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Seasonal Baskets in Consumer Price Indexes

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Abstract: This article examines the seasonal-basket approach to goods in consumer price indexes with seasonal patterns of purchase. This approach, which involves separate index baskets for each of the 12 months of the year, is preferable to the use of a single annual basket for all months. Seasonal-basket indexes should be calculated at base year rather than base month prices. At present, where a seasonal-basket approach is used in calculating official consumer price series, fixed basket shares are imposed at some level of aggregation; it would be more appropriate to apply the seasonal-basket formula at all levels of aggregation.

Key words: Axiomatic; chain price index; seasonality.

1. Introduction

Many goods and services are purchased more in some months of the year than in others. This is due to three determining causes: weather conditions, traditions and institutional arrangements; these causes interact to generate the seasonal pattern of economic activity.

Most official measures of the rate of price change are based on prices of a fixed basket of goods, although this basket is updated from time to time as it becomes irrelevant to the current economic situation. In most cases the basket represents expenditures for some reference year, and the same basket is used for all months of the year. But a price index is adversely affected if any seasonal good has the same basket share for all months of the year; the good will have an inappropriately small basket share in its in-season months, an inappropriately large one in its off-season months. Moreover, some seasonal goods disappear completely from the market for several months of the year, and in an index based on an annual basket, prices must be imputed for these goods in their out-of-season months.

This paper argues that a monthly index for seasonal goods should be constructed using different baskets for the 12 months of the year. Such an index would provide a representative measure of price change and would not require imputation for seasonally disappearing goods when they are out-of-season. Such an index should be calculated at annual rather than monthly base prices, so that it will collapse to a standard annual-basket index should all of its monthly baskets be equal.

Many official price indexes for seasonal

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goods are already calculated in this way, but there is no general acceptance of this approach. International practice with regard to seasonal goods in official price indexes is highly eclectic; there is probably no matter related to the calculation of both consumer and industrial price indexes where there is so great a diversity in approach. This eclecticism is unlikely to become a permanent state of affairs; in the future, there is reason to believe that the use of seasonal baskets will be generally accepted as the most suitable way to deal with changes in the relative importance of goods due to seasonality, just as chain linking of index numbers is now accepted as the appropriate way to deal with long run changes in the relative importance of goods.

For our purposes, a seasonal good is any good subject to significant seasonality in its volume of purchases, or quantities purchased, where the word "good" is understood in its most general sense to include services as well as physical goods. It may, but need not, be seasonally disappearing, which means it is not purchased at all in its out-of-season months. If a continuous monthly series for volumes purchased of a good is available, seasonal adjustment of the series using a program such as X11-ARIMA (Dagum 1980) will determine whether purchases have statistically significant seasonality.

The seasonal index number problem, which relates to seasonality in quantities, should be carefully distinguished from the problem of seasonally adjusting a price index, which relates to seasonality in prices. This paper is concerned only with the first problem, although it cannot completely ignore the related problem of seasonal adjustment. The weighting of price relatives required by an index formula may itself impose a seasonal pattern on the price index and this may influence the choice of a seasonal adjustment technique. By the same token, strong price seasonality accentuates the differences between aggregate series based on different formulas, and this will influence the choice of formula.

Turvey (1979) has shown how important the treatment of seasonal goods can be to measured rates of consumer price change. He asked statisticians responsible for the calculation of consumer price series to apply their country's methodology to estimate a price index for fresh fruit from January to December 1973 with a 1970 base period. All estimates were to be derived from imaginary price and quantity data he had devised for five fresh fruits covering the years 1970 to 1973. These data are shown in Table 1 below.

Turvey received estimates from statisticians in over twenty countries. The differences between their estimates were surprisingly large. The value of the June index number ranged between 129.1 and 169.5 and the peak month of the year varied from as early as June to as late as December. To some extent, these differences are misleading. In many cases a country's official methodology entailed seasonal adjustment of fruit prices (e.g., Denmark, the Netherlands, and Sweden) or of the fruit price index (e.g., France) and these indexes were compared to the unadjusted series of other countries. Nevertheless, Turvey's inquiry showed how very different the treatment of seasonal goods is in one country compared to another, and what a substantial effect the choice of treatment can have on measured price change.

Section 2 of this paper deals with the annual-basket formula generally used in consumer price series and outlines the drawbacks of annual-basket indexes for seasonal goods. Section 3 deals with the Rothwell formula, the seasonal-basket formula which is best suited to the needs of the consumer

Table 1. Prices and quantities purchased of fresh fruit (Turvey's hypothetical data)

		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
	1970												
Apples	p	1.14	1.17	1.17	1.40	1.64	1.75	1.83	1.92	1.38	1.10	1.09	1.10
	q	3,086	3,765	4,363	4,842	4,439	5,323	4,165	3,224	4,025	5,784	6,949	3,924
Peaches	p	–	–	–	–	–	3.15	2.53	1.76	1.76	1.94	–	–
	q	–	–	–	–	–	91	498	6,504	4,923	865	–	–
Grapes	p	2.48	2.75	5.07	5.00	4.98	4.78	3.48	2.01	1.42	1.39	1.75	2.02
	q	82	35	9	8	26	75	82	1,490	2,937	2,826	1,290	338
Strawberries	p	–	–	–	–	5.13	3.48	3.27	–	–	–	–	–
	q	–	–	–	–	700	2,709	1,970	–	–	–	–	–
Oranges	p	1.3	1.25	1.21	1.22	1.28	1.33	1.45	1.54	1.57	1.61	1.59	1.31
	q	10,226	9,656	7,940	5,110	4,089	3,362	3,396	2,406	2,486	3,222	6,958	9,762
	1971	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Apples	p	1.25	1.36	1.38	1.57	1.77	1.86	1.94	2.02	1.55	1.34	1.33	1.30
	q	3,415	4,127	4,771	5,290	4,986	5,869	4,671	3,534	4,509	6,299	7,753	4,285
Peaches	p	–	–	–	–	–	3.77	2.85	1.98	1.80	1.95	–	–
	q	–	–	–	–	–	98	548	6,964	5,370	932	–	–
Grapes	p	2.80	3.32	5.48	5.67	5.44	5.30	3.93	2.33	1.66	1.64	2.10	2.35
	q	85	32	10	8	53	80	94	1,583	3,021	2,984	1,308	254
Strawberries	p	–	–	–	–	5.68	3.72	3.78	–	–	–	–	–
	q	–	–	–	–	806	3,166	2,153	–	–	–	–	–
Oranges	p	1.35	1.36	1.37	1.44	1.51	1.56	1.66	1.74	1.76	1.77	1.76	1.50
	q	10,888	10,314	8,797	5,590	4,377	3,681	3,748	2,649	2,726	3,477	3,548	10,727

Table 1. Prices and quantities purchased of fresh fruit (Turvey's hypothetical data) (continued)

1972		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Apples	p	1.43	1.53	1.59	1.73	1.89	1.98	2.07	2.12	1.73	1.56	1.56	1.49
	q	3,742	4,518	5,134	5,738	5,498	6,420	5,157	3,881	4,917	6,872	8,490	5,211
Peaches	p	–	–	–	–	–	4.69	3.32	2.29	1.90	1.97	–	–
	q	–	–	–	–	–	1.04	604	7,378	5,839	1,006	–	–
Grapes	p	3.20	4.03	6.06	6.59	6.01	5.94	4.61	2.79	1.94	1.95	2.46	2.92
	q	88	34	11	8	70	87	103	1,668	3,118	3,043	1,441	373
Strawberries	p	–	–	–	–	6.21	3.98	4.30	–	–	–	–	–
	q	–	–	–	–	931	3,642	2,533	–	–	–	–	–
Oranges	p	1.56	1.53	1.55	1.62	1.70	1.78	1.89	1.91	1.92	1.95	1.94	1.64
	q	11,569	10,993	9,621	6,063	4,625	3,970	4,078	2,883	2,957	3,759	8,238	11,827

