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Small Area Estimation in the Presence of Correlated Random Area Effects

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This article is a contribution to the discussion on the utility of spatial models in the context of Small Area Estimation (SAE) (see Cressie 1991; Pfeffermann 2002; Saei and Chambers 2003, 2005; Singh et al. 2005; Pratesi and Salvati 2008). The attention is on the Fay–Herriot model and its Mean Squared Error (MSE) when a common autocorrelation parameter among small areas is included. Firstly, we discuss the extent to which the spatial effects in data used for SAE motivate the introduction of an autocorrelation parameter in the Fay–Herriot model. Secondly, the performance of MSE estimators is discussed through a simulation study where the joint effect of the area level sampling variance and of the parameter estimation is shown. The importance of the strength of spatial autocorrelation among small areas is confirmed. The results are tenable for different sampling variance patterns. A case study with spatial dependence in the data is presented and estimates at small area level are provided.

Key words: Small area estimation; Fay–Herriot model; spatial correlation; Simultaneously Autoregressive (SAR) model; Spatial Empirical Best Linear Unbiased Predictor (SEBLUP).

1. Introduction

This article is a contribution to the discussion on the utility of spatial models in the context of SAE (see Cressie 1991; Pfeffermann 2002; Saei and Chambers 2003, 2005; Singh et al. 2005; Pratesi and Salvati 2008). It is well-known that in many situations the location of the small areas can be so important as to cast doubt on the assumption of spatial independence in the Fay–Herriot model (1979). In the literature related to SAE this problem, however, is not new. In fact, Clayton and Kaldor (1987) suggested the use of spatial models, as against spatial independence, in disease mapping and Cressie and Chan (1989) applied spatial models in a pioneering work on socio-economic data. More specifically, in the context of SAE, the idea of extending the Fay–Herriot model to include spatial correlation was first proposed by Cressie (1991), although it was not completely developed for SAE. More recently, Pfeffermann (2002) has shown that the loss in efficiency in SAE can be substantial when correlation between small areas is ignored, but noted that unless this correlation is quite strong, Fay–Herriot models with a correlation parameter do not lead to significant efficiency gains. Other contributions, such as those of Saei and Chambers (2003, 2005), have emphasized the importance of time and area effects in SAE.

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Very recently the problem of introducing a common autocorrelation parameter among small areas through the Simultaneously Autoregressive (SAR) process has been taken into account and extended to the Fay–Herriot model (Salvati 2004; Singh et al. 2005; Petrucci et al. 2005; Petrucci and Salvati 2006; Pratesi and Salvati 2008).

This article addresses two research questions. Firstly, we analyse the extent to which the spatial effects in data used for SAE justify the introduction of an autocorrelation parameter in the Fay–Herriot model. In practice, strong spatial autocorrelations are rarely encountered in survey data applications although some exceptions in the fields of geology, agriculture, environmental science, health and epidemiology have been encountered. Our prime intention here is to compare the performance of the Best Linear Unbiased Predictor (BLUP) with that of the Spatial Best Linear Unbiased Predictor (SBLUP) by moving from weak to strong spatial correlation in the target variable. Secondly, we focus our attention on the estimation of the MSEs of the Empirical Best Linear Unbiased Predictor (EBLUP) and Spatial EBLUP (SEBLUP) proposed in Singh et al. (2005). This is carried out via a simulation experiment which also provides evidence on the joint effects on the width of the confidence intervals for the small area parameter estimates of the following components: the known sampling variances (φ_i) of the direct estimators of the small area means, the common variance (σ_u^2) of the random area effects in the model and the spatial autocorrelation parameter (ρ).

The article is organized as follows. Section 2 defines the general area level spatial model and the point estimation problem. Section 3 is devoted to the MSE estimation problem. The performance of the MSE estimators is discussed in Section 4 through a simulation study; the properties of various estimators are evaluated using the results in Sections 3 and 4. Section 5 illustrates a survey data application where there are spatial effects in data used for SAE. Section 6 concludes with our final comments and recommendations for future research.

2. Small Area Estimation Models

There are several alternatives for introducing spatial autocorrelation in SAE and for producing reliable estimates for the target variables (Petrucci et al. 2005). We will focus our attention on the introduction of the SAR process in the Fay–Herriot model.

Let $\boldsymbol{\vartheta}$ be the $m \times 1$ vector of the parameters of inferential interest (small area total y_i, small area mean $\bar{y}_i$ with $i = 1...m$) and assume that the direct estimator $\hat{\boldsymbol{\vartheta}}$ is available and design unbiased, i.e.,

$$\hat{\boldsymbol{\vartheta}} = \boldsymbol{\vartheta} + \mathbf{e} \tag{2.1}$$

where $\mathbf{e}$ is a vector of independent sampling errors with mean vector $\mathbf{0}$ and known diagonal variance matrix $\mathbf{R} = diag(\varphi_i)$, φ_i representing the sampling variances of the direct estimators of the area parameters of interest. Usually φ_i is unknown and is estimated by a "generalized variance function" applied not only to the specific area sample, but to the whole sample as well; for details see Wolter (1985, Chapter 5) and Wang and Fuller (2003). The result is treated as a good estimate of the sampling variance, which is a very strong assumption. We do not explore its limits here because in our study the φ_i patterns are given, but we are aware that the issue requires further research.

