the other correlations for only one combination (j, k, u, v), the effect to $\mathrm{var}(X^{(t)})$ and $\mathrm{var}(X^{(t+1)} - X^{(t)})$ will depend on the position of the correlation in the matrix. If $u = v = t$, an increase in the correlation will increase both $\mathrm{var}(X^{(t)})$ and $\mathrm{var}(X^{(t+1)} - X^{(t)})$. If $u \neq t$ (i.e., $u = t$ and $v = t + 1$) on the other hand, there will be no effect on $\mathrm{var}(X^{(t)})$, whereas $\mathrm{var}(X^{(t+1)} - X^{(t)})$ will decrease. If a correlation decreases the effects will be in the opposite direction. In a matrix where more correlations differ from an average ρ, most effects will cancel out.

From an empirical standpoint, we studied correlation matrices of expenditures on six products in a period of two quarters (26 weeks). One of these correlation matrices (for the product 'eggs') can be found in the appendix. The variances of the estimators were computed in two ways: using the complete correlation matrix, and using the average correlation. Table 1 shows factors between the variances obtained by the two methods. These are the factors that should be applied to the variances composed using the average correlation, in order to obtain the actual variances. For the product 'eggs' $\mathrm{var}(X^{(t)})$ is overestimated by 11%, $\mathrm{var}(X^{(t+1)})$ is underestimated by 10%, and $\mathrm{var}(X^{(t+1)} - X^{(t)})$ is underestimated by 3%. We may conclude that on the whole the errors due to the assumption of equal correlations are small.

The relationships (3) and (4) hold when the respondents are required to fill in the questionnaire during all M weeks of subsequent periods t and $t+1$. In the case where the respondents are only required to fill in the questionnaire during m out of M weeks we can distinguish two different models, corresponding to two different panel designs. For the first design we assume that a fraction $q = 1 - p$ drops out of the panel immediately after the measurement. In the weeks when no measurement with respect to the $X_{ij}^{(t)}$ takes place there is no attrition. This corresponds to a single-purpose panel, in which the panel is dedicated for the budget survey. In a multi-purpose panel, on the other hand, the panel is used for more than the budget survey alone, and the attrition continues in the weeks when no measurement with respect to $X_{ij}^{(t)}$ takes place. This is the typical case of a Telepanel, which may be used for many purposes. Both models lead to slightly generalised versions of Equations (3) and (4). Our definition of $X_j^{(t)}$ is thus generalized to

$$X_j^{(t)} = Mm^{-1}Nn^{-1}\sum_{i=1}^{n} X_{ij}^{(t)} \tag{6}$$

so that

$$X^{(t)} = Mm^{-1}Nn^{-1}\sum_{i=1}^{n}\sum_{j} X_{ij}^{(t)}$$

Table 1. Average correlations and factors by which variances are over/underestimated by assuming equal correlations between different pairs of weeks for six products

Product	Average correlation ρ	Over/underestimation factor obtained due to the assumption of equal correlations		
		$\mathrm{var}(X^{(t)})$	$\mathrm{var}(X^{(t+1)})$	$\mathrm{var}(X^{(t+1)} - X^{(t)})$
eggs	0.24	1.11	0.90	0.97
poultry	0.16	0.96	1.05	1.02
red meat	0.24	1.03	0.95	0.93
pork	0.21	0.98	0.99	0.92
meat products	0.33	0.98	1.01	0.95
meat and meat pr.	0.45	0.98	1.00	0.89

where j takes the values of only those weeks in which respondent i fills in a questionnaire. Then for the single-purpose panel we have

$$\mathrm{var}(X^{(t)}) = \frac{M^2N^2\sigma^2}{nm^2}\left(m + \frac{2\rho(m-1)p}{1-p} - \frac{2\rho p(1-p^{m-1})}{(1-p)^2}\right) \tag{7}$$

and

$$\begin{aligned}&\mathrm{var}(X^{(t+1)} - X^{(t)})\\ &= \frac{2M^2N^2\sigma^2}{nm^2}\left(m + \frac{2\rho(m-1)p}{1-p} - \frac{2\rho p^2(1-p^{m-1})}{(1-p)^2} - \frac{\rho p(1-p^m)^2}{(1-p)^2}\right)\end{aligned} \tag{8}$$

These expressions are identical to Equations (3) and (4), except that the total period M is replaced by the observed period m, and that there is a new factor M^2/m^2. For the multi-purpose model we have

$$\mathrm{var}(X^{(t)}) = \frac{MN^2\sigma^2}{nm}\left(M + \frac{2\rho(m-1)p}{1-p} - \frac{m-1}{M-1}\frac{2\rho p(1-p^{m-1})}{(1-p)^2}\right) \tag{9}$$

and

$$\begin{aligned}&\mathrm{var}(X^{(t+1)} - X^{(t)})\\ &= \frac{2MN^2\sigma^2}{nm}\left(M + \frac{2\rho(m-1)p}{1-p} - \frac{m-1}{M-1}\frac{2\rho p^2(1-p^{m-1})}{(1-p)^2}\right.\\ &\left. - M\rho p^M - \frac{2\rho(m-1)p^{M+1}}{1-p} - \frac{m-1}{M-1}\frac{2\rho p^{M+2}(1-p^{M-1})}{(1-p)^2}\right)\end{aligned} \tag{10}$$

provided that the sampling design is balanced with respect to first- and second-order inclusions of the weeks in the sample, i.e., every week j and every combination (j,k) of weeks appear in the sample with the same frequency. The proof of this statement can be found in the appendix.

