Journal of Official Statistics
Vol. 3, No. 3, 1987, pp. 223–233

Seasonal Adjustment of the Money Stock in the United States

Dennis E. Farley and Yueh-Yun C. O'Brien[1]

Abstract: The monetary aggregates in the United States are seasonally adjusted using X-11-ARIMA for monthly series and a model-based procedure for weekly series. Where monthly and weekly versions of the same series are published, a balancing procedure ensures that the monthly and weekly factors are consistent. Before annual publication, usually in February, seasonal adjustments are carefully reviewed by Federal Reserve analysts familiar with money stock data. Attempts are made, however, to incorporate all relevant information about the series into the forecasting models (or into prior adjustments to the data), thereby limiting subjective elements in the seasonal factors. There remain several unanswered questions concerning seasonal adjustment of the money stock.

Key words: Seasonal adjustment; money stock; time series; U.S. Federal Reserve; Census X-11; X-11-ARIMA.

1. Introduction

In the United States there are three principal measures of the money stock, known as M1, M2 and M3.[2] With its emphasis on currency and checkable deposits, M1 is the aggregate most closely associated with the idea from economic theory of a transactions demand for money. As the aggregates broaden through M2 to M3, each one augmenting the one before, more non-transaction components are included, reflecting money as a store of value rather than a medium of exchange. At the level of M3 there are several components, such as large-denomination time deposits

[1] Economists, Division of Research and Statistics, Board of Governors of the Federal Reserve System, Washington, DC, 20551, U.S.A. We would like to acknowledge the helpful comments of our colleagues Thomas Simpson, Brian Madigan, David Jones and David Pierce. Comments by two anonymous referees also improved this article. Views expressed are personal and do not necessarily represent the views of the Federal Reserve System.

[2] M1 consists of currency, travelers cheques, demand deposits and other checkable deposits. Each component is seasonally adjusted separately and then added to the others to form seasonally adjusted M1. M2 includes M1 plus savings and small-denomination time deposits. M3 takes in M2 plus large-denomination time deposits. All of the non-M1 components of M2 are summed and then seasonally adjusted as a whole. The non-M2 components of M3 are also summed and seasonally adjusted as a whole. Several, but not all, of these non-M1 components are also seasonally adjusted separately. Thesebrief definitions are not complete since many other components, involving repurchase agreements, Eurodollar accounts and money market mutual funds, are parts of the monetary aggregates. In addition to M1, M2 and M3, two other measures, known as L and Debt, are currently reported. These measures are very broad, including assets of varying degrees of liquidity that are not usually thought of as money.

The interested reader is referred to any issue of the monthly *Federal Reserve Bulletin.* In Table 1.21 of the statistical appendix the footnotes provide a good introduction to the composition of money stock measures. More details about any component are available by writing to the Banking Section, Division of Research and Statistics, Board of Governors of the Federal Reserve System, Washington, D.C. 20551, U.S.A.

(those with face values of $100 000 or more and maturity longer than a week) that fall under the heading of managed liabilities, meaning that depository institutions adjust the amounts outstanding of these components in line with their needs for funds. There is, therefore, a transition from M1 and most of M2, which are often viewed as mostly demand-determined, to the remainder of M2 and M3, which are seen as mostly supply-determined.

Reported by the Federal Reserve System each week of the year,[3] the money stock measures find their way into newspapers, financial newsletters, the computer networks of private information services and, occassionally, television news shows. In the past most of the attention paid by the public to these numbers has been focused on seasonally adjusted M1. Less attention has been given to seasonally adjusted M2 or M3 or their components, either seasonally adjusted or not seasonally adjusted. The broader measures have been attracting more attention in large part because M1 has recently tracked the economy less well than in previous years. Along with the aggregate measures of the money stock, weekly Federal Reserve publications contain about fifty not seasonally adjusted monthly components.[4] Of these, about twenty-five are also published seasonally adjusted. While every component, seasonally adjusted or not, appears on a monthly basis, just twenty-three components appear on a weekly basis not seasonally adjusted, and only seven of those appear seasonally adjusted.[5]

Considerable resources are devoted to developing seasonal adjustment techniques and to seasonally adjusting the money stock, mainly because the aggregate measures, M1, M2 and M3, have been of such importance for policy purposes. Policymakers at the Federal Reserve set objectives for growth in the seasonally adjusted money stock measures, while the private sector uses these series both to monitor policy decisions and to forecast the effects of those decisions on other economic variables.