1973		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Apples	p	1.67	1.79	1.85	1.94	2.06	2.13	2.22	2.25	1.95	1.87	1.88	1.73
	q	4,051	4,909	5,567	6,253	6,101	7,023	5,671	4,187	5,446	7,377	9,283	4,955
Peaches	p	–	–	–	–	–	6.10	4.08	2.80	2.06	2.01	–	–
	q	–	–	–	–	–	111	653	7,856	6,291	1,073	–	–
Grapes	p	3.52	4.67	6.48	7.34	6.51	6.43	5.00	3.07	2.20	2.19	2.74	3.13
	q	91	37	11	9	80	92	97	1,754	3,208	3,199	1,646	391
Strawberries	p	–	–	–	–	6.89	4.32	4.91	–	–	–	–	–
	q	–	–	–	–	1,033	4,085	2,877	–	–	–	–	–
Oranges	p	1.68	1.66	1.70	1.85	1.95	2.03	2.10	2.12	2.07	2.13	2.14	1.79
	q	12,206	11,698	10,438	6,593	4,926	4,307	4,418	3,165	3,211	4,007	8,833	12,558

price index. Section 4 deals with the Balk formula, another seasonal-basket formula. Section 5 deals with the aggregation of indexes for seasonal goods and non-seasonal goods in the consumer price index. It argues for the use of the Rothwell formula at all levels of aggregation. Section 6 discusses the chaining of consumer price indexes where there are seasonal goods in the basket and advocates the use of annual rather than monthly links. Section 7 concludes.

2. The Annual-Basket Index

Table 2 provides a comparison of alternative formulas for the treatment of seasonal goods in consumer price series. Most official consumer price indexes (CPIs), for seasonal or non-seasonal goods, are based on some variant of the first formula, which is for an annual-basket (AB) index at base year prices. The formula shown is for the index for the mth month of year y with base year 0 and an index basket representing expenditures of year c. (It is assumed that the consumer price series are calculated on a monthly basis, but the same principles apply if they are calculated on a quarterly basis.) If year c corresponds to year 0 then the index would be a Laspeyres index with an annual basket. Summation is over n goods. The j symbol stands for goods but it is not employed as a variable superscript to reduce the burden of notation. $p_{y,m}$ is the price of a given good in month m of year y, $\bar{q}_c$ is the mean monthly quantity purchased for a given good in year c, and $\bar{p}_o$ is its base year mean price.

The formula for an annual AB index is simply

$$p^a_{y/0} = \sum_j \bar{p}_y \bar{q}_c \Big/ \sum_j \bar{p}_0 \bar{q}_c \qquad (2.1)$$

where the annual prices are means of the monthly prices. An equivalent way of obtaining the annual AB index is to take the mean of its monthly price indexes.

An AB index at base year prices (hereafter simply an AB index, unless otherwise specified) has many nice properties. It satisfies all of the four axioms required of a price index by Eichhorn and Voeller (1983): monotonicity, proportionality, price dimensionality, and commensurability. The most important of these axioms for our purposes is proportionality, since it is violated by many of the index numbers to be discussed. The proportionality axiom states that if the level of prices in period t is k times as great as in period 0, then the value of the index will also be k. Moreover, the ratio of any two index numbers in an AB series is itself an AB index, and therefore also satisfies the same four axioms. This is important because it is not a property that many other price indexes possess. For example, a Paasche index satisfies the same four axioms but the ratio of two Paasche index numbers for a pair of consecutive months does not; that ratio violates the proportionality axiom.

Eichhorn and Voeller have shown that any index which satisfies their four basic axioms also has a number of other useful properties. Among others, it satisfies the mean value test, which states that the value of the index lies between the minimum and maximum price relatives of the goods that it represents.

The attractive properties of the AB index account for its almost universal use in official CPIs. In spite of this, it is poorly suited to measuring the rate of price change of seasonal goods. It lacks the property that Drechsler (1973, p. 19), in a study of spatial price indexes, calls "characteristicity," but which we shall call "representativeness." Here representativeness requires that the weights used in any comparison of price levels are related to the volume of purchases

Table 2. Comparison of alternative price index formulas for seasonal goods

Formula		Properties								
		Reduction	Proportionality				Representativity			
			Index	Annual Ratio	12-Month Ratio	Monthly Ratio	Index	Annual Ratio	12-Month Ratio	Monthly Ratio
Annual-basket at base year prices	$\sum_j p_{y,m}\bar{q}_c / \sum_j \bar{p}_0\bar{q}_c$	if $c = 0$, reduces to a Laspeyres index	×	×	×	×	×			
Seasonal-basket at base year prices (Rothwell formula)	$\sum_j p_{y,m}q_{c,m} / \sum_j p_0 q_{c,m}$	If, for each good, $q_{c,m} = \bar{q}_c$, reduces to AB index at base year prices	×	×	×		×	×	×	×
Seasonal-basket at base month prices	$\sum_j p_{y,m}q_{c,m} / \sum_j p_{0,m}q_{c,m}$	If, for each good, $q_{c,m} = \bar{q}_c$, reduces to AB index at base month prices	×	×	×		×	×	×	×
12-month m.a. of Rothwell index		reduces to 12-month m.a. of AB index at annual prices		×				×		
Balk price index	$\sum_j \hat{p}_{y,m}q'_{c,m} / \sum_j \hat{p}_0 q'_{c,m}$	If, for each good, $q_{y,m} = q_{c,m}$, $\hat{p}_0$ equals p_0, reduces to Rothwell index	×	×			×	×	×	×

Table 3. Relative importance of expenditures for fresh fruit by item based on monthly quantities at 1970 annual prices

Month	Apples	Peaches	Grapes	Strawberries	Oranges
January	23.34		0.75		75.91
February	28.35		0.32		71.34
March	35.86		0.09		64.05
April	49.08		0.10		50.82
May	42.99		0.30	17.85	38.86
June	33.34	0.75	0.56	44.69	20.66
July	31.00	4.86	0.73	38.61	24.80
August	20.28	53.61	11.27		14.85
September	24.47	39.23	21.47		14.83
October	42.92	8.41	25.21		23.46
November	45.34		10.12		44.54
December	28.21		2.92		68.87
Year	32.99	10.34	6.77	8.67	41.22

in the periods of comparison. The goods that are most heavily purchased should have the largest index weights; goods that are not purchased at all in either the given or base period should not be part of the index.

As an example of unrepresentativeness of an AB index in the case of seasonal goods, consider Turvey's fresh fruit data. Table 3 shows the annual expenditure shares for the year 1970 of the five fruits in his data series. These are quite different from the monthly expenditure shares, where expenditures are expressed in 1970 constant prices. It can be seen that oranges have the largest share of the annual basket, followed by apples, the two fruits accounting for almost three quarters of annual purchases. But strawberries are the dominant fruit in June as are peaches in August and September, although neither of these fruits has so much as an eighth of the annual basket. Grapes have approximately a fifth of the September basket and a quarter of the October basket, but only a fifteenth of the annual basket. This means that for comparisons of price levels between the same months of different years, i.e., 12-month changes, the AB index will not provide a representative measure of price change.