The spatial dependence among small areas is introduced by specifying a linear mixed model with spatially correlated random area effects for ϑ, i.e.,

$$\boldsymbol{\vartheta} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{v} \tag{2.2}$$

where $\mathbf{X}$ is the $m \times p$ matrix of the specific area auxiliary covariates $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{ip})$, $\boldsymbol{\beta}$ is the $p \times 1$ vector of regression parameters, $\mathbf{Z}$ is an $m \times m$ matrix of known positive constants and $\mathbf{v}$ is an $m \times 1$ vector of spatially correlated random area effects given by the following autoregressive process with spatial autoregressive coefficient ρ and $m \times m$ spatial interaction matrix $\mathbf{W}$ (see Cressie 1993; Anselin 1992):

$$\mathbf{v} = \rho\mathbf{W}\mathbf{v} + \mathbf{u} \Rightarrow \mathbf{v} = (\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{u} \tag{2.3}$$

where $\mathbf{u}$ is an $m \times 1$ vector of random area effects, independent of $\mathbf{e}$**,** with zero mean and constant variance σ_u^2 and $\mathbf{I}$ is an $m \times m$ identity matrix.

The $\mathbf{W}$ matrix describes the spatial interaction structure of the small areas, usually defined through the neighbourhood relationship between areas; generally speaking, $\mathbf{W}$ has a value of 1 in row i and column j if areas i and j are neighbours. The autoregressive coefficient ρ defines the strength of the spatial relationship among the random effects associated with neighbouring areas. Generally, for ease of interpretation, the spatial interaction matrix is defined in row standardized form, in which the row elements sum to one; in this case ρ is called a spatial autocorrelation parameter (Banerjee et al. 2004).

Combining (2.1) and (2.2), the estimator with spatially correlated errors can be written as:

$$\hat{\boldsymbol{\vartheta}} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}(\mathbf{I} - \rho\mathbf{W})^{-1}\mathbf{u} + \mathbf{e} \tag{2.4}$$

The error terms $\mathbf{v}$ have the $m \times m$ Simultaneously Autoregressive (SAR) covariance matrix:

$$\mathbf{G} = \sigma_u^2\left[\left(\mathbf{I} - \rho\mathbf{W}^T\right)\left(\mathbf{I} - \rho\mathbf{W}\right)\right]^{-1} \tag{2.5}$$

and the covariance matrix of $\hat{\boldsymbol{\vartheta}}$ is given by:

$$\mathbf{V} = \mathbf{R} + \mathbf{Z}\mathbf{G}\mathbf{Z}^T$$

Under Model (2.4), the Spatial Best Linear Unbiased Predictor (SBLUP) estimator of ϑ_i is (using $\mathbf{X}^T$ to indicate the transpose of matrix $\mathbf{X}$):

$$\tilde{\vartheta}_i^S\left(\sigma_u^2, \rho\right) = \mathbf{x}_i\tilde{\boldsymbol{\beta}} + \mathbf{b}_i^T\mathbf{G}\mathbf{Z}^T\left\{\mathbf{R} + \mathbf{Z}\mathbf{G}\mathbf{Z}^T\right\}^{-1}\left(\hat{\boldsymbol{\vartheta}} - \mathrm{X}\tilde{\boldsymbol{\beta}}\right) \tag{2.6}$$

where $\tilde{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{V}^{-1}\hat{\boldsymbol{\vartheta}}$ and $\mathbf{b}_i^T$ is a $1 \times m$ vector $(0, 0 \ldots 0, 1 \ldots 0)$ with value 1 in the ith position. The predictor is obtained from Henderson's 1975 results for general linear mixed models involving fixed and random effects. The SBLUP, when $\rho = 0$, reduces to the BLUP, i.e., an independent random specific area effects model.

The SBLUP estimator $\tilde{\vartheta}_i^S\left(\sigma_u^2, \rho\right)$ in (2.6) depends on the unknown variance component σ_u^2 and spatial autocorrelation parameter ρ. Substituting their asymptotically consistent estimators $\hat{\sigma}_u^2, \hat{\rho}$, obtained either by Maximum Likelihood (ML) or Restricted Maximum Likelihood (REML) methods based on the normality assumption of the random effects, the following two-stage estimator $\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2, \hat{\rho}\right)$, called the SEBLUP, is obtained:

$$\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2, \hat{\rho}\right) = \mathbf{x}_i\hat{\boldsymbol{\beta}} + \mathbf{b}_i^T\hat{\mathbf{G}}\mathbf{Z}^T\left\{\mathbf{R} + \mathbf{Z}\hat{\mathbf{G}}\mathbf{Z}^T\right\}^{-1}\left(\hat{\boldsymbol{\vartheta}} - \mathbf{X}\hat{\boldsymbol{\beta}}\right) \tag{2.7}$$

The ML estimators of σ_u^2 and ρ can be obtained iteratively using the "Nelder-Mead" algorithm (Nelder and Mead 1965) and the "scoring" algorithm (Rao 2003) in sequence. The use of these procedures sequentially is necessary because the log-likelihood function has a global maximum as well as some local maximums; for more details see Singh et al. (2005) and Pratesi and Salvati (2008).