In practice the weekly attrition q is rather small. Realistic values for q range from 0.1% to 5%, depending on the subject matter and the time horizon. Such values of q are sufficiently small to justify a linear and quadratic approximation of Equation (7) through (10) for reasonable values of m. An illustration of this is presented in Figure 1. Throughout this chapter we will use approximations in order to change complex relationships into simpler ones. In some cases optimal values of m are intractable for the variance functions that we study, but not for their linear approximations. Figure 1 shows Equation (8) and two approximations in a single purpose-panel with 1% attrition a week in a period of $M = 52$ weeks for a product with $\rho = 0.7$.

In a time interval of a quarter $\mathrm{var}(X^{(t+1)} - X^{(t)})$ and its approximations are almost equal. Because in the case of a single-purpose model M only serves as a cut-off value, this shows that approximations are good if $M = 13$. We now give the equations for the first order approximation (first and second order approximations are derived in the appendix). The linear approximation equation for $\mathrm{var}(X^{(t)})$ in the case of a single-purpose panel is

$$\mathrm{var}(X^{(t)}) \approx \frac{N^2M^2\sigma^2}{nm}(1 + (m-1)\rho - (m^2-1)\rho q/3) \tag{11}$$

The second term of Equation (11) represents the well-known cluster effect due to the fact that repeated measurements take place on the same respondents (see e.g., Kish 1965). The third term shows a decrease in the cluster effect because of the attrition: an expected proportion of q panel members is replaced every week. For $m > 0$ Equation (11) is a decreasing function in m. Clearly then, it is optimal to have as many measurements as possible.

The first order approximation of the variance of $\mathrm{var}(X^{(t+1)} - X^{(t)})$ in the case of a single-purpose panel, is given by Equation (12) (the derivation of which can be found in the appendix).

$$\mathrm{var}(X^{(t+1)} - X^{(t)}) \approx \frac{2N^2M^2\sigma^2}{nm}(1 - \rho + (2m^2 + 1)\rho q/3) \tag{12}$$

The second term represents the gain from correlations using panel data. The third term shows that these gains are reduced by the attrition. By differentiating Equation (12) with respect to m, we can find where this approximation takes a minimum value. The minimum exists, because there is an implicit relationship between the number of budget measurements and the attrition in a period of M weeks. Finding such optimal values of m will be the topic for the next section in which we will study relationships between response burden and attrition. In the formulation of a multi-purpose panel there is no such relationship between the quarterly attrition and m. This is reflected in the approximation

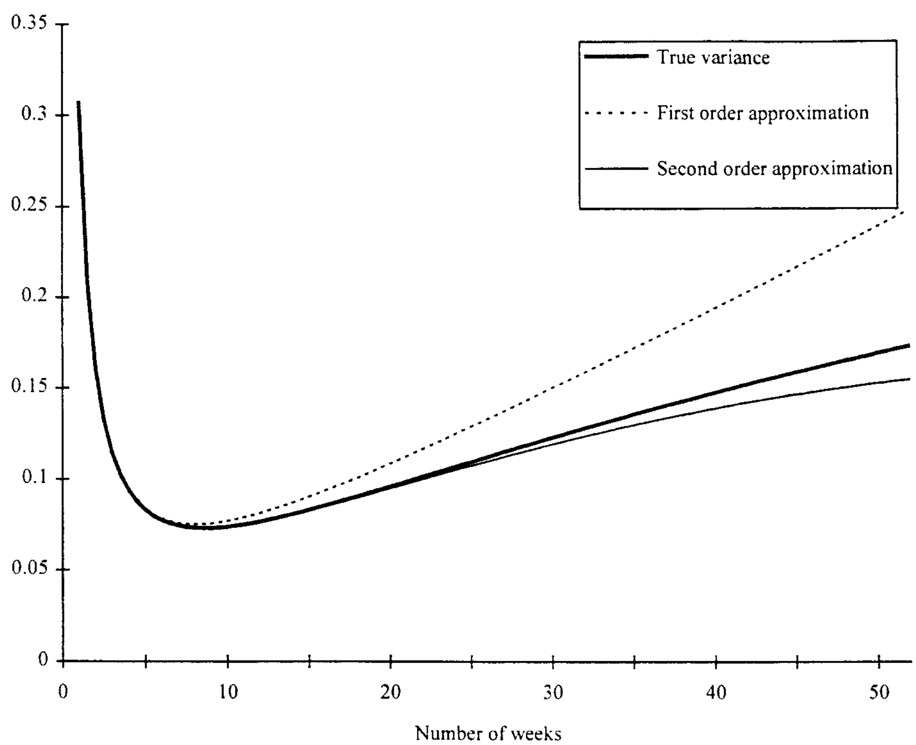

Fig. 1. Var($X^{(t+1)} - X^{(t)}$) and its first- and second-order approximations for q in the single-purpose panel for a product for which $\rho = 0.7$ and a panel with attrition rate $q = 0.01$

formulas for the variances (which are derived in the appendix), namely

$$\mathrm{var}(X^{(t)}) \approx \frac{N^2M^2\sigma^2}{nm}\left(1+(m-1)\rho\left(1-\frac{M+1}{3}q\right)\right) \tag{13}$$

and

$$\mathrm{var}(X^{(t+1)}-X^{(t)}) \approx \frac{2N^2M^2\sigma^2}{nm}(1-\rho+Mm\rho q) \tag{14}$$

Both Equation (13) and (14) are decreasing functions in m: our measurements are more accurate when we observe more weeks. This changes, however, if there is a relationship between q and m.