Now look at representativeness in terms of measuring annual price movements. Three of the five fruits have most of their purchases in just two months of the year. Even apples, the least seasonal of the fruits, have November purchases that are more than double those of January. But the annual AB index is based on yearly prices of these fruits calculated as simple averages and so does not provide a representative measure of the annual rate of price change, underweighting in-season prices and overweighting off-season prices.

The problem is made worse because a couple of the fruits are seasonally disappearing. Strawberries are unavailable from August to April, and peaches from November to May. There are two ways of treating such seasonally disappearing goods in an AB index. Either they are omitted from the index altogether, making it less representative, or they are retained in the index, imputing prices for the goods in their out-of-season months. Since there are no purchases of an out-of-season good, its imputed price does not approximate a transaction price, or any kind of market price. But in calculating September index numbers for fresh fruit, these imputed prices for strawberries will have a more important weight than the actual prices for grapes. And in calculating a yearly AB index the imputed prices for strawberries and peaches will have a greater influence on the index than the actual prices, although they correspond to months of zero purchases.

Grapes, which are continuously available in Turvey's data set, would not be continuously priced in many countries given their seasonal pattern of purchase. For example, the Republic of Korea, which adopts an AB approach to seasonal goods, will price a good only in months where purchases are greater than or equal to one third of the monthly mean, imputing a price movement in non-pricing months (Turvey 1979, p. XVII). Using the Korean rule, grapes would be priced only from July to October. Other countries, without formalizing their pricing rules, limit their pricing seasons at least as restrictively in practice.

Sometimes it is infeasible to obtain reliable price estimates for a seasonal good in some off-season months of the year, and these are appropriately made non-pricing months. One of the drawbacks of the AB approach is that statisticians are motivated to impute a price movement, not only in such months, but in all off-season months, in order to reduce the volatility of higher level aggregates. Notice that the large increase in grape prices every March dominates the movement of the AB index for that month, although grapes have less than 0.1% of the March basket. This could be avoided by restricting the pricing frequency of grapes and imputing a price movement for March, but this involves a loss of price information that would be annoying to index users specifically interested in grape prices.

It is not sufficiently appreciated that when there are seasonally priced or seasonally disappearing goods, the AB index may no longer possess the properties of a price index except in a formal sense. For example, any price index that satisfies the four basic axioms also satisfies the mean value test, which states that the price index always lies between the maximum and minimum price relatives of the index. In an AB index with imputation for seasonally priced goods, this is still formally true, if one considers the property as applying to all prices, actual or imputed. It is not generally true if one is considering only the price ratios of goods priced in both months.

For seasonal goods, the AB index is best considered an index partially adjusted for seasonal variation. It is based on annual quantities, which do not reflect the seasonal fluctuations in the volume of purchases, and on raw monthly prices, which do incorporate seasonal price fluctuations. Zarnowitz (1961, pp. 256–257) calls it an index of "a hybrid sort." Being neither of sea nor of land, it does not provide an appropriate measure either of monthly or 12-month price change. The question that an AB index answers with respect to price change from January to February say, or January of one year to January of the next, is "What would the change in consumer prices have been if there were no seasonality in purchases in the months in question, but prices nonetheless retained their own seasonal behaviour?" It is hard to believe that this is a question that anyone would be interested in asking. On the other hand, the 12-month ratio of an AB index based on seasonally adjusted prices would be conceptually valid, if one were interested in eliminating seasonal influences.

Because the AB approach is not well-suited to highly seasonal goods, where it is employed there is a tendency for such goods to be underrepresented in consumer price surveys, supposing, as is generally the case, that there is judgmental selection of items to be priced. This is the case with the Canadian CPI, which now contains no highly seasonal fresh produce items in its monthly price survey. By contrast, prior to 1973, when monthly baskets were used for seasonal food groups, fresh strawberries, peaches, grapes, and fresh corn were all priced items in the index.

Turvey (1979, p. XVII) notes that Australia, Denmark, and Ireland, countries which follow an AB approach towards seasonal goods, only include continuously available goods in the index basket. Since then, the Australian Bureau of Statistics has revised its policy of excluding seasonally available items from the index (see Castles 1987, pp. 43–46). However, starting in 1980, with its 1976-based indexes, the Federal Republic of Germany only includes continuously available goods in its official consumer price series (see OECD 1981).

Particularly for items of fresh produce the exclusion of highly seasonal items from

Table 4. Comparison of price indexes for fresh fruit (1970 = 100)

Date	Annual-Basket		Seasonal-Basket		
	I	II	III	IV	V
1970					
January	87.8	87.0	93.7	93.7	99.2
February	87.5	86.8	90.8	90.8	97.6
March	92.3	91.6	88.3	88.3	95.9
April	99.3	98.4	96.3	96.3	97.7
May	108.4	113.4	114.5	114.5	99.8
June	112.9	117.7	108.8	108.8	102.1
July	116.2	116.5	111.2	111.2	100.2
August	118.1	114.3	111.4	111.4	101.8
September	101.8	99.6	98.0	98.0	104.3
October	95.0	94.6	92.7	92.7	99.6
November	95.0	94.6	99.2	99.2	100.0
December	85.7	85.4	93.1	93.1	101.8
Year	100.0	100.0	100.0	100.0	100.0
1971					
January	93.7	93.3	98.5	98.6	104.4
February	98.7	98.3	100.7	100.7	108.2
March	105.5	105.0	101.5	101.5	109.3
April	114.1	113.7	110.6	110.6	111.5
May	121.9	127.9	128.3	128.3	112.1
June	126.0	132.2	118.2	118.2	112.3
July	128.3	129.5	124.6	124.6	112.2
August	129.3	125.8	124.0	124.0	112.3
September	114.4	111.6	108.0	108.0	115.3
October	108.6	107.2	107.1	107.1	114.0
November	109.2	107.8	110.3	110.3	115.4
December	99.4	98.2	107.7	107.7	117.6
Year	112.4	112.5	111.9	111.6	112.0

(cont.)

Table 4. Comparison of price indexes for fresh fruit (1970 = 100) (continued)

Date	Annual-Basket		Seasonal-Basket		
	I	II	III	IV	V
1972					
January	107.7	106.4	113.6	113.5	120.6
February	111.8	110.4	113.3	113.3	122.0
March	119.8	118.2	115.5	115.5	123.6
April	127.9	126.2	123.3	123.2	123.4
May	133.9	138.7	139.9	141.1	123.3
June	139.3	148.5	128.5	128.4	123.4
July	142.4	144.8	138.4	138.9	125.6
August	139.7	137.0	139.6	139.8	125.5
September	126.3	123.1	118.6	118.5	127.7
October	122.5	120.0	121.7	121.7	128.8
November	123.5	121.0	130.7	130.7	130.7
December	111.7	109.4	119.6	119.3	130.7
Year	125.5	125.3	125.3	125.4	125.4
1973					
January	120.0	117.6	124.6	124.6	132.4
February	125.9	123.4	125.7	125.7	135.7
March	134.0	131.3	129.4	129.3	137.9
April	144.5	141.6	139.5	139.5	139.1
May	149.4	153.2	155.9	157.1	138.2
June	154.2	168.8	140.8	140.8	136.7
July	155.5	161.1	154.4	154.8	140.4
August	151.9	151.3	160.9	161.0	143.6
September	139.0	135.8	131.0	131.0	141.9
October	138.8	135.4	138.3	138.3	145.1
November	140.9	137.4	149.6	149.7	148.9
December	124.8	121.7	132.5	132.6	144.6
Year	139.9	139.9	140.2	140.5	140.4

price surveys or from the index basket can be potentially dangerous. Suppose, for example, that because seasonally priced items are excluded from the price sample for fresh fruit, apples are substantially overweighted in the fresh fruit CPI. If an early frost hits the apple-growing regions or a blight ravages the orchards, the fresh fruit CPI will show a steep increase, although prices of other fresh fruit may not be greatly affected.