3. The Mean Squared Error Problem

The MSE of the SBLUP depends on the unknown variance component σ_u^2 and on the spatial autocorrelation parameter ρ and can be written as (Singh et al. 2005; Pratesi and Salvati 2008):

$$MSE\left[\tilde{\vartheta}_i^S\left(\sigma_u^2,\rho\right)\right] = g_{1i}\left(\sigma_u^2,\rho\right) + g_{2i}\left(\sigma_u^2,\rho\right) \tag{3.1}$$

where the first term, $g_{1i}\left(\sigma_u^2,\rho\right)$, is due to the estimation of the random effects and is of order $O(1)$ and the second term, $g_{2i}\left(\sigma_u^2,\rho\right)$, is due to the estimation of $\boldsymbol{\beta}$ and is of order $O(m^{-1})$ for large m (Singh et al. 2005). The details of the calculation are reported in Appendix A.

Under regularity conditions, together with the assumption of a large m and ignoring the terms of the order $O(m^{-1})$, the following second-order approximation to the $MSE\left[\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2,\hat{\rho}\right)\right]$ of the SEBLUP is obtained (Singh et al. 2005; Pratesi and Salvati 2008):

$$MSE\left[\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2,\hat{\rho}\right)\right] \approx g_{1i}\left(\sigma_u^2,\rho\right) + g_{2i}\left(\sigma_u^2,\rho\right) + g_{3i}\left(\sigma_u^2,\rho\right) \tag{3.2}$$

where the variance component σ_u^2 and the spatial autocorrelation coefficient ρ, assuming normality of the random effects, can be estimated by either Maximum Likelihood (ML) or Restricted Maximum Likelihood (REML) methods, and $g_{3i}\left(\sigma_u^2,\rho\right)$ can be obtained by following the results of Kackar and Harville (1984), Prasad and Rao (1990), Datta and Lahiri (2000), Singh et al. (2005) and Pratesi and Salvati (2008); see Appendix A for details of the computation of $g_{3i}\left(\sigma_u^2,\rho\right)$.

In practical applications, an approximately unbiased estimator of the MSE of $\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2,\hat{\rho}\right)$ is given by (Singh et al. 2005; Pratesi and Salvati 2008):

$$mse\left[\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2,\hat{\rho}\right)\right] \approx g_{1i}\left(\hat{\sigma}_u^2,\hat{\rho}\right) + g_{2i}\left(\hat{\sigma}_u^2,\hat{\rho}\right) + 2g_{3i}\left(\hat{\sigma}_u^2,\hat{\rho}\right) \tag{3.3}$$

when $\hat{\sigma}_u^2$ and $\hat{\rho}$ are REML estimators. On the other hand, if the ML procedure is used for estimating the parameters, the $mse\left[\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2,\hat{\rho}\right)\right]$ is given by:

$$\begin{aligned} mse\left[\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2,\hat{\rho}\right)\right] \approx\ & g_{1i}\left(\hat{\sigma}_u^2,\hat{\rho}\right) - \mathbf{b}_{ML}^T\left(\hat{\sigma}_u^2,\hat{\rho}\right)\nabla g_{1i}\left(\hat{\sigma}_u^2,\hat{\rho}\right) + g_{2i}\left(\hat{\sigma}_u^2,\hat{\rho}\right) \\ & + 2g_{3i}\left(\hat{\sigma}_u^2,\hat{\rho}\right) \end{aligned} \tag{3.4}$$

The extra term $\mathbf{b}_{ML}^T\left(\hat{\sigma}_u^2,\hat{\rho}\right)\nabla g_{1i}\left(\hat{\sigma}_u^2,\hat{\rho}\right)$ is due to the bias of $g_{1i}\left(\hat{\sigma}_u^2,\hat{\rho}\right)$ (see Pratesi and Salvati 2008).

Expressions (3.3) and (3.4) are strictly valid only if $\mathbf{V}$ has a linear covariance structure (Rao 2003); however, from the results of Harville and Jeske (1992) and Zimmerman and

Cressie (1992), Expressions (3.3) and (3.4) could be taken as approximate unbiased estimators of the *MSE* (Pratesi and Salvati 2008).

For large m and neglecting all terms of order $O(m^{-1})$, Singh et al. (2005) proposed yet another expression for the estimator of the *MSE* based on subtracting from Expressions (3.3) and (3.4) an extra term $g_{5i}(\hat{\sigma}_u^2, \hat{\rho})$ depending on the presence of spatial autocorrelation which makes the covariance structure of V nonlinear. The component $g_{5i}(\hat{\sigma}_u^2, \hat{\rho})$ up to order $O(m^{-1})$ is reported in Appendix A.

As mentioned above, we focus our attention in this article on the performance of Estimator (3.3) with and without the introduction of the extra term $g_{5i}(\hat{\sigma}_u^2, \hat{\rho})$ suggested in Singh et al. (2005) in order to provide evidence on the empirical effect of the spatial autocorrelation. The properties of the estimators are evaluated via the simulation experiments in Section 4.

4. The Simulation Experiments

The finite-sample performances of the MSE estimators of the SBLUP and SEBLUP have been investigated through Monte Carlo experiments carried out for the area level estimator (2.4) with an intercept and one covariate variable: $\hat{\vartheta}_i = 1 + 2x_i + v_i + e_i$. The x-values were generated from a Chi-square distribution with 20 degrees of freedom.