In the following discussion, the seasonal adjustment techniques employed at the Federal Reserve are described and illustrated with an example. The method to ensure consistency between monthly and weekly results is also discussed. The last section presents some unresolved issues relating to seasonal adjustment of the monetary aggregates.

[3] The weekly release is entitled "Money Stock, Liquid Assets and Debt Measures" and is generally known as "the H.6", after its code in the catalogue of Federal Reserve statistical releases. The release is normally issued on Thursday at 4:30 p.m., Eastern Time in New York City by the Federal Reserve Bank of New York and, simultaneously, in Washington, D.C. by the Board of Governors of the Federal Reserve System. When holidays are on Thursdays, it is published on Fridays.

The basic data are compiled from reports of deposits submitted weekly by most large depository institutions in the U.S. Estimates are made for some small institutions, which report less frequently (either quarterly or annually). More detail on the reporting system is available from the Banking Section at the address cited above.

[4] The H.6 release is published weekly, but not all series are observed weekly. If a particular component is only observed monthly, then only monthly data are reported on the weekly release.

[5] They are: the currency component of M1, the travelers cheque component of M1, the demand deposit component of M1, the other checkable deposit component of M1, the commercial bank savings deposit component of M2, the commercial bank small-denomination time deposit component of M2 and the commercial bank large-denomination time deposit component of M3.

Weekly seasonally adjusted M1 is calculated as the sum of its seasonally adjusted components. Since some components of M2 and M3 are not available weekly, no weekly versions of these aggregates are published.

2. Methodology

2.1. Monthly seasonal adjustment procedures

For monthly data the seasonal adjustment procedures used by the Federal Reserve are based on the method of X-11-ARIMA[6] (Dagum (1979) and Board of Governors (1981)). For a particular series, time series models are used to provide forecasts for at least a year beyond the last month of actual data. These forecasts are then reviewed. In extreme cases some forecasts are modified judgmentally if it is clear from other information that the model has failed to capture an obvious pattern. An extended sample of data, actual and forecasted, is then processed through Census X-11 (Shiskin et al. (1967)) yielding seasonally adjusted levels and seasonal factors. The seasonal factors estimated by X-11 for the forecast period are used to seasonally adjust future observations as they arrive. Since seasonal adjustments are typically revised once a year, the length of the forecast should be at least twelve months to provide enough projected seasonal factors for use until the next estimation. The Census X-11 program can produce its own set of projected seasonal factors for twelve months beyond the last data point, in the absence of ARIMA or other forecasts of the underlying series, but these factors are not used. The whole purpose of the X-11-ARIMA approach is to obtain projected seasonal factors that are better (closer to the final revised factors) than those projected by X-11 alone.

The models used to generate forecasts for a series need not be ARIMA models. In fact, the models used for money stock components are mixtures of both regression and time-series approaches. Deterministic influences such as fixed seasonalities, trading-day variation, outliers, interventions, and special holiday or other calendar effects, are estimated via least-squares regression on the appropriate explanatory variables (mainly dummy variables or periodic functions of time such as sines and cosines, though economic or institutional variables could also be included). The residuals from this regression are then modeled as ARIMA processes.

The model form most often used for money stock data is:

$$\phi(B)(1-B)(w_t - f_t) = \theta(B)a_t$$

where w_t are the data, or frequently the natural logarithms of the data, and f_t is a regression component representing fixed (deterministic) effects. The residuals, $w_t - f_t$, are what remain after fixed effects are removed, and these residuals are modeled as an ARIMA process, possibly with terms at seasonal lags to capture seasonality in the data not adequately modeled as a fixed effect (Pierce (1978)). B is a back-shift operator such that $B^k w_t = w_{t-k}$. The factor $(1-B)$, therefore, denotes the first differencing, which is usually sufficient to induce stationarity in our data. The autoregressive and moving-average parts of the model are represented as polynomials in the operator B, denoted by ϕ and θ respectively. The error term a_t is assumed to be white noise, or a sequence of independent and identically distributed variates with mean 0 and variance σ^2.

In practice, the parameters of the combined model – namely, the coefficients in the polynomials ϕ and θ, the variance of a_t and the coefficients in the regression part, f_t – are estimated simultaneously by nonlinear least squares. The important point is that forecasts are used to extend the sample available for seasonal adjustment. These forecasts could just as well come from an econometric model or a judgmental projection as from an ARIMA model.