Table 4 provides a comparison of different indexes based on Turvey's fresh fruit data. Series I is an AB index which excludes the seasonally disappearing items strawberries and peaches. Series II includes them, and price movements are extrapolated for their out-of-season months based on those of the continuously available fruits. It can be seen that the two series do not tend to diverge greatly over time. This is partly because the two seasonally disappearing items have opposite price trends: from 1970 to 1973 peach prices increased more rapidly and strawberry prices less rapidly than fresh fruit prices in general. But even in this example, there are substantial divergences between the price movements of the two AB series for particular months. Examining the 12-month changes, the largest difference is for June 1973. In that month peach prices rose 30%, so that series II registers a 13.7% rate of increase as opposed to a 10.7% increase for series I. Note that in June both peaches and strawberries are in-season so the difference between the rates of price change for the two series is little influenced by the procedure used to impute their price movements over the out-of-season months.

Note also that from May to June 1973, series II rises by 10.2%, which is more than twice the highest rate of increase of any fruit available in both May and June. The AB series owes its high rate of increase to peaches, which come into season in June. Conceptually the price of peaches must have fallen from some demand reservation price in excess of the observed June price; otherwise there would have been purchases in May. But the extrapolated May price is well below the June price, which creates the large jump in the index in June. Thus, the AB series, although formally consistent, is logically inconsistent with the mean value test. It only lies between the maximum and minimum price relatives of its component series because of an unrealistically large value for the relative of peaches, based on an imputed May price. Using a realistic value for that relative (i.e., a value less than one) the monthly ratio of the AB index exceeds the largest price relative of any of its components.

3. The Rothwell Index

3.1. The annual index

Let us consider how one might obtain a more representative measure of annual price change for seasonal goods than is provided by an AB index. One proposed index, which is compatible with a number of different monthly indexes, has the following formula

$$P^s_{y/0} = \sum_m \sum_j p_{y,m} q_{c,m} \Big/ \sum_m \sum_j p_{0,m} q_{c,m} \tag{3.1}$$

where $p_{0,m}$ is the price of a given good in month m of base year 0 and $q_{c,m}$ is the quantity purchased of the given good in month m of basket year c.

If annual base year prices are defined as

$$p_0 = \sum_m p_{0,m} q_{c,m} \Big/ \sum_m q_{c,m} \tag{3.2}$$

and if annual prices for the given year p_y are similarly defined, then (3.1) can be rewritten as

$$P^s_{y/0} = \sum_j p_y q_c \Big/ \sum_j p_0 q_c \tag{3.3}$$

where $q_c = \Sigma_m q_{c,m}$.

Note carefully that (3.2) is not a formula for a unit value, since it represents a weighted average based on purchases for the basket year and not the given year.

From (3.3) it is obvious that (3.1) is equivalent to an annual AB index with annual prices defined as weighted rather than simple averages of monthly prices. Thus, both the index and its annual ratios will satisfy the four basic axioms of a price index. At the same time, for any good, prices of in-season months are weighted more heavily than those of off-season months, and for any month, prices of goods with a large volume of purchases are weighted more heavily than those with a lesser volume. If a good is seasonally disappearing then it is assigned a zero weight in its out-of-season months so there is no need to make imputations for missing prices as there would be in a standard AB index.

Provided that there is a stable seasonal pattern of purchase for the goods whose prices the index measures, an index based on (3.1) will give a more representative measure of price change than an AB index.

Note also that if purchases of each good are the same in every month of the basket year, then the annual prices in (3.2) are simply mean monthly prices and (3.3) reduces to (2.1), the formula for an annual AB index.

3.2. *The monthly index*

One of the formulas in Table 2 represents a seasonal-basket (SB) index at base year prices, also known as a Rothwell index. Rothwell (1958) recommended it as the best formula for the calculation of consumer price series for seasonal goods. However, it was originally proposed much earlier by Bean and Stine (1924), in a pathfinding paper which anticipated much of the later work on seasonal goods.

The annual Rothwell index is defined by (3.1); it is an average of the monthly Rothwell indexes weighted by basket month expenditures at base year prices. If purchases of each good are the same in each month of the basket year then the Rothwell index reduces to a monthly AB index.

The monthly Rothwell index, like any SB index, is undefined if basket month purchases are nil for every good in the index. This does not create any problems for the calculation of annual indexes, since for any month whose index is undefined, the basket month expenditures for that month would be zero. In a table of monthly index numbers, one could simply leave a blank entry for a month in which an SB index is undefined, and this is, in fact, the publication policy of the Netherlands Central Bureau of Statistics (1978) with regard to seasonal goods in its consumer price series.

It can be seen that the Rothwell index is formulaically similar to a Paasche index. For a Paasche index, the index basket always relates to the given month, for a Rothwell index, to the same calendar month of the basket year. If seasonality has a predominant influence on changes in the pattern of relative expenditures then the Rothwell index will closely approximate a Paasche index. In the limiting case, where the only factor causing changes in an item's relative share of purchases is a seasonal pattern characterised by constant seasonal factors, the Rothwell and Paasche series will be identical. This is a real advantage of the Rothwell index, since in actual statistical work consumer price series are often used as deflators rather than the Paasche series one would like to have. For example, if quarterly expenditure on fresh produce is being deflated, but regular quarterly expenditure estimates on the component series are unavailable, the consumer price series for fresh produce will generally be used as a deflator. In such a situation, a Rothwell price index will provide a much closer approximation than an AB index to the desired Paasche deflator.

The Rothwell index satisfies the four basic axioms of a price index, since it is, like the AB index, a fixed-basket index, but with a monthly rather than an annual basket. Similarly, the ratio of two Rothwell indexes 12 months apart is also a fixed-basket index; it is defined as

$$\sum_j p_{y,m} q_{c,m} \Big/ \sum_j p_{y-1,m} q_{c,m}$$

and satisfies the four axioms of a price index, including proportionality. Assuming that there is a stable seasonal purchase pattern, the above index will also provide a more representative measure of 12-month change than its AB counterpart.

The Rothwell index is not the only monthly index compatible with the annual index defined by (3.1). Another is the SB index at base month prices, whose formula is shown in Table 2. The formula and the Rothwell formula are identical in their numerators, so both indexes provide the same measure of monthly price change. The annual index for an SB index at base month prices, which as said is identical to the Rothwell annual index, is calculated as a weighted average of its monthly values, with weights based on basket month expenditures at basket month prices. (See Section 3.6.)

3.3. *The monthly ratios*

The ratio of two Rothwell index numbers for consecutive months will be

$$P^s_{y,m/y,m-1} = \frac{\sum_j p_{y,m} q_{c,m}}{\sum_j p_{y,m-1} q_{c,m-1}} \div \frac{\sum_j p_0 q_{c,m}}{\sum_j p_0 q_{c,m-1}} \tag{3.4}$$

which does not satisfy the proportionality axiom of a price index.