It is reasonable to assume that the spatial estimator performs better when the spatially correlated random effects model provides a good fit rather than an imprecise one, so we generated 90,000 data sets using the spatial regression model (3.3) with random area effects of neighbouring areas correlated according to the SAR dispersion matrix with an established spatial autoregressive coefficient. The experiment was designed following Datta et al. (2005). Letting $\sigma_u^2 = 1$, independent random variables $\mathbf{v} = [v_1, v_2, \ldots, v_m]^T$ from a $MVN\left(\mathbf{0}, \sigma_u^2\left[\left(\mathbf{I} - \rho\mathbf{W}^T\right)\left(\mathbf{I} - \rho\mathbf{W}\right)\right]^{-1}\right)$ and $\mathbf{e} = [e_1, e_2, \ldots, e_m]^T$ from a $MVN(\mathbf{0}, diag(\varphi_i))$ were generated for specified sampling variances φ_i. The number of small areas m is set to 42 and divided into five groups $G_1, G_2, \ldots, G_5$, with the first four groups containing eight areas and group G_5 containing 10 areas. The φ_i's within each group are held constant. In particular, three different φ_i -patterns were chosen: (a) 0.7, 0.6, 0.5, 0.4, 0.3; (b) 2.0, 0.6, 0.5, 0.4, 0.2; (c) 4.0, 0.6, 0.5, 0.4, 0.1. Hence, for example, the φ_i's in the group G_1 are set to 0.7 for pattern (a), to 2.0 for pattern (b), to 4.0 for pattern (c) and likewise for the other groups. As can be seen, the variability in the sampling variance values φ_i is largest in pattern (c) and least in pattern (a). Intermediate values are found in pattern (b).

The SAR dispersion matrix was generated with $\rho = \pm 0.25, \pm 0.5, \pm 0.75$ and neighbourhood matrix structure $\mathbf{W}$ defined randomly by assigning neighbourhoods to each area as follows: for the fixed number of small areas $m = 42$, the spatial weight w_{ij} is assigned a value of 1 if the random number drawn from a uniform distribution [0,1] is greater than 0.5 and zero otherwise. The maximum number of neighbours for each area was set to 5, and the $\mathbf{W}$ matrix was standardized by rows so that we can refer to ρ as an autocorrelation parameter. The $\mathbf{W}$ matrix was kept fixed for the whole simulation experiments.

For each sample drawn, the mean of each small area was estimated using (i) the SBLUP estimator (Expression (2.6)), (ii) the SEBLUP estimator (Expression (2.7)), (iii) the BLUP estimator and (iv) the EBLUP estimator; the last two estimators, BLUP and EBLUP, may be easily obtained from (2.6) and (2.7) by setting $\rho = 0$. Further references may be found in Rao (2003, Sections 6.2.1 and 6.2.3).

Let y_{it} denote the simulated mean for small area i in the tth simulation, $t = 1, \ldots, T$, and $\hat{y}_{it}$ be the BLUP, SBLUP, EBLUP, and SEBLUP estimates. The performances of the estimates $\hat{y}_{it}$ have been summarized by their Average Relative Root MSE (*ARRMSE*) and Average Relative Bias (*ARB*), defined as follows:

$$ARRMSE = \frac{1}{m}\sum_{i=1}^{m} \frac{\left[MSE(\hat{y}_i)^{1/2}\right]}{\bar{y}_i} \times 100 \text{ with } MSE(\hat{y}_i) = \frac{1}{T}\sum_{t=1}^{T}\left[\hat{y}_{it} - y_{it}\right]^2 \text{ and}$$

$$\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$$

$$ARB = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{1}{T}\sum_{t=1}^{T}\frac{\left[\hat{y}_{it} - y_{it}\right]}{y_{it}}\right| \times 100$$

The performance of the EBLUP and SEBLUP estimators $\tilde{y}_{it}$ has been evaluated also on the basis of their Average Empirical Mean Squared Error (*AEMSE*):

$$AEMSE = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{T}\sum_{t=1}^{T}\left[\hat{y}_{it} - y_{it}\right]^2$$

The purpose of *ARB* and *ARRMSE* is to compare the performance of estimators, while the purpose of *AEMSE* is to provide a benchmark for comparing the performance of estimators of mean squared errors. The MSE_i of SEBLUP in each area is estimated by Expression (3.3) (mse_{1i}). Its average over the T simulations and m areas is called $Amse_1$. The estimation is performed also by including the term $g_{5i}\left(\sigma_u^2, \hat{\rho}\right)$ indicated in Equation (A.1.6). The resulting estimator is called mse_{2i} and its average is $Amse_2$. The MSE_i of EBLUP is estimated by the well-known Expression (6.2.36) in Rao (2003). Its average over the T simulations and m areas is called the Average estimated mean squared error (*Amse*).

The whole simulation experiment was carried out with the use of a series of new programmes running under the R environment. The main results are summarized in Tables 1, 2, and 3 and Figure 1.

4.1. Comparison of BLUP vs SBLUP (σ_u^2 and ρ known)

Table 1 reports the values of the *ARRMSE (%)* and *ARB (%)* of the BLUP and SBLUP estimators, distinguishing between the three φ_i -patterns (a), (b) and (c).

The results offer an insight into the average bias and mean squared error of the estimators for varying values of the spatial correlation in the target variable.