3. Models for Response Burden and Attrition

In this section we assume a relationship between the response burden and the attrition. We will study the behaviour of $\mathrm{var}(X^{(t+1)}-X^{(t)})$ as a function of the attrition. The behaviour of $\mathrm{var}(X^{(t)})$ is trivial, because this variance is minimal when we have as many as possible independent observations; consequently, it decreases when attrition increases. This makes the behaviour of $\mathrm{var}(X^{(t)})$ less interesting, not only from the substantive point of view, but from the statistical point of view as well. When we measure differences between intervals, however, it is well known that dependent observations may yield higher precision than independent observations (see Kish 1965).

We define the response burden (m) to be the number of measurements in a given period with M possible measurements. For the single-purpose panel, such a relationship is already implied by definition, as each measurement causes a fraction $q = 1 - p$ to leave the panel. By differentiating Equation (12) with respect to m, it is easily shown that the approximated variance is minimised by

$$m_0 = \sqrt{\frac{3}{2}\frac{(1-\rho)}{\rho q}+\frac{1}{2}} \tag{15}$$

So the optimal response burden m_0 decreases both with increasing ρ and q. This makes sense. When q approaches zero, it is desirable to have a large m since attrition is low and we can take advantage of the fact that we have correlated single-source data which are well suited for measuring differences. When ρ tends to zero, we have no reason to be careful to keep single-source data for measuring differences, so we may just as well measure as often as possible, regardless of the attrition. If, on the other hand, ρ approaches one, the optimum value of m becomes less than one, so $m = 1$ is the best possible value. From this one observation we can predict with certainty the differences between $X_{ij}^{(t)}$ at other points of time.

So far it has been assumed that there is no direct relationship between q and m. A reasonable assumption is that q may have the form of

$$q = \lambda m^{\alpha} \tag{16}$$

where both λ and α are parameters, that can be determined empirically. Such a model comes from the theory of magnitude estimation, where one tries to relate the magnitude of a number to the magnitude of the sensation produced by a stimulus magnitude (see e.g., Hamblin 1974). In our case the stimulus magnitude is m, the number of budget questionnaires held in a period, and the sensation caused by it, is the response burden perceived by the panel members. If we assume that the probability of leaving the panel is proportional to the perceived response burden, we obtain Equation (16).

For the single-purpose panel the attribution model implies that the expected attrition in M weeks (q small) is of order $\alpha + 1$, i.e., linear when $\alpha = 0$, quadratic when $\alpha = 1$, etc. To explore the consequences of this assumption we substitute for q in Equation (12). This leads to

$$\mathrm{var}(X^{(t+1)} - X^{(t)}) \approx \frac{2N^2M^2\sigma^2}{nm}(1 - \rho - \lambda(2m^{\alpha+2} + m^{\alpha})\rho/3) \tag{17}$$

This equation has analytic minima for $\alpha = 0$ (when $\lambda = q$), $\alpha = 1$ and $\alpha = 2$. For $\alpha = 1$ the minimum is

$$m_0 = \left(\frac{3}{4}\frac{(1-\rho)}{\lambda\rho}\right)^{\frac{1}{3}} \tag{18}$$

and for $\alpha = 2$ the minimum m_0 satisfies

$$m_0 = \sqrt{\frac{-1}{12} + \sqrt{\frac{1}{144} + \frac{1-\rho}{2\lambda\rho}}} \tag{19}$$

The interpretation of Equations (18) and (19) is similar to the interpretation of Equation (15), where λ has taken the role of q. By taking the respective roots, the values of m_0 decrease when α increases. For small values of ρ the optimum value m_0 is high. If ρ tends to 1 then m_0 approaches zero. Since the effect of the response burden increases with α, m_0 decreases with α.

In the case of a multipurpose panel, the response burden comes not only from the budget survey, but also from the other questionnaires. This leads to a multivariate version of (16) in which the attrition q has the form

$$q = \lambda m^{\alpha} s^{\beta} \tag{20}$$

Here m is the number of budget surveys, s is the number of questionnaires on other topics, and λ, α and β are parameters that may be estimated from experiments. Suppose that s is fixed, then the expected attrition in M weeks (q small) is of order α, i.e., linear when $\alpha = 1$, quadratic when $\alpha = 2$, etc. Substitution for q in Equation (14) and differentiation with respect to m leads to the optimal number of observed weeks

$$m_0 = \left(\frac{1-\rho}{M\alpha\lambda_1\rho}\right)^{\frac{1}{\alpha+1}} \tag{21}$$

where $\lambda_1 = \lambda s^{\beta}$. Note the similarity with Equation (18) if $\alpha = 2$. Here too m_0 decreases with λ: i.e., the more the attrition associated with the budget survey, the smaller the optimal number of weeks to observe.

Table 2. Attrition from the Telepanel

Period	m	s	Mean fraction of attrition per week
1st quarter 1993	2	24	0.0078
2nd–4th quarter 1993	4	22	0.0109
1994	13	13	0.0176

4. Practical Considerations

It is now appropriate to discuss the practical implications of the previously derived theoretical relationships. We will propose an optimal survey design for the budget survey of the Telepanel. The Telepanel is a multipurpose panel. Every week two questionnaires are administered. In 1993 the budget survey was conducted once in the first quarter and twice in each of the last three quarters. Since respondents were supposed to report on all purchased products, no second questionnaire could be administered (so it is as if a budget measurement takes two questionnaires). In 1994, the budget survey was conducted every week, but respondents only reported their purchases of meat, poultry and eggs. During that period the budget survey was administered in a parallel fashion with a questionnaire on some other topic (we take therefore a budget measurement to be one questionnaire).