The second factor in (3.4), the ratio at base year prices of purchases in base month m to those in the previous month, can, as Zarnowitz (1961, pp. 271–272) notes, be considered a seasonal adjustment factor, adjusting the ratio of prices from one month to the next for the change in monthly baskets. The seasonal factor interpretation is the more apt since the geometric mean of these ratios equals one for the 12 months of the year. Decomposing the first factor on the right hand side of (3.4) one obtains

$$P^s_{y,m/y,m-1} = \frac{\sum_j p_{y,m} q_{c,m-1}}{\sum_j p_{y,m-1} q_{c,m-1}} \times \left\{ \frac{\sum_j p_{y,m} q_{c,m}}{\sum_j p_{y,m} q_{c,m-1}} \div \frac{\sum_j p_0 q_{c,m}}{\sum_j p_0 q_{c,m-1}} \right\} \tag{3.5}$$

The first factor is a price index that satisfies the axioms of Eichhorn and Voeller, while the expression in parentheses represents the ratio of a volume index at given month prices to the same index at base period prices. It can be seen that the latter ratio will reduce to one in the special case where given month prices for all goods are the same fixed multiple of base period prices. In general, the monthly ratio of the Rothwell index will approximate that of its related price index the more closely the smaller the change in the structure of relative prices between the base period and the given period.

Note that the price index factor represents the formula for a link in a monthly-rebased chain index with the formula

$$P^{ch}_{y,m/0} = P^{ch}_{y,m-1/0} \times \sum_j p_{y,m} q_{c,m-1} \Big/ \sum_j p_{y,m-1} q_{c,m-1}. \tag{3.6}$$

Formula (3.6) defines a quasi-Laspeyres chain index; it differs from a true chain Laspeyres index because the chain link for a given month of year y is based on the basket for the previous month of basket year c rather than given year y. The percent changes of such a quasi-Laspeyres chain index represent the price change component of the Rothwell index's monthly percent change. The difference represents that part of the index change due to the interaction between price and quantity change. It can be seen that in the special case where all monthly baskets are equal for all goods, the expression in parentheses in (3.5) will reduce to one, and (3.6) will reduce to the formula for an AB index.

If official consumer price series for seasonal goods were based on the Rothwell formula, the monthly ratios of (3.6) would provide a useful analytical adjunct, allowing the statistician to better interpret the monthly movements of the official series. However such ratios, while satisfying the definition of a price index, do not necessarily provide representative measures of price change, since baskets for consecutive months can sometimes differ dramatically.

If one were interested in a representative measure of monthly price change, unadjusted for seasonal variation, a better choice would be

$$P^{ch}_{y,m/0} = P^{ch}_{y,m-1/0} \times \sum_j p_{y,m} q_{c,*} \Big/ \sum_j p_{y,m-1} q_{c,*} \quad (3.7)$$

where

$$q_{c,*} = (q_{c,m-1} \times q_{c,m} \times ((q_{c,m-1} + q_{c,m})/2))^{1/3}. \quad (3.8)$$

This is a variant of the log-change index defined by Theil (1973). Note that the quantity weight defined by (3.8) is set to zero if a good is not purchased in either the given or base month of the chain link. Note also that in the special case where all monthly baskets are identical for every good, (3.7) will reduce to the formula for an AB index. Even this index will not provide a representative measure of price change if there are many goods coming into season or going out of season in month m. Furthermore, it will be undefined if this is so for every good in the index. It is a sad fact that for some seasonal commodity groups monthly price changes are not meaningful, whatever the choice of formula.

3.5. *The divergence between AB and Rothwell indexes*

Bortkiewicz has shown that the sign of the difference between Laspeyres and Paasche price indexes is determined by the correlation between price and volume changes from the base period to the given period (Allen 1975, pp. 62–65). If there is a negative (positive) relationship between prices and quantities, the Laspeyres index will be greater than (less than) the Paasche index. Unfortunately there is no such clear relationship between AB and Rothwell indexes. This is because the two indexes diverge due to differences in base period prices as well as differences in index baskets. Even if the index baskets were the same, one would expect AB and Rothwell indexes to be somewhat different. For example, an AB index for June would generally differ from a Rothwell index for the same month even if the annual and June baskets were identical. The differences in annual movements between the AB and Rothwell indexes are also indeterminate, whatever the correlation between prices and quantities.

Fortunately, one can say something about the signs of the differences between the 12-month changes of AB and Rothwell indexes, since for these measures the base period prices of the original indexes are irrelevant. It can readily be shown that the 12-month change of the Rothwell index will exceed that of the AB index if goods with higher basket shares in its basket also have above average rates of price change (see Baldwin 1988, pp. 20–21).

The rest of this section discusses the divergence between the AB and Rothwell series shown in Table 4. Series III is the Rothwell index, unadjusted for seasonal variation. Let us compare its 12-month changes with those of series II, which is more comparable with the Rothwell index than series I since it includes seasonally disappearing fruits in its basket. For quite a few months there are large differences between the 12-month rates of change of the two series, with the AB series showing the larger rate of increase in some months, the Rothwell series in others. From June 1971 to June 1972, the AB series increases by 12.3%, the Rothwell series by only 8.6%. This was due mainly to the low price increase for strawberries, since they are the most important fruit in the June basket, and to the large price increase for peaches, which have a negligible share of the same basket.

In other months, the 12-month change for the Rothwell series exceeds that of the AB series. The most dramatic example of this is August 1973, when the Rothwell series shows a 15.2% increase as opposed to a 10.4% increase for the AB series. This was largely because of the 23% increase in the price of peaches, the dominant fruit in the August basket, but also to the low price increase for apples, which have a smaller share of the August than of the annual basket.

Note that the large monthly increases in the AB series every March due to a seasonal surge in grape prices are not found in the Rothwell index. This is not surprising, since grapes have less than 0.1% of the index basket in March. Because the volatile prices of highly seasonal goods in their off-season months are not unduly weighted in a Rothwell index, there is not the same temptation to exclude such prices from the index, either by imputing a price movement for the good in its off-season months or by assigning a proxy index.

The differences between the annual changes of the AB and Rothwell series are small, not exceeding 0.6 percentage points for any year. Those for the individual fruits are much larger, especially for peaches, strawberries, and grapes, but they are also partially offsetting. Since the annual basket for the AB and Rothwell series are the same, the only difference between the two series lies in the component annual indexes, which for the AB series are means and for the Rothwell series are weighted averages of the monthly indexes. The differences between the annual AB and Rothwell series would probably be larger if they were based on actual price and quantity data. Turvey's data, although derived from consumer price information for the Federal Republic of Germany, is more regular and consistent than data that an official statistician is likely to encounter. Baldwin (1987) compared the AB and Rothwell formulas using actual data for fresh vegetables in Canada. He found the annual rates of change of AB and Rothwell series to differ by as much as three percentage points.

3.6. The seasonally-adjusted Rothwell index

Most of the countries that use seasonal baskets in their official consumer price series calculate SB indexes at base month prices, or at "trend-adjusted" base month prices; only a few use the Rothwell formula (ILO 1984; Turvey 1979). In spite of this, the Rothwell formula is clearly the better formula. An SB index at base month prices does not reduce to a monthly AB index, as does a Rothwell index. If one is interested in a deflator, an SB index at base month prices is also a poorer proxy for a Paasche index than the Rothwell index. It does not reduce to a Paasche index in the special case where only a constant seasonal pattern accounts for the differences in the relative purchases of the goods in an aggregate.