One may well ask why, after having been specified with some care, a model is not used

[6] Auto-Regressive Integrated Moving Average, See Box and Jenkins (1976).

directly for seasonal adjustment. Why not decompose the model into seasonal and non-seasonal parts and eliminate the X-11 step entirely? On the one hand, it is possible in many cases to do this successfully and provide a model-based seasonal adjustment (Pierce (1978)). On the other hand, the abandonment of X-11 as the official method of seasonal adjustment is a difficult step to take. Census X-11 embodies an empirical ratio-to-moving-average approach to seasonal adjustment that goes back at least to Macaulay (1931). This approach has attained widespread acceptance by government agencies and by private producers and users of statistics. As a computer program, X-11 is fast, easy to use, and gives good results most of the time. There is now a vast body of experience behind X-11 as a seasonal adjustment procedure. An unfortunate consequence of this widespread acceptance is that the X-11 approach has acquired a reputation for objectivity that is not warranted.

Many users of statistics believe that there is a unique, identifiable seasonal component in a data series, for example in monthly money stock data, and that Census X-11 removes it. Most producers of statistics know, however, that the decomposition of a time series into seasonal and nonseasonal parts is not unique, or even identifiable, until some assumptions have been made about the shapes of those parts. In other words, models (either as implicit mental pictures or as explicit statistical forms) are essential to achieving a useful seasonal adjustment. The X-11 procedure itself depends implicitly upon models, as shown in an important article by Cleveland and Tiao (1976). But this fact is not as widely appreciated outside the academic community as it should be. Explicit model-based seasonal adjustment is under serious consideration at the Federal Reserve, but for this approach to be adopted officially, more experience with the models for money stock data will be needed. At the same time a stronger effort at educating users of these data is desirable so that public confidence will not be eroded by a shift away from Census X-11.

2.2. *Weekly seasonal adjustment procedures*

Attempts have been made to develop a weekly seasonal adjustment procedure in the spirit of Census X-11, that is, as a ratio-to-moving-average, without explicit consideration of a model for weekly time series. On this side of the Atlantic efforts were made at the U.S. Bureau of the Census in the late 1960s, and the U.S. Bureau of Labor Statistics and the Board of Governors of the Federal Reserve System in the 1970s. The main references are Somer (1969), Plewes and Altschuler (1977), Zeller (1972) and Nickelsburg (1973).

Until 1985, the Federal Reserve used the procedure described in Nickelsburg (1973) which contained some of the moving-average features of X-11. Monthly seasonally adjusted levels were placed in the middle of each month and then interpolated with straight lines. At weekly intervals the interpolated values were divided into not seasonally-adjusted weekly levels (located on the last day of the week) to get initial estimates of weekly seasonal factors. These initial factors were then interpolated linearly to obtain factors for days of the year on which weeks did not end. This whole set of initial factors for every day of every year in the sample was then smoothed, across years, with moving averages. The resulting estimates of weekly seasonal factors were then adjusted to average the monthly factors from X-11, taking into account that some weeks were partly in one month and partly in the next month.

The weekly seasonal factors obtained from the foregoing procedure often were not adequate to smooth out recurring within-month movements in the data. Judgmental changes resulting from many hours of careful review were needed before the final set of weekly seasonal factors could be published. In the early 1980s efforts were made to develop a weekly model-based procedure for seasonal adjustment. The main references are Pierce et al. (1984) and Cleveland and Grupe (1981). It was hoped that a weekly time series model would better capture the withinmonth patterns of money stock data. Weekly results would still have to be made consistent with monthly results from X-11, but the initial set of weekly seasonal factors would reduce the time spent on judgmental review. These efforts bore fruit in 1985 when the Federal Reserve began using model-based estimates as the starting point for further analysis. Since then experience has shown that the desired savings in time and the desired improvements in the final weekly seasonal factors are being realized.

Instead of constraining weekly seasonal factors to monthly seasonal factors as was done before 1985, the current method of weekly seasonal adjustment constrains a weighted average of weekly levels in each month to the level for that month, both seasonally adjusted (s.a.) and not seasonally adjusted (n.s.a.). The weighting scheme for the s.a. data is different from that for the n.s.a. data. It is assumed that seasonal movements are removed during seasonal adjustment, so that the s.a. weekly data contain only trend and noise. The deviations of noise from trend are further assumed to have a zero mean. Under these assumptions, weights for the weekly s.a. data are set equal to the number of days in each week that are in a given month, divided by the total number of days in that month. For example, the weekly average levels (for Tuesday to Monday banking weeks) for the fourth quarter of 1985 are located on October 7, 14, 21, 28, November 4, 11, 18, etc. The weighted average of weekly levels for October is $(1/31)\,(7x_1+7x_2+7x_3+7x_4+3x_5)$ where x_1 is the level for the week ending October 7, x_2 is the level for the week ending October 14, ..., x_5 is the level for the split week ending November 4. The week ending November 4 has three days in October.