The data we used on attrition from the Telepanel originate from 68 weeks: all weeks of 1993 and the first 16 weeks of 1994. Table 2 shows the values of m and s in the different

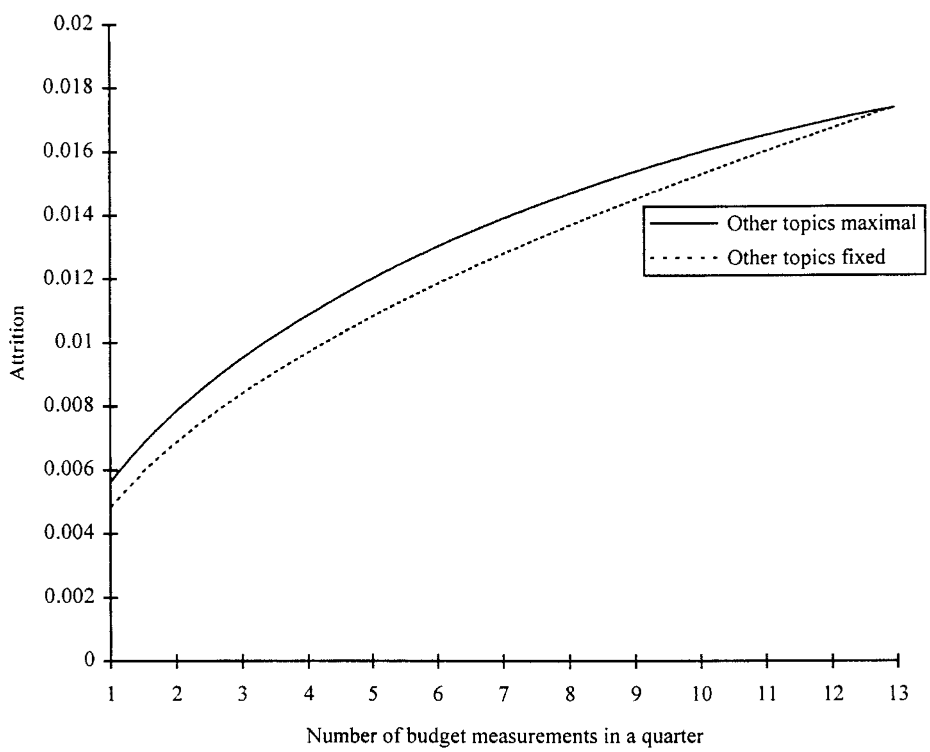

Fig. 2. The weekly attrition q as a function of the number of budget questionnaires in a quarter

periods in compliance with the above explanation, together with the mean attrition per week. We see that the mean attrition per week increased from 0.78% to 1.76% due to a higher response burden caused by more budget measurements.

Fitting the multivariate attrition Model (20) to the Telepanel data by use of nonlinear regression, gave us the following values (and asymptotic standard errors) of the parameters: $\lambda = 0.00245$ (a.s.e. $= 0.00943$), $\alpha = 0.518$ (a.s.e. $= 0.433$) and $\beta = 0.251$ (a.s.e. $= 1.064$). The fact that α is larger than β suggests that budget questionnaires cause greater response burden than those on other topics, although this statement is insufficiently supported by the empirical data. In our data the number of questionnaires on other topics is maximal ($s = 26 - m$), and the attrition in terms of m is shown as the solid line in Figure 2 (note that it is actually a step function).

In order to optimise the research design for the budget survey we will fix s, the number of questionnaires on other topics, to the constant number of 13. The attrition as a function of m for this situation can also be seen in Figure 2 (dashed line). It is below the line with a maximal number of questionnaires on other topics because of the lower response burden. We will use this function for the attrition in order to find an optimal value of m for the variance $\mathrm{var}(X^{(t+1)} - X^{(t)})$.

Having estimated the parameters of the attrition model, we find the number of budget measurements that minimises the variance for a given product by application of Equation (21). Figure 3 shows the number of measurements as a function of ρ. We see that if the