One argument for an SB index at base month prices is that it is appropriate to consider the month of purchase as a suitable criterion for the definition of a commodity, a contention which Balk (1980a, p. 30) has convincingly rebutted. Another argument for the index is that it eliminates much of the seasonal price variation of the goods for which the index is calculated. In fact, an SB index at base month prices is equivalent to a Rothwell index that has been seasonally adjusted using a set of fixed seasonal factors, namely, the ratios of base year prices to base month or trend-adjusted base month prices. This is an advantage only if one wants to eliminate the seasonal price profile from the price series, which is not necessarily so. Even if this were so, the use of a fixed set of seasonal factors constitutes a crude method of seasonal adjustment indeed; surely the seasonal adjustment of a Rothwell index using X11-ARIMA or a similar program would provide a superior seasonally adjusted series.

A backward 12-month moving average of a Rothwell index constitutes another crude way of calculating a seasonally adjusted SB series. The December values of such a series would be equivalent to the annual Rothwell index. France and Luxemburg have based their CPIs for seasonal goods on a similar formula. More recently, Diewert (1983) has advocated the application of a 12-month moving average to an SB index. But the French no longer favour their current approach, and in 1993 will be calculating their seasonal series as SB indexes at base year prices (INSEE 1989).

Series V in Table 4 is the Rothwell index adjusted for seasonal variation using the X11-ARIMA program (Dagum 1980). Its annual averages are the means of the monthly index numbers and do not match the weighted annual averages of the raw series (series III). This series, or more precisely its 1973 values, provides an interesting comparison with the national series shown in Turvey's study. It is close in level and movement to several of the indexes based on seasonal baskets at base month prices (e.g., Belgium and the Netherlands). It is also similar to the seasonally adjusted series calculated from the two AB series shown in Table 4, which are not shown here, but are available from the author on request.

Section 3.3 dealt with the unsuitability of the Rothwell index as an indicator of monthly price change, due to distortions caused by changes in the monthly baskets. It is worth remembering that to some extent these distortions are removed by seasonal adjustment, and that monthly, three-month or other short term measures of price change based on a seasonally adjusted Rothwell index are unlikely to differ greatly from those based on a seasonally adjusted AB index.

The only disadvantage of seasonally adjusting a Rothwell index using a program such as X11-ARIMA is that the data for the most recent years will be revised as new data become available. The use of such crude seasonal adjustment methods as fixed seasonal factors or a 12-month backward moving average at least ensures that the initial seasonally adjusted estimates never have to be revised. Given the importance of the CPI in most countries in contract escalation, it is desirable that the official series either be non-revisable or revisable only for the most recent months.

But surely there is no need to have a non-revision policy for seasonally adjusted consumer price series. The official consumer price series should be based on raw prices and unadjusted for seasonal variation. These would be the only indexes that would have official sanction for escalation purposes, and should be subject to a non-revision or a limited revision policy. In addition to these raw series, seasonally adjusted CPIs should be produced as there is a demand for them. These indexes should be revisable over several years so that they can be adjusted using the best methods available.

4. The Balk Index

The annual prices used in the Rothwell index are averages of monthly prices weighted by monthly quantities for the basket year. One might prefer to define an annual index in terms of unit value prices, that is

$$P^b_{y,0} = \sum_j \hat{p}_y q_c \Big/ \sum_j \hat{p}_0 q_c \qquad (4.1)$$

where

$$\hat{p}_y = \sum_m p_{y,m} q_{y,m} \Big/ \sum_m q_{y,m}$$

and

$$\hat{p}_0 = \sum_m p_{0,m} q_{0,m} \Big/ \sum_m q_{0,m}.$$

The monthly index corresponding to (4.1) is shown in Table 2 as the Balk price index. It was developed by the Dutch economist Bert M. Balk, who has also suggested several other indexes for seasonal goods (Balk 1980a, 1980b, 1980c, 1981). The quantity weights in the index are obtained by rescaling the quantities purchased in the given year to sum to the basket year purchases

$$q'_{c,m} = \left(q_{y,m} \Big/ \sum_m q_{y,m}\right) \times q_c.$$

In this way, the index reflects the seasonal purchase pattern of the given year, but there is still a constant annual basket. The Balk index provides a more representative measure of price change than the Rothwell index if there is substantial variation in seasonal patterns from one year to another, or if the series are highly erratic. In calculating a Rothwell price index, one would still have to impute prices for seasonal goods that disappeared from the market in months when they are not out-of-season in the basket reference year. Not so with the Balk index. If a good is not purchased in a given month, for whatever reason, it is not part of the basket for that month, so there is never any need to impute a price for a good that has disappeared from the market. In fact, series with a strong irregular component might be better calculated using the Balk formula, even if the goods are non-seasonal.

Despite its many good features, there are two reasons to believe that the Balk index will probably never be used to calculate official consumer price series. First, the index requires purchase data at a fine commodity level of detail for every month of the estimation period. Not many statistical agencies currently supply such data, or would be capable of doing so without a major commitment of resources. Second, monthly indexes require quantitites for the complete calendar year. Initial monthly estimates would have to be based on some proxies for these quantities (Balk 1980a, pp. 35–36) and revised when quantity data became available. Even if price and quantity data were estimated at the same time, the most favourable assumptions for the use of the Balk formula, the index would still require an 11-month revision period, since the January index would be revised with the publication of initial December estimates. It is hard to imagine any statistical agency agreeing to revise their official consumer price series over a period of 11 months or longer. The Balk formula is more likely to be useful in constructing industry price indexes, where revision of series for a year or more may be admissible.

If there is generally a negative correlation between price change and quantity change one would expect the Rothwell index to be greater than the Balk index, where both indexes have the same base year and basket reference year (Balk 1980a, pp. 39–40; Baldwin 1988, p. 30–31). In Table 4, series III is the Rothwell index and series IV the Balk index for fresh fruit. Note that the differences between annual and monthly series are very slight. As expected, the Rothwell index is greater than the Balk index in 1971, but it is less than the Balk index in 1972 and 1973.

Note the substantially smaller increase of the Balk index from November 1970 to November 1971 compared to the Rothwell index, and its substantially larger increase from November 1971 to November 1972. This is largely due to the unusually low November purchases of oranges in 1971 compared to other years. There are only three fruits in-season in November, the price of oranges exceeds the weighted average price of apples and grapes, and there is a basket shift away from oranges in November 1971 which is reversed a year later.

For November 1972, the 12-month change of the Balk index is 18.1%, which is higher than the percent change of any of its component indexes. This illustrates how the 12-month ratios of the Balk index, unlike the Rothwell index, will sometimes violate the mean value test.

5. Seasonal-Basket Series in the Total CPI

There are great differences in the scope of the SB approach in consumer price series among those countries that have adopted it. For example, Israel and the United Kingdom have only seasonal baskets for fresh fruits and vegetables, while the Netherlands uses seasonal baskets for these and other food groups, as well as for plants and clothing. But however seasonal groups are determined, most groups in the total CPI are non-seasonal, and their consumer price series will be calculated using the AB approach. This raises the question of how AB and SB components are to be aggregated to create the total index.

The simplest and probably the best way to proceed is to calculate the total CPI and all of its subaggregates as Rothwell indexes. For non-seasonal goods, the monthly quantities are the same for all 12 months of the year, that is

$$q_{c,m} = \bar{q}_c; \quad m = 1 \text{ to } 12.$$

The Rothwell indexes for non-seasonal groups reduce to AB indexes, but there is seasonal weighting for all seasonal groups and of all aggregates that contain seasonal groups as subindexes.