The average bias and mean squared errors of the BLUP and SBLUP estimators, for varying values of the spatial correlation coefficient, are notably different. Whereas for the SBLUP estimator positive values of ρ have only a slightly greater effect on relative bias and MSE than their negative counterparts, for the BLUP estimator the results are very

Table 1. Comparison BLUP, SBLUP: ARB (%), ARRMSE (%) for $m = 42$, $\sigma_u^2 = 1$, $\rho = \pm 0.25, \pm 0.5, \pm 0.75$ and φ_i -patterns (a), (b) and (c)

	Pattern					
	(a)		(b)		(c)	
Spatial Correlation	*ARB*	*ARRMSE*	*ARB*	*ARRMSE*	*ARB*	*ARRMSE*
SBLUP						
0.75	0.04	1.52	0.04	1.59	0.04	1.59
0.5	0.03	1.51	0.04	1.57	0.04	1.56
0.25	0.03	1.51	0.04	1.56	0.04	1.54
−0.25	0.04	1.51	0.04	1.57	0.04	1.54
−0.5	0.04	1.52	0.04	1.58	0.04	1.56
−0.75	0.04	1.53	0.04	1.60	0.04	1.59
BLUP						
0.75	0.25	11.38	0.25	10.46	0.25	10.15
0.5	0.06	2.51	0.06	2.48	0.06	2.51
0.25	0.04	1.58	0.04	1.64	0.04	1.64
−0.25	0.04	1.55	0.04	1.61	0.04	1.60
−0.5	0.04	1.80	0.04	1.86	0.05	1.90
−0.75	0.06	2.74	0.05	2.65	0.06	2.84

Table 2. Comparison EBLUP, SEBLUP: Average of the empirical Mean Squared Error (AEMSE), Average of the estimated Mean Squared Error (Amse, $Amse_1$, $Amse_2$) for $m = 42$, $\sigma_u^2 = 1$, $\rho = \pm 0.25, \pm 0.5, \pm 0.75$

	SEBLUP			EBLUP	
Spatial Correlation	*AEMSE*	$Amse_1$	$Amse_2$	*AEMSE*	*Amse*
φ_i -pattern (a)					
0.75	0.346	0.346	0.344	0.388	0.401
0.5	0.346	0.342	0.337	0.357	0.358
0.25	0.348	0.344	0.336	0.343	0.338
−0.25	0.351	0.347	0.337	0.345	0.337
−0.5	0.354	0.348	0.339	0.356	0.349
−0.75	0.357	0.351	0.344	0.372	0.367
φ_i-pattern (b)					
0.75	0.402	0.412	0.408	0.475	0.485
0.5	0.395	0.399	0.389	0.409	0.411
0.25	0.392	0.394	0.381	0.383	0.382
−0.25	0.393	0.393	0.377	0.384	0.380
−0.5	0.400	0.397	0.384	0.401	0.398
−0.75	0.409	0.407	0.397	0.429	0.427
φ_i-pattern (c)					
0.75	0.441	0.464	0.455	0.533	0.538
0.5	0.417	0.434	0.419	0.429	0.431
0.25	0.404	0.415	0.397	0.393	0.394
−0.25	0.403	0.404	0.386	0.392	0.393
−0.5	0.413	0.413	0.396	0.414	0.416
−0.75	0.431	0.432	0.418	0.453	0.455

Table 3. Comparison EBLUP, SEBLUP:ARB (%), ARRMSE (%) for $m = 42$, $\sigma_u^2 = 1$, $\rho = \pm 0.25, \pm 0.5, \pm 0.75$ and φ_i -patterns (a), (b) and (c)

	Pattern					
	(a)		(b)		(c)	
Spatial Correlation	*ARB*	*ARRMSE*	*ARB*	*ARRMSE*	*ARB*	*ARRMSE*
SEBLUP						
0.75	0.06	1.54	0.07	1.62	0.08	1.63
0.5	0.06	1.54	0.07	1.61	0.07	1.60
0.25	0.06	1.54	0.06	1.60	0.08	1.58
−0.25	0.06	1.55	0.06	1.61	0.08	1.57
−0.5	0.06	1.55	0.06	1.62	0.08	1.60
−0.75	0.06	1.56	0.06	1.64	0.08	1.63
EBLUP						
0.75	0.06	1.63	0.06	1.74	0.07	1.76
0.5	0.06	1.56	0.06	1.63	0.06	1.62
0.25	0.06	1.54	0.06	1.60	0.06	1.57
−0.25	0.06	1.55	0.06	1.60	0.06	1.58
−0.5	0.06	1.56	0.06	1.63	0.06	1.61
−0.75	0.06	1.59	0.07	1.67	0.07	1.66

asymmetric. For example, the average bias of the SBLUP estimator for pattern (a) is 0.04 when $\rho = \pm 0.75$, but the corresponding values for the BLUP estimator are 0.06 when $\rho = -0.75$ and 0.25 when $\rho = 0.75$. Similar results are obtained for the φ_i -patterns (b) and (c).