For the n.s.a. data the weighting scheme is similar except for the "split weeks" that overlap two months. For most M1 components existing daily n.s.a. data, particularly for the split weeks, show definite intra-weekly patterns. Where historical daily data are available, weights (called split factors) for the two portions of split weeks are estimated according to the number of days in that week in each month. In the example of the preceding paragraph, there are two split factors calculated for the split week ending November 4, 1985. One represents the weight given to the first three days of the week (last three days of October) and the other represents the weight given to the remaining four days of the week (first four days of November). The former, f_1, is derived as the ratio of the average of daily n.s.a. levels for the first three days of the week to the average n.s.a. level of that week. The latter, f_2, is estimated as the ratio of the average of daily n.s.a. levels for the remaining four days of the week to the average n.s.a. level of that week. Thus, the weighted average of these two split factors should equal unity, using as weights the number of days included in each split factor divided by seven: $(3/7)f_1 + (4/7)f_2 = 1$. The weighted average of weekly n.s.a. levels for October 1985 is then $(1/31)\ (7x_1+7x_2+7x_3+7x_4+3f_1x_5)$ where x_1 through x_5 are the same as above.

To meet the constraints, the weekly levels are balanced to monthly levels by the algorithm in the appendix. The balancing program produces a set of weekly changes that have the least sum of squared deviations from a set of designated weekly changes, subject to the constraints described above. For historical periods, the balanced weekly s.a. levels are divided into actual weekly n.s.a. levels to derive the final weekly seasonal factors. For forecast periods, the balanced weekly s.a. levels are divided into balanced weekly n.s.a.

levels to get weekly seasonal factors, which are subject to judgmental review.[7]

If the reviewer determines that a particular n.s.a forecast is unlikely to occur under expected normal conditions, that forecast is revised. Iterations between balancing and judgmental review are continued until the reviewer is satisfied with the weekly seasonal factors. Judgmental review is subjective, but it is not arbitrary. There are certain influences, such as corporate tax dates and government transfer payments, whose effects on money stock components are significant, but are not yet incorporated in our weekly models. In such cases the judgment of an experienced analyst is indispensable in obtaining a useful set of weekly seasonal factors.

3. An Example – The Currency Component of M1

3.1. Monthly seasonal adjustment of currency

In this section the currency component of M1 is used to illustrate the procedures employed for 1987 seasonal adjustment. As in Section 2.1 monthly average currency levels[8] are modeled with regression components for fixed seasonal and trading-day effects, and with an ARIMA specification for the residuals.[9] This model, estimated over the period January 1975 – December 1986 is used to forecast 15 monthly n.s.a. levels for the period January 1987 – March 1988. The forecasts are then appended to historical currency data for January 1970 – December 1986, adjusted for trading-day effects,[10] and the extended sample is fed through the Census X-11 program to yield initial estimates of monthly seasonal factors for currency.

These initial results are examined on year-over-year plots (tier charts) of such measures as monthly n.s.a. levels, monthly n.s.a. changes, or the estimated seasonal factors themselves. As with most modeling, some experimentation is needed until the appropriate specification, based on goodness of fit and the reviewer's judgment about the reasonableness of the forecast, is obtained. In extreme cases, a forecast is modified judgmentally if it is clear that important information has not been incorporated in the model.

3.2. Weekly seasonal adjustment of currency

The weekly model for currency includes sines and cosines to capture most periodic fluctuations, eight dummy variables for holiday

[7] For historical periods actual n.s.a. levels and model-based s.a. levels provide the sets of designated, or desired, weekly changes used by the balancing program. For forecast periods model-based forecasts of n.s.a. levels supply one set of desired changes. The other set is not obtained from model-based forecasts of s.a. levels, but is just a set of constant weekly changes. These changes are determined by the difference between the last forecast and last historical monthly s.a. levels, divided by the number of weeks in between. In the forecast period we assume that proper seasonal adjustment would result in a smooth weekly series if there were no other influences on the data such as noise or changes in trend from policy decisions. The balancing program, therefore, begins at some starting point near the end of the historical period and constructs, in the forecast period, weekly s.a. changes that cumulate to given monthly s.a. levels and are as nearly constant as possible. Detailed description is given in Section 3.2.

[8] These are averages of daily levels within the month.