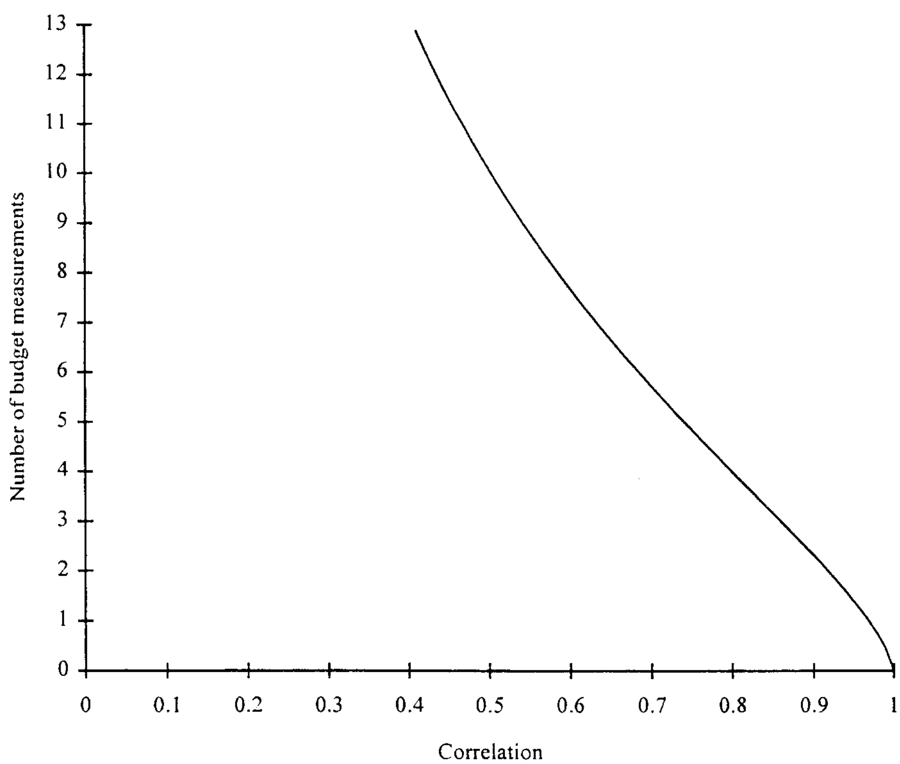

Fig. 3. The optimal number of budget measurements in the Telepanel as a function of ρ

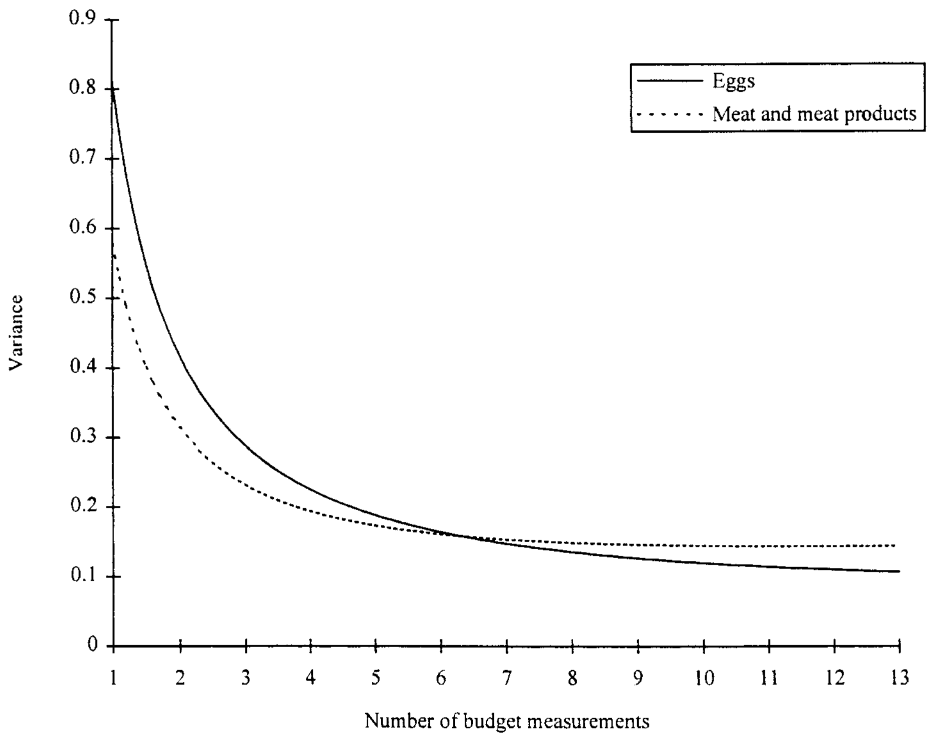

Fig. 4. $Var(X^{(t+1)} - X^{(t)})$ in the Telepanel for 'eggs' and 'meat and meat products' as a function of m

correlation of the product is low, it is optimal to measure on a continuing basis, i.e., take measurements in all 13 weeks of a quarter. If correlations are higher than 0.40, it is optimal to take fewer measurements.

For most of the products we presented in Table 1, the optimal design is to take measurements on a continuing basis. For the product 'meat and meat products,' which exhibits a correlation of 0.45, it is desirable to take 11 measurements in a quarter. Although this suggests that a design with continuing measurements is the best, we like to point out that there is a large region in which the obtained results are very close to being optimal.

Figure 4 shows $\mathrm{var}(X^{(t+1)} - X^{(t)})$ as a function of m both for the product 'eggs' ($\rho = 0.24$) and for 'meat and meat products' ($\rho = 0.45$). As mentioned before, for $\rho = 0.45$ the minimum is near $m = 11$. Note further that for $\rho = 0.24$ the variance keeps decreasing with m. Yet the picture shows clearly that there would be a very small increase in variance if the number of measurements were fixed at 7. This minimum, however, is rather flat. It follows then that a sub-optimal design does not give results that are much worse.

5. Summary and Conclusions

In panel research, especially with a Telepanel, research design is a complex issue, as there are many factors to be considered. Even in a perfect world, where all respondents happily fill in their questionnaires without measurement error and without being bored by the many detailed questions, there are complex optimisation problems arising because products are bought in different patterns, for which different sampling schemes are optimal. In this chapter we restricted ourselves to global product categories. The justification for this is that such categories follow more or less the same patterns each week. We did not assume, however, a perfect world, but instead we considered that respondents may become overloaded with their task. Under some very strict model assumptions we were able to calculate the variance of difference scores between two periods. Under even stricter model assumptions we derived expressions for m_0, the optimum number of weeks to be observed in a quarter or a year. In our practical situation it seemed reasonable to have a design where respondents fill in a questionnaire for one of every two weeks.