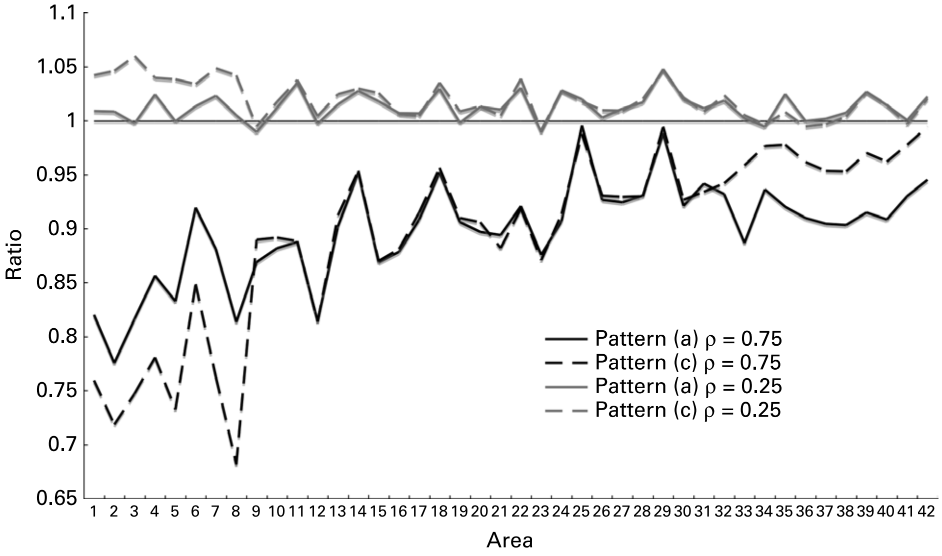

Fig. 1. Ratio between the Empirical MSE_i ($EMSE_i$) of SEBLUP and EBLUP for $m = 42$, $\sigma_u^2 = 1$, $\rho = \pm 0.25, \pm 0.75$ and φ_i -pattern (a) and (c)

What is interesting to note is that there seems to be very little variation between the three φ_i -patterns. An important and reassuring overall result that emerges from Table 1 is that for the same value of ρ the relative bias and MSEs are larger for the BLUP estimator, with the differences between the two estimators increasing rather notably together with increases in the positive values of ρ.

4.2. *Comparison of EBLUP vs SEBLUP (σ_u^2 and ρ not known)*

Figure 1 contains our main results on the behaviour of the empirical MSE_i of the EBLUP and the SEBLUP in each of the $m = 42$ areas. Table 2 compares the performance of the estimators on the basis of the *Average Empirical MSE*. In Table 3 the estimators are compared in terms of *ARB* and *ARRMSE*.

We begin by focussing our attention on the ratio of the empirical MSE of the SEBLUP and EBLUP estimators (Figure 1). The objective is to see whether – and if so when – the two estimators achieve similar MSE values. Our expectation is that the two estimates should be different in the case of relevant spatial correlation and achieve common levels when spatial correlation is low. The results obtained under φ_i -pattern (c) (dashed line) and φ_i -pattern (a) (solid line) are shown in Figure 1 in order to evidence any dependency on the sampling variability φ_i in the small areas.

We observe that the ratio is always lower than 1 in the presence of a high positive spatial correlation ($\rho = 0.75$), suggesting that the SEBLUP is always more accurate than the EBLUP in this case. This is most probably due to the ability of the spatial Fay–Herriot model to capture the presence of spatial effects in data used in the SAE. Given a high ρ, we note that the lowest values of the ratio correspond to areas included in the group G_1 where the sampling variability φ_i is highest. The dashed and solid lines follow the same trajectory when the sampling variances in the areas are the same, i.e., areas 9 to 32, which fall in Groups 2, 3, and 4. On the other hand, when the spatial correlation is weak ($\rho = 0.25$) the ratio, as expected, is very close to 1 for all the 42 areas.

Table 2 shows the average of the empirical MSE_i (*AEMSE*) and the $Amse_1$, $Amse_2$, $Amse$ estimators for $m = 42$, $\sigma_u^2 = 1$, $\rho = \pm 0.25, \pm 0.5, \pm 0.75$ and all three φ_i -patterns.

Two results stand out in Table 2. The first refers to all the *MSE* estimates whose values increase along with the sampling variability in the area groups and independently of the values of ρ; in fact, the MSE estimates are at their lowest levels for φ_i -pattern (a), where the sampling variability in the area groups is at its lowest, and at their highest for φ_i -pattern (c), where the sampling variability in the area groups is at its highest.

The second result of interest concerns the behaviour of the estimators of the target. As expected, the *AEMSE* and the *AMSE* of the EBLUP are noticeably larger than the *AEMSE* of the SEBLUP when spatial correlation between areas is high, but these indexes are very similar in value when ρ is not relevant ($\rho = \pm 0.25$).

With regard to the average MSE_i estimators of the SEBLUP, we note that the $Amse_2$ is almost always inferior in value to that of the *AEMSE*; the only exceptions to this result occur in φ_i -patterns (b) and (c) and when $\rho = 0.75$.

Both the $Amse_1$ and $Amse_2$ estimators are obtained from approximations (Expressions (3.3) and (A.1.6)), which behave well for very large m and might not hold for the m-value in our simulations; remember that here $m = 42$. From the results reported in Table 2, we note an appreciable difference between the values of $Amse_2$ and $Amse_1$. In fact, mse_{2i} is a subtraction, from Expression (3.3), of the term $g_{5i}\left(\hat{\sigma}_u^2, \hat{\rho}\right)$ which in our experiments has always been positive, thereby resulting in a final estimate of the MSE very often inferior, rather than being similar, to that of the *AEMSE*.

In Table 3 the estimators are compared in terms of *ARB* and *ARRMSE*. The average bias and mean squared errors of the EBLUP and SEBLUP estimators, for varying values of the spatial correlation coefficient, are not notably different. It seems that the estimation of the variance component σ_u^2 and/or the spatial correlation ρ by REML procedure has a confounding effect on the performance of the estimators. However, the positive and negative values of spatial correlation have an effect on relative bias and MSE, reducing them especially when $\rho = -0.75$ or $\rho = 0.75$. As in Table 1, the variation between the three φ_i -patterns is very small. For the same value of ρ the relative bias and MSEs are still larger for the EBLUP estimator, with the differences between the two estimators increasing together with increases in the positive values of ρ.