[9] The monthly currency model may be written as $(1-B)(w_t-f_t) = \theta_0 + (1-\theta_1 B)a_t$ where w_t is the log of monthly currency and the other symbols are as shown in Section 2.1 of the text. The weekly model for currency is slightly more complicated in that it involves a breakdown of the residuals from the regression into two components. One component is the stochastic nonseasonal and the other the stochastic seasonal part of the residual. Details of these models are available from the authors.

[10] Five years of monthly data are added to the beginning of the seasonal adjustment sample in order to avoid end-point problems in early 1975, which is the starting point for the weekly data. The model-based estimates of trading-day effects are used as a prior adjustment before running Census X-11.

effects, and an ARIMA specification for the residuals.[11] Estimation is over the period January 6, 1975 – January 5, 1987 with forecasts to April 4, 1988.[12] The results consists of initial sets of weekly n.s.a. levels, weekly seasonal factors and, therefore, weekly s.a. levels.

The weekly sample is then divided into two parts for the balancing program: one for January 6, 1975 – June 30, 1986 and the other covering July 7, 1986 – April 4, 1988. July 1986 is the latest month before the forecast period whose first week (Tuesday to Monday) ends on the seventh of the month. There is no overlapping split week between June and July 1986, so that these two subsamples may be balanced independently.

For the earlier period the initial model-based seasonal factors are used to adjust the actual n.s.a. levels. The resulting weekly s.a. levels are balanced to monthly s.a. levels, using the weighting scheme described in Section 2.2. Finally, these balanced weekly s.a. levels are divided into the actual n.s.a. data to yield final weekly seasonal factors.

For the later period more elaborate procedures are followed to derive the final weekly seasonal factors. For the weeks from July to December 1986, the initial model-based seasonal factors are used to adjust the actual weekly n.s.a. levels, yielding weekly s.a. levels and weekly s.a. changes. For the forecast weeks from January 1987 through March 1988, the weekly s.a. changes are set equal to a constant. This constant is the difference between monthly s.a. levels for March 1988 and December 1986, divided by the number of weeks (65) in between. In other words, we assume that, in the absence of noise and changes in policy, the path of the weekly s.a. data in the forecast period will be smooth and will be closely approximated by a constant change each week. The whole series of weekly s.a. changes (for July 7, 1986 – April 4, 1988) then becomes the designated, or desired, set of weekly changes used in the balancing program. The output from that program is another set of weekly s.a. changes that satisfy the monthly averaging constraints and are as close as possible to the desired changes.

These balanced weekly changes are then cumulated from a starting point (the s.a. level for June 30, 1986) to construct balanced weekly s.a. levels for July 7, 1986 – April 4, 1988. In the forecast period these levels are multiplied by the model-based seasonal factors to get new n.s.a. forecasts. Combining these with actual n.s.a. levels for July 7, 1986 – January 5, 1987, yields a series of n.s.a. weekly levels that can be balanced to given (actual and forecasted) monthly n.s.a. levels. The balanced weekly n.s.a. levels are then divided by the balanced weekly s.a. levels to obtain final weekly seasonal factors.

Judgmental review begins at this point by evaluating the (balanced) weekly n.s.a. levels and associated seasonal factors for the forecast period (January 12, 1987 – April 4, 1988). The process of revising particular weekly forecasts, and rebalancing them to the monthly constraints is iterated until the reviewer is satisfied.

4. Issues for the Future

Seasonal adjustment is far from being a cut-and-dried topic. To prove the point, we offer some unresolved issues facing the Federal Reserve that concern seasonal adjustment. We hope to generate discussion of these issues and to hear from other practitioners with similar problems. First, concurrent seasonal

[11] Since there are not exactly 52 weeks in a year, the use of 52 dummy variables to capture a periodic annual fluctuation (seasonality) in weekly data is not appropriate. The holidays represented are New Year's Day, President's Day (February), Easter (in March or April), Memorial Day (May), Independence Day (July 4), Labor Day (September), Thanksgiving Day (November) and Christmas Day.

[12] Weeks are dated on Monday, the last day of the seven-day (Tuesday to Monday) banking statement week in the United States.

adjustment seems an attractive alternative to the present method of developing seasonal factors once a year and using projected factors until the next year. In the concurrent approach seasonal adjustment factors are re-estimated, as each new datum arrives, using all the observations up to that point. Several studies (McKenzie (1984), Pierce and McKenzie (forthcoming), and Wolter (1986)) suggest that concurrent seasonal factors are closer to "final" seasonal factors than the projected factors are, where "final" values are those estimated after several years of data are available beyond the month of interest.