There is still a world to be discovered in this field. On the one hand the assumption that all respondents drop out of the panel with equal probability is not very realistic. It is more likely that there is a group of faithful respondents who, if it were up to them, would stay in the panel for life, and another group who drop out very rapidly. On the other hand there are products that are bought with longer intervals than one week. When the purchasing process of such products is fitted to some statistical model, a pattern of correlations between different weeks may come out that would not necessarily lead to untractable results. Furthermore other classes of estimators (like composite estimators) may improve the results we have provided here. Models to correct for measurement error (which have not been considered in this article), may also have an impact on sampling design and estimation. Finally, time series models from which empirical Bayes estimators can be derived, can be used for optimal estimation of consumption.

6. Appendix

6.1. Design of the multipurpose panel

Let the n individuals be randomly assigned to L groups $G_1, G_2, \ldots, G_L$. The group of individual i we denote by $G(i)$. The groups correspond to the sets of weeks $\Gamma_1, \Gamma_2, \ldots, \Gamma_L$. Each set Γ_l consists of m out of M weeks. The design is such that for every week j the number of individuals that have week j in their set $|i : j \in \Gamma_{G(i)}| = nm/M$, and for every combination (j, k) of weeks we have $|i : j, k \in \Gamma_{G(i)}| = nm(m-1)/\,(M(M-1))$. In other words in each week the budget survey is conducted for a fraction m/M of the respondents and in each combination of weeks for a fraction $m(m-1)/(M(M-1))$ of the respondents. Now let $X_{ij}^{(t)}$ be the amount of purchases by household i in week j of period t if $j \in \Gamma_{G(i)}$. This definition is a slight generalisation of the definition in the main text. Now let us calculate the variances and the covariances of the $X_j^{(t)}$'s, defined according to Equation (6),

$$\text{var}(X_j^{(t)}) = \frac{N^2M^2}{n^2m^2} \sum_{i:j\in\Gamma_{G(i)}} \text{var}(X_{ij}^{(t)}) = \frac{N^2M}{nm}\sigma^2$$

and for $k > j$

$$\text{cov}(X_j^{(t)}, X_k^{(t)}) = \frac{N^2M^2}{n^2m^2} \sum_{i:j,k\in\Gamma_{G(i)}} \text{cov}(X_{ij}^{(t)}, X_{ik}^{(t)}) = \frac{N^2M(m-1)}{nm(M-1)}\rho\sigma^2 p^{k-j}$$

The covariances between weeks in two successive periods are similar, except for a factor p^M for the attrition in M weeks. So we find for $k > j$

$$\mathrm{cov}(X_j^{(t)}, X_k^{(t+1)}) = \frac{N^2 M(m-1)}{nm(M-1)} \rho \sigma^2 p^{M+k-j}$$

from which identities (3) and (4) can be easily derived.

6.2. *Correlation matrix of weekly purchases of eggs in first half year of 1994*

	.04	.20	.21	.23	.18	.29	.26	.18	.18	.25	.13	.25	.12	.21	.25	.29	.14	.25	.19	.24	.21	.26	.21	.22	.22
.04		.14	.18	.16	.21	.22	.16	.16	.15	.17	.16	.10	.17	.18	.20	.18	.22	.15	.32	.07	.31	.19	.19	.29	.22
.20	.14		.11	.17	.23	.15	.21	.19	.20	.23	.18	.25	.16	.32	.21	.20	.24	.18	.22	.32	.29	.22	.15	.16	.25
.21	.18	.11		.15	.21	.21	.24	.27	.28	.17	.21	.27	.25	.17	.26	.14	.19	.24	.26	.17	.32	.28	.20	.24	.18
.23	.16	.17	.15		.14	.20	.23	.17	.26	.28	.22	.26	.21	.27	.26	.28	.23	.28	.24	.26	.27	.28	.22	.22	.21
.18	.21	.23	.21	.14		.25	.28	.24	.23	.25	.32	.26	.22	.36	.25	.28	.29	.27	.39	.23	.34	.19	.31	.22	.21
.29	.22	.15	.21	.20	.25		.18	.16	.33	.27	.14	.22	.16	.25	.28	.28	.12	.37	.23	.20	.20	.32	.16	.25	.22
.26	.16	.21	.24	.23	.28	.18		.20	.32	.20	.29	.30	.18	.19	.30	.29	.30	.35	.31	.31	.32	.19	.27	.29	.28
.18	.16	.19	.27	.17	.24	.16	.20		.09	.22	.13	.21	.24	.18	.23	.32	.23	.24	.29	.24	.22	.21	.29	.18	.22
.18	.15	.20	.28	.26	.23	.33	.32	.09		.14	.30	.19	.29	.21	.30	.15	.23	.33	.30	.18	.29	.23	.21	.22	.22
.25	.17	.23	.17	.28	.25	.27	.20	.22	.14		.14	.30	.23	.37	.28	.29	.24	.30	.23	.27	.20	.19	.26	.15	.25
.13	.16	.18	.21	.22	.32	.14	.29	.13	.30	.14		.17	.19	.23	.32	.17	.26	.32	.28	.28	.30	.16	.20	.23	.18
.25	.10	.25	.27	.26	.26	.22	.30	.21	.19	.30	.17		.12	.22	.21	.22	.22	.20	.24	.28	.23	.24	.21	.27	.24
.12	.17	.16	.25	.21	.22	.16	.18	.24	.29	.23	.19	.12		.15	.27	.20	.23	.29	.18	.21	.26	.22	.17	.22	.23
.21	.18	.32	.17	.27	.36	.25	.19	.18	.21	.37	.23	.22	.15		.20	.33	.31	.34	.35	.33	.32	.34	.24	.27	.21
.25	.20	.21	.26	.26	.25	.28	.30	.23	.30	.28	.32	.21	.27	.20		.18	.26	.37	.33	.27	.38	.28	.28	.35	.22
.29	.18	.20	.14	.28	.28	.28	.29	.32	.15	.29	.17	.22	.20	.33	.18		.23	.27	.30	.39	.29	.29	.27	.30	.30
.14	.22	.24	.19	.23	.29	.12	.30	.23	.23	.24	.26	.22	.23	.31	.26	.23		.20	.34	.22	.31	.23	.28	.23	.29
.25	.15	.18	.24	.28	.27	.37	.35	.24	.33	.30	.32	.20	.29	.34	.37	.27	.20		.21	.39	.29	.34	.31	.26	.25
.19	.32	.22	.26	.24	.39	.23	.31	.29	.30	.23	.28	.24	.18	.35	.33	.30	.34	.21		.16	.42	.36	.37	.37	.27
.24	.07	.32	.17	.26	.23	.20	.31	.24	.18	.27	.28	.28	.21	.33	.27	.39	.22	.39	.16		.23	.26	.18	.24	.34
.21	.31	.29	.32	.27	.34	.20	.32	.22	.29	.20	.30	.23	.26	.32	.38	.29	.31	.29	.42	.23		.21	.34	.44	.30
.26	.19	.22	.28	.28	.19	.32	.19	.21	.23	.19	.16	.24	.22	.34	.28	.29	.23	.34	.36	.26	.21		.13	.26	.31
.21	.19	.15	.20	.22	.31	.16	.27	.29	.21	.26	.20	.21	.17	.24	.28	.27	.28	.31	.37	.18	.34	.13		.25	.30
.22	.29	.16	.24	.22	.22	.25	.29	.18	.22	.15	.23	.27	.22	.27	.35	.30	.23	.26	.37	.24	.44	.26	.25		.19
.22	.22	.25	.18	.20	.21	.22	.28	.22	.22	.25	.18	.24	.23	.21	.22	.30	.29	.25	.27	.34	.30	.31	.30	.19	