5. A Case Study

The data are from the Farm Structure Survey (FSS – ISTAT 2003). The survey is carried out once every two years. The sample is selected by means of a stratified one-stage design with self-representation of larger farms (agricultural holdings). The sample size is 55,030 farms for Italy and 2,504 for Tuscany. The survey is carried out in order to produce accurate estimates of agricultural production at national and regional levels (Ballin and Salvi 2004). In this case study, the target parameter is the farm production of olives in quintals at a subregional level in Tuscany.

Tuscany is divided into 53 Agricultural Zones (AZs). They are defined on a geographical basis and are very useful small areas in economic studies on sectors of economic activity. They are determined following the administrative boundaries of the 287 Municipalities of Tuscany. All the AZs are represented in the regional FSS sample. The average sample size per AZ results in $\bar{n} = 45.2$ (s.d. 37.3). The area level sampling variances, $\hat{\varphi}_i$, have been obtained by estimating the sampling variances of the small area direct estimators (Ballin and Salvi 2004). The values of estimated sampling variances range from 0.48 to 3,100.

5.1. Exploratory Analysis

The exploratory analysis firstly tested the presence of the spatial dependence in the data. Essential to this are the definitions of the spatial location of the AZs and the spatial interaction matrix (**W**). The centroid of each AZ is considered to be the spatial reference for all the units (farms for AZs) residing in the same small area and it is defined to be the location of the small area. The Atlas of Coverage of the Tuscany Region maintained by the Geographical Information System of the Regione Toscana provided all the information on coordinates, extensions and positions of the small areas of interest (UTM system). The Population Census and Agricultural Census databases provided all the auxiliary information related to the average farm production of olives (quintals per farm) and their covariates at small area level.

The spatial interaction matrix (**W**) for each location specifies which other locations in the system affect the value of the farm production of olives at that location. The elements of **W** are nonstochastic and exogenous to the SAE model. In our definition the elements of **W** take nonzero values (they are equal to 1) only for those pairs of AZs, which are contiguous to each other (first-order contiguity).

Spatial autocorrelation in the target variables and in the auxiliary variables has been checked by the two best-known test statistics for spatial autocorrelation: Moran's *I* and Geary's *C* (Moran 1950; Cliff and Ord 1981). The best explanatory variable for the target variable is the agricultural surface utilized for the production of olives (measured in hectares).

The results in Table 4 show spatial dependence in the covariate and in the target variable. For the covariate, the Moran's *I* statistics are significant at the 1% level, indicating that similar values are more spatially clustered than what might be caused purely by chance. This is consistent with the estimated values for Geary's *C*. Spatial dependence in the target variable is weaker, but still statistically significant.

5.2. *Small Area Estimation Results*

The per farm production of olives was modelled by the Spatial Fay–Herriot model and by the more traditional Fay–Herriot model. For the spatial model the value of the estimated spatial autoregressive coefficient $\hat{\rho}$ was 0.686 (s.e. = 0.319) and the value of the estimated variance component $\hat{\sigma}_u^2$ was 0.792 (s.e. = 0.604) when we used the REML procedure. Table 5 summarizes the results. Columns (a) and (b) report the average and the median of the small area estimates. The accuracy of the estimates is measured by the coefficients of variation. They are computed as follows:

$$CV_1 = \left(\sum_i \sqrt{mse_{1i}/m}\right)\bar{\tilde{\vartheta}}^{-1} \times 100, CV_2 = \left(\sum_i \sqrt{mse_{2i}/m}\right)\bar{\tilde{\vartheta}}^{-1} \times 100$$

where $\bar{\tilde{\vartheta}}$ indicates the mean or the median of the small area estimates.

The mean of the point estimates suggests a production of olives of about six quintals per farm with a slightly lower median value obtained in both the SEBLUP and EBLUP procedures. This is not a surprise as the distribution of the target variable in the population is skewed and concentrated on small production units. The average accuracy of the estimates is not appreciable: the *CV* is about 30% of the estimates. This can be mainly due to the high dispersion of the sample size in the areas and to the skewness of the distribution

Table 4. Spatial dependence in the covariates and in the target variables (Standard Errors in parentheses)

Covariate	Moran's *I*	Geary's *C*
Surface area for production of olives (ha)	0.528 (0.002)	0.439 (0.002)
Target variable per farm average production of olives (q)	0.263 (0.040)	0.634 (0.008)

Table 5. Case study: Spatial Fay–Herriot and traditional Fay–Herriot estimates

Estimate	$\tilde{\vartheta}$ (quintals)	Q_2(quintals)
SEBLUP	6.04	5.56
$CV_1\%$	(33.22)	(20.34)
$CV_2\%$	(32.27)	(19.02)
EBLUP	6.13	5.39
$CV\%$	(34.22)	(19.69)

of the target variable. EBLUP on average is slightly more variable, even though its performance is in line with that of SEBLUP.