The questions about concurrent seasonal adjustment are practical rather than theoretical. Since all seasonal factors are re-estimated each month, how far back should revisions be published? In a concurrent X-11-ARIMA approach should new forecasted seasonal factors be published each month? An extension of the concurrent approach to weekly series would raise additional problems. Existing procedures would have to be streamlined as there would be no time for careful review of the results. A greater burden of realism would be placed on the models for weekly series so that they could assume, in an automatic way, functions now performed judgmentally. A possible benefit of the concurrent approach is that it would highlight the uncertainty in published money stock numbers. Users would see the data had been revised each week or month instead of once a year. There might even be a demand for publishing standard errors along with seasonally adjusted levels or growth rates. The Federal Reserve since 1984 has been publishing a concurrent X-11-ARIMA version of seasonally adjusted M1, on a monthly basis only and with revisions covering the previous fifteen months. So far there has been limited reaction from users, perhaps because the results have no official status.

Another issue concerns the objective of seasonal adjustment. If the important component of a series is its trend, one might attempt to filter out all short-period noise, not just the seasonal. Those who object to such adjustments usually contend that procedures for deciding what is trend and what is not are too arbitrary, whereas the seasonal component is well-defined. Further filtering beyond seasonal adjustment is viewed as unjustifiable "cooking" of the figures. Of course, such views are not logical, since the seasonal component is as arbitrary as any other unobserved component, but they do exist. In a sense, seasonality is easy to explain, but difficult to define. There are immediately understandable notions stemming from the earth's journey around the sun, the resulting changes in temperature, rainfall, daylight, etc., and the fluctuations in human activity in response to these changes. Seasonality, however, is almost never defined by relating it to its ultimate causes. Usually, it is defined in an empirical fashion as certain fluctuations, of (or near) a particular periodicity, which may be in the data; and such an approach can be applied equally to any other components which may be in the data, such as the trend, the trading-day effect, or the (irregular) noise. In an effort to smooth out noise in the weekly data, the Federal Reserve actually publishes a 4-week and a 13-week unweighted moving average of weekly M1, seasonally adjusted. These constructs are *ad hoc* (four weeks being about one month and thirteen weeks being about one quarter) and are not explicitly designed to obtain certain desired characteristics. A better approach might be to publish a trend estimate based on a model for the series, or, on a monthly basis, the trend estimate from Census X-11.

A third issue we will mention is what is called the policy seasonal (Poole and Lieberman (1972), Board of Governors (1976) and Ghysels (1984)). This issue concerns the seasonal adjustment of a series over which policymakers have some control. In the extreme case of perfect control there is no point to seasonal adjustment except as a descriptive technique for historical data. The future behavior of the series, including any seasonal movement, becomes a matter of choice. In real-world situations with imperfect control, failure to include the controller in a model of seasonality may lead to unintended results. Using its available instruments (open market operations, discount rates, and reserve requirements), the Federal Reserve can influence the trend component of the money stock. The measurement of trend, however, depends on the technique used for estimating the seasonal (and other) components. Census X-11 and similar techniques usually assume that the unobserved components of interest are mutually orthogonal, but this is manifestly false in the present example. Instead, a feedback loop is established between trend and seasonality with the Federal Reserve as controller. In this case estimates of one component affect the behavior of the other, or possibly of both, which then result in new estimates to further affect the behavior of one, or both components, and so on. In principle, such a scheme could lead to unintended movements in the money stock. Whether or not it does is an open question whose answer awaits the development of more comprehensive models of seasonality as part of the monetary control process.

The last issue we will mention is the proper level, for seasonal adjustment, of aggregation over time. In Section 2.2 above a procedure is described that forces weekly s.a. data to agree with a given monthly s.a. result, but more work remains to be done here. One could go the other way and construct the monthly results from the weekly results, thereby not using X-11 on monthly data. The Federal Reserve does this for M1 on an experimental basis and publishes the results along with the concurrent X-11-ARIMA version mentioned at the beginning of this section. But why stop with weekly data? We are just now acquiring enough daily data (collected on a consistent basis from several thousand of the largest depository institutions in the U.S.) to think about seasonally adjusting major components of M1, M2, and M3 daily. If satisfactory techniques can be developed, then seasonally adjusted averages over a week, a month, or even a quarter, can be built up from the days, and most consistency or "balancing" issues can be avoided. At present we have almost no experience with models for daily data. Future research efforts, however, should be directed toward this area because it offers, in principle, a solution to the vexing problem of making seasonally adjusted levels for different sampling intervals consistent with one another.