6.3. *First- and second-order approximations in the single-purpose panel design*

We start from Equation (7). Writing $q = 1 - p$ and $c_1 = N^2 M^2 \sigma^2 / n$ we can write for the variance of $X^{(t)}$

$$\begin{aligned}
\mathrm{var}(X^{(t)}) &= \frac{c_1}{m^2}\left(m + \frac{2\rho(m-1)(1-q)}{q} - \frac{2\rho(1-q)^2(1-(1-q)^{m-1})}{q^2} \right) \\
&= \frac{c_1}{m^2}\Bigg(m - 2\rho(m-1) - 2\rho(1-(1-q)^{m-1}) \\
&\quad + \frac{2\rho(m-1) + 4\rho(1-(1-q)^{m-1})}{q} - \frac{2\rho(1-(1-q)^{m-1})}{q^2} \Bigg)
\end{aligned}$$

$$
\begin{aligned}
&\approx \frac{c_1}{m^2}\Bigg(m - 2\rho(m-1) - 2\rho\left\{\binom{m-1}{1}q - \binom{m-1}{2}q^2\right\}\\
&\quad + \frac{1}{q}\left[2\rho(m-1) + 4\rho\left\{\binom{m-1}{1}q - \binom{m-1}{2}q^2 + \binom{m-1}{3}q^3\right\}\right]\\
&\quad - \frac{1}{q^2}\left[2\rho\left\{\binom{m-1}{1}q - \binom{m-1}{2}q^2 + \binom{m-1}{3}q^3 - \binom{m-1}{4}q^4\right\}\right]\\
&= \frac{c_1}{m}\left(1 + (m-1)\rho - \frac{1}{3}(m-1)(m+1)\rho q + \frac{1}{12}(m-1)(m-2)(m+1)\rho q^2\right)
\end{aligned}
$$

In a similar way we can approximate the covariance between $X^{(t)}$ and $X^{(t+1)}$

$$
\begin{aligned}
\mathrm{cov}(X^{(t)}, X^{(t+1)}) &= \frac{c_1}{m^2}\rho(1-q)\{1-(1-q)^m\}^2/q^2\\
&= \frac{c_1}{m^2}\left(\frac{-\rho\{1-(1-q)^m\}^2}{q} + \frac{\rho\{1-(1-q)^m\}^2}{q^2}\right)\\
&\approx \frac{c_1}{m^2}\Bigg(\frac{-\rho}{q}\left\{\binom{m}{1}q - \binom{m}{2}q^2 - \binom{m}{3}q^3\right\}^2\\
&\quad + \frac{\rho}{q^2}\left\{\binom{m}{1}q - \binom{m}{2}q^2 - \binom{m}{3}q^3\right\}^2\Bigg)\\
&= \frac{c_1}{m}\left(m\rho - m^2\rho q + \frac{1}{12}m(m-1)(7m+1)\rho q^2\right)
\end{aligned}
$$

This enables us to compute the second order approximation of the variance of $X^{(t+1)} - X^{(t)}$:

$$
\mathrm{var}(X^{(t+1)} - X^{(t)}) \approx \frac{2c_1}{m}\left(1 - \rho + \frac{1}{3}(2m^2+1)\rho q - \frac{1}{12}(m-1)(6m^2+2m+2)\rho q^2\right)
$$