To clarify the implications of this procedure, consider the calculation of a total CPI for which fruit and vegetables are the only categories treated as seasonal groups. The food series is then a Rothwell index, since it includes fruit and vegetables as components, which have different value weights in each month of the year. For meat, fish, seafood, and other food subgroups, the value weight is the same in every month, that is

$$\bar{p}_0 q_{c,m} = \bar{p}_0 \bar{q}_c; \quad m = 1 \text{ to } 12 \qquad (5.1)$$

but the relative weight

$$\bar{p}_0 \bar{q}_c \Big/ \sum_j \bar{p}_0 q_{c,m}$$

changes from one month to another, since the value weights for the fruit and vegetable groups differ from month to month. Similarly, the total series is a Rothwell index, since it includes food as a major group. Although the value weight for housing, clothing or the non-food groups is the same in every month, the relative weight, due to the changing purchases for the food group, is not.

It can also be seen that the annual indexes for all aggregates with seasonal subgroups will be weighted averages of the corresponding monthly indexes. Since the Rothwell index is consistent in aggregation, the identical estimate of the annual total CPI can be calculated from major group indexes, basic aggregates or any exhaustive and mutually exclusive set of component series at or above the basic aggregation level, as a weighted average of annual goods indexes or of monthly indexes for the total CPI. (The basic aggregation level is the lowest level of aggregation to which the chosen index formulas apply; the construction of basic aggregates themselves may be carried out in a different way (see Szulc 1987).)

This approach is operationally as well as conceptually simple. Assume that the calculation of consumer price series is an automated process. One computer program can calculate all indexes at or above the basic aggregation level as Rothwell series, since an AB index is only a Rothwell index with the same monthly baskets for all 12 months of the year. If, from one updating of the index basket to the next, a group has its classification changed from seasonal to non-seasonal, or vice-versa, there is no need to change the production system; updated monthly value weights for the group will ensure that its series is calculated as an AB index.

In spite of these nice properties, the above approach has not been adopted by any country that calculates consumer price series as SB indexes; instead, seasonal groups are assigned fixed basket shares for all months of the year at some level of aggregation. For example, Japan calculates SB consumer price series for fresh fish, fresh fruits and fresh vegetables, but each of these groups has a constant basket share within the overall CPI.

The two approaches are equivalent in the special case where (5.1) holds, that is, where basket month expenditures at base year prices are the same for all basket months. This is a theoretical rather than a practical possibility but one may find seasonal groups for which there are only slight differences in total expenditures for the different months of the basket year, even among groups that contain highly seasonal items. Generally, this is because a seasonal group usually contains non-seasonal as well as seasonal goods; also, to at least some degree the monthly purchase patterns of its seasonal goods will be mutually offsetting.

Nevertheless, a group may contain mostly series with strong and congruent seasonal patterns so that its total purchases are highly seasonal. The drawbacks to assigning a fixed basket share to such a group in the overall CPI are the same as those associated with assigning a fixed basket share to a highly seasonal good in a consumer price series. One does not obtain a representative measure of price change, and imputed price movements or highly erratic price movements of the seasonal group may unduly influence the movement of the aggregate series.

To be fair, an index maker who assigns constant basket shares to a seasonal good can ensure that the total purchases for the group are not highly seasonal by pooling the group with less seasonal groups or groups with offsetting seasonal patterns. But here is another weakness of this approach: it tends to distort the classification of groups by introducing extraneous considerations into the decision-making process.

Calculating the total CPI as an AB series also creates a problem of consistency in aggregation over goods. Suppose women's and men's wear each have fixed basket shares in the CPI and that there are monthly baskets within each clothing group. One might wish to calculate separate SB series for footwear and clothing excluding footwear that would straddle the boundaries of these two groups. A clothing series based on a weighted average of the latter series would not equal the official clothing index. In consequence, a total CPI derived from these two series or their aggregate would not match the official total index. By contrast, there is consistency in aggregation if the total CPI is calculated as a Rothwell index.

Given the useful properties of an AB index, one might argue that it is unreasonable to calculate the total CPI as an SB index, solely because of a few seasonal groups whose relative importance in the overall index is not large. If an SB approach is adopted for seasonal groups, should it not either be made compatible with an AB approach at higher levels of aggregation, or else not contemplated at all?

This line of reasoning is not persuasive if one considers that the monthly ratio for the total CPI does not, strictly speaking, satisfy the axioms of a price index if any of its components are SB series. Since the monthly ratio of the SB component does not satisfy the proportionality axiom, neither does the monthly ratio of the total CPI. The latter series can only be said to satisfy the axioms of a price index if one assigns fixed basket shares to seasonal groups and arbitrarily defines these groups to be basic aggregates. Surely, little is gained by making the monthly ratios of the total CPI conform to the definition of a price index in such an artificial way.

6. Chaining of Seasonal-Basket Series

All official consumer price series have their baskets updated on a regular or irregular basis; i.e., they are chain price indexes. In countries such as France, Sweden, and the United Kingdom, a new index basket is linked in every 12 months. In Canada, the basket is changed every four years, and in Japan every five years. In these and other countries, for many possible periods of comparison CPI ratios will not satisfy the axioms of a price index because of chain linking. Nevertheless, all official indexes involve the calculation of linked series because the first priority of official agencies is to have an index that is properly representative of the goods consumers purchase. The parallel with the SB approach is obvious.

One can link in a new index basket at the year or at the month, but most consumer price series are linked at the month. Linking at the month, say at December of year $y - 1$, preserves the monthly movement of the series. There is then no discontinuity between December and January, and the January monthly ratio satisfies the axioms of a price index. On the other hand, the annual ratio for year y does not satisfy these axioms due to the change in baskets. If instead one links at year $y - 1$, the annual ratio for y satisfies the axioms of a price index but not the January monthly ratio. The CPI contains important seasonal groups, whose annual movements can be adequately measured using AB or SB indexes, but whose monthly movements, it was argued in Section 3.3, cannot be well measured either way. Therefore, it makes more sense to link consumer price series at the year, so that all annual ratios satisfy the axioms of a price index.

Consider the chain index constructed by linking SB series. Suppose the old index basket for year c is replaced by an updated basket for year n, using year r as the link year. Then the formula for the monthly chain index is

$$P^{ch}_{y,m/0} = P^{ch}_{r/0} \times \sum_j p_{y,m} q_{n,m} \Big/ \sum_j \bar{p}_r q_{n,m}; \quad y = r + 1, r + 2, \ldots, \tag{6.1}$$

where

$$\bar{p}_r = \sum_m p_{r,m} q_{n,m} \Big/ \sum_m q_{n,m}.$$

The formula for the annual chain index is

$$P^{ch}_{y/0} = P^{ch}_{r/0} \times \sum_j \bar{p}_y q_n \Big/ \sum_j \bar{p}_r q_n$$

where

$$\bar{p}_y = \sum_m p_{y,m} q_{n,m} \Big/ \sum_m q_{n,m}.$$

One might also wish to calculate an annual index for a fiscal year that overlaps the link year and year $r + 1$, the first year for the new basket. Suppose, for example, that the fiscal year runs from April 1 to March 31, then the appropriate formula for the index for fiscal year r is

$$P^{ch}_{r,3/0} \times \left(\sum_j \sum_{m=4}^{12} p_{r,m} q_{c,m} \Big/ \sum_j \sum_{m=1}^{12} p_{r,3} q_{c,m} \right) + P^{ch}_{r,12/0} \times \left(\sum_j \sum_{m=1}^{3} p_{r+1,m} q_{n,m} \Big/ \sum_j \sum_{m=1}^{12} p_{r+1,m} q_{n,m} \right)$$

where $P^{ch}_{r,3/0}$ represents the chain index for March of year r, and $P^{ch}_{r,12/0}$ the index for December. It can readily be seen that if $q_{c,m} = \bar{q}_c$ and $q_{n,m} = \bar{q}_n$ for every good, the formula simplifies to a simple average of the monthly chain index numbers.