Appendix

Algorithm of Program to Constrain Weekly and Monthly Levels[1]

Given a vector of finalized monthly levels, $\mathbf{m}$, and a vector of designated weekly changes, $\mathbf{c}$, the balancing program solves for a vector of weekly changes, $\mathbf{x}$, that has the least sum of squared deviations from $\mathbf{c}$ subject to a constraint. The constraint is that a weighted average of weekly levels in each month must be equal to the level for that month. Since weekly levels can be expressed as the sums of an initial weekly level plus the accumulated weekly changes, the constraint can be stated as follows. The sum of an initial weekly level plus the weighted weekly changes in each month must be equal to the level for that month. Thus, we can write the logic of the balancing program as follows:

$$\underset{\mathbf{x}}{\text{minimize}} \; ||\, \mathbf{x} - \mathbf{c} \,||^2$$

$$\text{subject to } \mathbf{a} + \mathrm{K}\,\mathbf{x} = \mathbf{m} \qquad (1)$$

where $\mathbf{a}$ is a vector with each element equal to the level for the week prior to the first week covered by the balancing program and K is a matrix of weights that constrain weekly changes to monthly levels.

This is a constrained ordinary least squares (OLS) estimation problem, which can be solved by standard techniques.

Step 1: If $\mathbf{a} + \mathrm{K}\,\mathbf{x} = \mathbf{m}$ then $\mathbf{x}$ can be expressed as

$$\mathbf{x} = \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) - (\mathrm{I} - \mathrm{K}^+\mathrm{K})\,\mathbf{y} \qquad (2)$$

for some vector $\mathbf{y}$, where I is the identity matrix and K^+ denotes the generalized inverse of the matrix K.

Step 2: From step 1, we may write

$$\begin{aligned} ||\, \mathbf{x} - \mathbf{c} \,||^2 &= ||\, \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) - (\mathrm{I} - \mathrm{K}^+\mathrm{K})\,\mathbf{y} - \mathbf{c} \,||^2 \\ &= ||\, [\mathrm{K}^+ (\mathbf{m} - \mathbf{a}) - \mathbf{c}] - (\mathrm{I} - \mathrm{K}^+\mathrm{K})\,\mathbf{y} \,||^2 \\ &= ||\, \mathbf{b} - \mathrm{H}\,\mathbf{y} \,||^2 \end{aligned}$$

where $\mathbf{b} = \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) - \mathbf{c}$ (3)

and $\mathrm{H} = (\mathrm{I} - \mathrm{K}^+\mathrm{K})$. (4)

Thus, the original constrained optimization problem (1) can be transformed into the following unconstrained OLS problem:

$$\underset{\mathbf{y}}{\text{minimize}} \; ||\, \mathbf{b} - \mathrm{H}\,\mathbf{y} \,||^2. \qquad (5)$$

Step 3: The solution to (5) is

$$\mathbf{y} = \mathrm{H}^+ \mathbf{b} + (\mathrm{I} - \mathrm{H}^+\mathrm{H})\,\mathbf{z} \qquad (6)$$

for some vector $\mathbf{z}$. Thus, substituting (3), (4), and (6) into (2), we obtain the solution $\mathbf{x}$ to the original problem (1) as follows:

$$\begin{aligned} \mathbf{x} &= \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) - \mathrm{H}\,[\mathrm{H}^+ \mathbf{b} + (\mathrm{I} - \mathrm{H}^+\mathrm{H})\,\mathbf{z}] \\ &= \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) - \mathrm{H}\mathrm{H}^+ \mathbf{b} - \mathrm{H}\,(\mathrm{I} - \mathrm{H}^+\mathrm{H})\,\mathbf{z} \\ &= \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) - \mathrm{H}\mathrm{H}^+ [\mathrm{K}^+ (\mathbf{m} - \mathbf{a}) - \mathbf{c}] \\ &\quad - \mathrm{H}\,(\mathrm{I} - \mathrm{H}^+\mathrm{H})\,\mathbf{z}. \end{aligned} \qquad (7)$$