6.4. First- and second-order approximations in the multi-purpose panel design

We start with Equation (9). Writing $c_2 = N^2 M\sigma^2/n$ we can write for the variance of $X^{(t)}$:

$$
\begin{aligned}
\mathrm{var}(X^{(t)}) &= \frac{c_2}{m}\left(M + \frac{2\rho(m-1)(1-q)}{q} - \frac{2\rho(m-1)(1-q)^2(1-(1-q)^{M-1})}{(M-1)q^2}\right)\\
&\approx \frac{c_2}{m}\Bigg(M - 2\rho(m-1)\Bigg\{\frac{1}{q} - 1\\
&\quad - \frac{1}{(m-1)q^2}\left[\binom{M-1}{1}q - \binom{M-1}{2}q^2 + \binom{M-1}{3}q^3 - \binom{M-1}{4}q^4\right]\\
&\quad + \frac{2}{(M-1)q}\left[\binom{M-1}{1}q - \binom{M-1}{2}q^2 + \binom{M-1}{3}q^3\right]\\
&\quad - \frac{1}{(M-1)}\left[\binom{M-1}{1}q - \binom{M-1}{2}q^2\right]\Bigg\}\Bigg)
\end{aligned}
$$

$$= \frac{Mc_2}{m}\left(1 + (m-1)\rho - \frac{1}{3}(m-1)(M+1)\rho q + \frac{1}{12}(m-1)(M-2)(m+1)\rho q^2\right)$$

In a similar way we can approximate the covariance between $X^{(t)}$ and $X^{(t+1)}$

$$\begin{aligned}\mathrm{cov}(X^{(t)}, X^{(t+1)}) &= \frac{c_2}{m}\left(M\rho(1-q)^M\right.\\ &\quad \left. + \frac{m-1}{M-1}\rho(1-q)^M\left[\frac{2(M-1)(1-q)}{q} - \frac{2(1-q)^2(1-(1-q)^{M-1})}{q^2}\right]\right)\\ &\approx \frac{Mc_2}{m}\left(m\rho - \left\{M + \frac{1}{3}(m-1)(4M+1)\right\}\rho q\right.\\ &\quad \left. + \left\{\frac{1}{2}M(M-1) + \frac{1}{12}(m-1)(11M^2 - 3M - 2)\right\}\rho q^2\right)\end{aligned}$$

so for the variance of $X^{(t+1)} - X^{(t)}$ we find:

$$\mathrm{var}(X^{t+1)} - X^{(t)}) \approx \frac{2Mc_2}{m}\left(1 - \rho + Mm\rho q - \frac{1}{6}M(5Mm - 2M - m - 2)\rho q^2\right)$$

7. References

Hamblin, R.L. (1974). Social Attitudes: Magnitude Measurement and Theory. In H.M. Jr. Blalock, Measurement in the Social Sciences, Theories and Strategies. Macmillan Press, London, 61–120.

Harrison, R. (1991). Respondent Burden and Respondent Fatigue in the 1988–89 Australian Household Expenditure Survey. Paper prepared for the Workshop on Diary Surveys, February, Stockholm, Sweden.

Hausman, J.A. and Wise, D.A. (1979). Attrition Bias in Experimental and Panel Data: The Gary Income Maintenance Experiment. Econometrica, 47, 455–473.

Kish, L. (1965). Survey Sampling. New York, Wiley.

Lindström, H.L., Lindkvist, H., and Näsholm, H. (1989). Design and Quality of the Swedish Family Expenditure Survey. Proceedings of the Fifth Annual Research Conference, U.S. Bureau of the Census, 501–514.

Lyberg, L. (1991). Reducing Nonresponse Rates in Family Expenditure Surveys by Forming Ad Hoc Task Forces. Paper prepared for the Workshop on Diary Surveys, February, Stockholm, Sweden.

Nevraumont, U. (1991). Evaluation of the Canadian Food Diary Survey. Paper prepared for the Workshop on Diary Surveys, February, Stockholm, Sweden.

Ribe, M.G. (1991). A Study of Errors in Swedish Consumption Data. Paper prepared for the Workshop on Diary Surveys, February, Stockholm, Sweden.

Saris, W.E., Prastacos, P., and Recober, M.M. (1992). CASIP: A Complete Automated System for Information Processing in Family Budget Research. In New Technologies and Techniques for Statistics. Proceedings of the Conference, Bonn, February, 80–87.

Silberstein, A.R. and Jacobs, C.A. (1989). Symptoms of Repeated Interview Effects in the Consumer Expenditure Interview Survey. In Kasprzyk, D., Duncan, G., Kalton, G., and Singh, M.P. (eds.) Panel Surveys. New York, Wiley, 289–303.

Stasny, E.A. (1987). Some Markov-Chain Models for Nonresponse in Estimating Gross Labor Force Flows. Journal of Official Statistics, 3, 359–373.

Statistics Netherlands (1992). Gebruikershandboek budgetonderzoek. Voorburg/Heerlen, Centraal Bureau voor de Statistiek. (In Dutch).

Van de Pol, F. (1989). Reducing Panel Bias. A Review of Sampling Designs. Kwantitatieve Methoden, 32, 41–63.

Van de Pol, F. (1993). Weighting Panel Survey Data. Paper presented at Symposium on Analysis of Longitudinal Data, Tampere, Finland.

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