Suppose again that the year n basket is replacing the year c basket but that the link is made at December of year r. Then we see that

$$P^{ch}_{y,m/0} = P^{ch}_{r,12/0} \times \sum_j p_{y,m} q_{n,m} \Big/ \sum_j p_{r,12} q_{n,m}. \tag{6.2}$$

The second factor on the right hand side of (6.2) defines an index at prices of December of link year r. Note that if a good is out-of-season in December but in-season in month m, it is necessary either to impute link month prices for that good or to drop it from the basket. In Section 3, it was mentioned that one of the great advantages of the SB approach is that it is unnecessary to impute prices for out-of-season items. This is a good reason for not linking using a formula such as (6.2). If linking at December, a more appropriate formula would be

$$P^{ch}_{y,m/0} = P^{ch}_{r,12/0} \times (P^{s}_{y,m/n} / P^{s}_{r,12/n}) \tag{6.3}$$

where

$$P^{s}_{y,m/n} = \sum_j p_{y,m} q_{n,m} \Big/ \sum_j p_n q_{n,m}.$$

This involves calculating a direct SB index with a year n basket and base period and linking it to the chain index at December of year r. Note that the formula is much more complicated than (6.1), the one for linking at the year.

There are a couple of other benefits associated with linking at the year rather than the month. First, in most countries, linking at the year is consistent with the practice of the System of National Accounts. Both implicit price indexes and constant dollar series are linked at a year instead of a month or quarter. Second, weighting diagrams that are published in reference documents on price indexes should ideally show the quantities purchased in the basket month or basket year at link period prices. Value weights at December link-month prices can be quite misleading to index users since the structure of relative prices for December can be atypical. Value weights at link-year prices are more useful to the user who wishes to obtain an idea of the relative importance of different goods in the CPI over all months of the year.

7. Conclusion

In summary, this paper argues for the adoption of the following procedures in calculating official consumer price series for seasonal goods.

1. Monthly consumer price series for seasonal goods should be based on an SB formula at base year prices, the Rothwell formula (see Section 3.2).
2. The annual indexes for seasonal goods should be calculated as weighted averages of the monthly indexes (Section 3.2).
3. A monthly-rebased quasi-Laspeyres chain index should be calculated as an analytical adjunct of the Rothwell series. Its monthly ratio corresponds to the price change factor of the monthly ratio of the Rothwell index (Section 3.3).
4. The official indexes for seasonal goods should be based on raw prices, i.e., prices unadjusted for seasonal variation. If separate seasonally adjusted indexes are produced, they should not have any official sanction for purposes of escalation so that the most recent years' data may be revised as required (Section 3.6).
5. The total CPI should be calculated as a

Rothwell index. The monthly value weights for non-seasonal goods can be set equal for all months of the year. In this way, the indexes for non-seasonal goods will in effect be based on the AB formula (Section 5).

6. When the basket for consumer price series is updated, the new series should be linked at a year rather than at a month (Section 6).

The diversity of methods now used to compute price indexes for seasonal goods may intrigue index makers, but it should also embarrass us. There should be, and there probably will be, a greater standardization of procedures by official agencies. When this happens, it will likely involve the adoption of the seasonal basket approach to seasonal goods by those countries that have not already done so, and its more rigorous application by those that have.

8. References

Allen, R.G.D. (1975). Index Numbers in Theory and Practice. London: MacMillan.

Baldwin, A. (1987). A Seasonal-Basket Price Index for Fresh Vegetables. Consumer Prices and Price Indexes (Catalogue 62-010), April-June 1987, Ottawa: Statistics Canada, 109–124.

Baldwin, A. (1988). Seasonal Goods in Consumer Price Indexes. Carleton University working paper, Carleton University, Ottawa.

Balk, B.M. (1980a). Seasonal Products in Agriculture and Horticulture and Methods for Computing Price Indices. Statistical Studies No. 24, The Hague: Netherlands Central Bureau of Statistics.

Balk, B.M. (1980b). Seasonal Commodities and the Construction of Annual and Monthly Price Indexes. Statistische Hefte, 21, 110–116.

Balk, B.M. (1980c). A Method for Constructing Price Indexes for Seasonal Commodities. The Journal of the Royal Statistical Society, ser. A, 143, 68–75.

Balk, B.M. (1981). A Simple Method for Constructing Price Indexes for Seasonal Commodities. Statistische Hefte 22, 72–78.

Bean, L.H. and Stine, O.C. (1924). Four Types of Index Numbers of Farm Prices. Journal of the American Statistical Association, XIX, 30–35.

Castles, I. (1987). The Australian Consumer Price Index: Concepts, Sources and Methods. Canberra: Australian Bureau of Statistics.

Dagum, E.B. (1980). The X11-ARIMA Seasonal Adjustment Method. (Catalogue 12-546E), Ottawa: Statistics Canada.

Diewert, W.E. (1983). The Treatment of Seasonality in the Cost-of-Living Index. In Price Level Measurement. Proceedings from a Conference Sponsored by Statistics Canada, eds. W.E. Diewert and C. Montmarquette, Ottawa: Statistics Canada, 1019–1045.

Drechsler, L. (1973). Weighting of Index Numbers in Multilateral International Comparisons. Review of Income and Wealth, 19, 17–34.

Eichhorn, W. and Voeller, J. (1983). Axiomatic Foundations of Price Indexes and Purchasing Power Parities. In Diewert and Montmarquette, op. cit., 411–450.

Institut National de la Statistique et des Etudes Economiques (INSEE) (1989). L'indice des prix à la consommation, conference of 11 May, 1989, Conseil National de l'Information Statistique.

International Labour Office (ILO) (1984). Statistical Sources and Methods: Volume 1 Consumer Price Indices. Geneva: ILO.

Netherlands Central Bureau of Statistics (1978). Maandstatistiek van de prijzen. The Hague: Central Bureau of Statistics.

Organisation for Economic Co-operation and Development (OECD) (1981). Main Economic Indicators: Sources and Methods. Germany, Paris: OECD.

Rothwell, D.P. (1958). Use of Varying Seasonal Weights in Price Index Construction. Journal of the American Statistical Association, XLIII, 66–67.

Szulc, B.J. (1987). Price Indices Below the Basic Aggregation Level. Bulletin of Labour Statistics, 1987.1, ix-xvi.

Theil, H. (1973). A New Index Number Formula. Review of Economics and Statistics, 55, 498–502.

Turvey, R. (1979). The Treatment of Seasonal Items in Consumer Price Indices. Bulletin of Labour Statistics, 1979.4, xiii-xxiii.

Zarnowitz, V. (1961). Index Numbers and the Seasonality of Quantities and Prices. Staff Paper No. 5 in the Price Statistics of the Federal Government, ed. G.P. Stigler, Washington, D.C.: National Bureau of Economic Research.

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