Step 4: From the properties of generalized inverses it may be shown that $\mathrm{H}^+ = \mathrm{H}$ and $\mathrm{H}\mathrm{H} = \mathrm{H}$. Also, $\mathrm{H}\mathrm{K}^+ = 0$. Thus, (7) is just

$$\begin{aligned} \mathbf{x} &= \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) - \mathrm{H}\mathrm{K}^+ (\mathbf{m} - \mathbf{a}) + \mathrm{H}\,\mathbf{c} \\ &\quad - \mathrm{H}\,(\mathrm{I} - \mathrm{H})\,\mathbf{z} \\ &= \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) + \mathrm{H}\,\mathbf{c} - (\mathrm{H} - \mathrm{H}\mathrm{H})\,\mathbf{z} \\ &= \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) + \mathrm{H}\,\mathbf{c} \\ &= \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) + (\mathrm{I} - \mathrm{K}^+\mathrm{K})\,\mathbf{c} \\ &= \mathrm{K}^+ (\mathbf{m} - \mathbf{a}) + \mathbf{c} - \mathrm{K}^+\mathrm{K}\,\mathbf{c}. \end{aligned} \qquad (8)$$

Hence, the balancing program calculates the solution for $\mathbf{x}$ based on (8).

[1] Derived by David S. Jones, senior economist, Division of Research and Statistics, Board of Governors of the Federal Reserve System.

5. References

Board of Governors of the Federal Reserve System (1976): Improving the Monetary Aggregates. Report of the Advisory Committee on Monetary Statistics. Washington, D.C.

Board of Governors of the Federal Reserve System (1981): Seasonal Adjustment of the Monetary Aggregates. Report of the Committee of Experts on Seasonal Adjustment Techniques. Washington, D.C.

Box, G.E.P. and Jenkins, G.M. (1976): Time Series Analysis: Forecasting and Control. Revised edition, Holden-Day, San Francisco.

Cleveland, W.P. and Grupe, M.R. (1981): Modeling Time Series when Calendar Effects are Present. Special Studies Paper, No. 162. Board of Governors of the Federal Reserve System, Washington, D.C.

Cleveland, W.P. and Tiao, G.C. (1976): Decomposition of Seasonal Time Series: A Model for the Census X-11 Program. Journal of the American Statistical Association, 71, pp. 581–587.

Dagum, E.B. (1979): The X-11-ARIMA Seasonal Adjustment Method. Catalogue No. 12-564E, Statistics Canada, Ottawa.

Ghysels, E. (1984): The Economics of Seasonality: The Case of the Money Supply. Ph. D. dissertation, Northwestern University, Evanston, Illinois.

Macaulay, F.R. (1931): The Smoothing of Time Series. National Bureau of Economic Research, Washington, D.C.

McKenzie, S.K. (1984): Concurrent Seasonal Adjustment with Census X-11. Journal of Business and Economic Statistics, 2, pp. 235–249.

Nickelsburg, G. (1973): Documentation of the Banking Section Weekly Seasonal Adjustment Program. Board of Governors of the Federal Reserve System, Division of Research and Statistics, Washington, D.C.

Pierce, D.A. (1978): Seasonal Adjustment when both Deterministic and Stochastic Seasonality are Present. Seasonal Analysis of Economic Time Series. Economic Research Report, ER-1, U.S. Bureau of the Census, Washington, D.C.

Pierce, D.A., Grupe, M.R., and Cleveland, W.P. (1984): Seasonal Adjustment of the Weekly Monetary Aggregates. Journal of Business and Economic Statistics, 2, pp. 260–270.

Pierce, D.A. and McKenzie, S.K. (forthcoming): On Concurrent Seasonal Adjustment. Journal of the American Statistical Association (applications).

Plewes, T.J. and Altschuler, B.R. (1977): Seasonal Adjustment of Weekly Series: The Revised Bureau of Labor Statistics Method. U.S. Department of Labor, Bureau of Labor Statistics, Washington, D.C.

Poole, W. and Lieberman, C. (1972): Improving Monetary Control. Brookings Papers on Economic Activity, No. 1, pp. 293–335.

Shiskin, J., Young, A.H., and Musgrave, J.C. (1967): The X-11 Variant of the Census Method II Seasonal Adjustment Program. Technical Paper No. 15, U.S. Department of Commerce, Bureau of the Census, Washington, D.C.

Somer, M. (1969): Seasonal Adjustment of the Weekly Retail Trade Series: Presented at the annual meeting of the American Statistical Association, New York.

Wolter, K.M. (1986): Current Research Problems Encountered at the U.S. Bureau of the Census. Journal of Official Statistics, 2, pp. 57–70.

Zeller, L. (1972): Weekly Seasonal Adjustment for the IBM S/360 Computer. Board of Governors of the Federal Reserve System, Division of Research and Statistics, Washington, D.C.

Received March 1987
Revised September 1987