The performances of EBLUP and SEBLUP are similar even though the spatial relationship appears to be of medium strength and significant. These results seem to be consistent with those of Table 3. Indeed, the low and nonsignificant value of the estimated variance component and the wide range of sampling variances could have produced the not relevant differences between the EBLUP and SEBLUP estimates and their accuracy.

5.3. Model-based Simulation on Real Data

The differences in the behaviour of the mse_{1i} and mse_{2i} estimators have been explored by a simulation experiment based on the case study data. Given the estimated sampling variances $\hat{\varphi}_i$, the spatial interaction matrix (**W**) and the estimated $\hat{\beta}$ and $\hat{\sigma}_u^2$, the sampling distribution of SEBLUP has been simulated for different spatial correlation values $\rho = \pm 0.25, \pm 0.5, \pm 0.75$ under the assumption of normality of **u** and **e**. Also here, as in Section 4, empirical mean squared error is considered as the "target" MSE_i.

For each sample drawn the MSE_i has been estimated also by Equation (3.2), given $\hat{\sigma}_u^2$ and ρ. The mean of $t = 1, \ldots, T$, simulations is called *second-order* (*so*) MSE_i:

$$soMSE_i = \frac{1}{T}\sum_{t=1}^{T} MSE_t\left[\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2, \hat{\rho}\right)\right]$$

Our expectation is that the behaviour of the empirical MSE_i and the $soMSE_i$ be the same under the different simulation settings.

The ratio between each estimator and the empirical MSE_i is computed for each AZ and summarized in Table 6, where the averaging is over the 53 AZs. The values for the 53 AZs are reported only for $\rho = \pm 0.75$ in Figures 2 and 3.

The differences in the performances of the mse_{1i} and mse_{2i} estimators observed in the simulations of Section 4 are confirmed here, i.e., when moving from low to relevant spatial correlation in the data, mse_{1i} overestimates the true MSE_i and produces conservative confidence intervals, whereas mse_{2i} shows a slight underestimation for low and medium values of $|\rho|$. The summary of Table 6 masks the variability of the results at area level. This can instead be seen in Figures 2 and 3, where we note that the behaviour of the estimators when spatial correlation is positive (Figure 2, $\rho = 0.75$) is replicated in the presence of negative spatial correlation (Figure 3, $\rho = -0.75$).

Table 6. Simulation study on real data: the average of the ratio of each MSE_i estimator ($soMSE_i$, mse_{1i}, mse_{2i}) on the empirical Mean Squared Error of SEBLUP estimator for $m = 53$, $\hat{\sigma}_u^2$, $\rho = \pm 0.25, \pm 0.5, \pm 0.75$

	Average Ratio		
Spatial Correlation	*soMSE*	mse_1	mse_2
0.75	1.041	1.131	1.013
0.5	1.055	1.169	0.982
0.25	1.057	1.197	0.977
−0.25	1.057	1.187	0.961
−0.5	1.046	1.178	0.983
−0.75	1.016	1.155	1.014

Regarding the properties of the second-order approximation of the true parameter, the results are in line with our expectations. We note that the approximation follows the empirical MSE_i and this is true for all the φ_i-patterns. The ratio suggests overestimation of MSE_i for positive ρ-values and underestimation of MSE_i for negative ρ-values. This "wave-like" behaviour is probably due to the heuristic nature of the solution proposed for the $g_{3i}\left(\sigma_u^2, \rho\right)$ term.

6. Final Remarks

In this article we have discussed to which extent the spatial effects in data used for SAE compromise the performance of the BLUP obtained under the area level

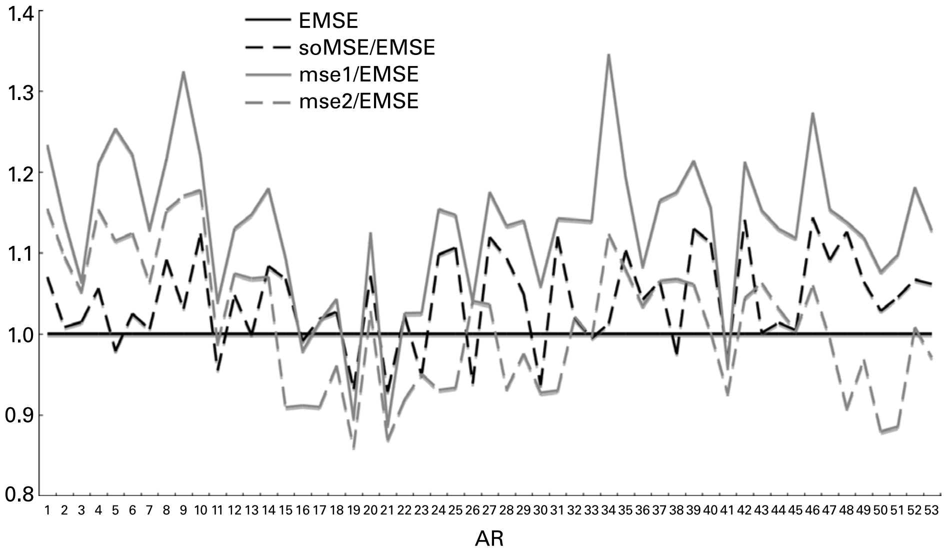

Fig. 2. Ratio of $soMSE_i$, mse_{1i} and mse_{2i} on Empirical MSE_i ($EMSE_i$) of SEBLUP for $m = 53$, $\hat{\sigma}_u^2$, $\rho = 0.75$ and $\hat{\varphi}_i$

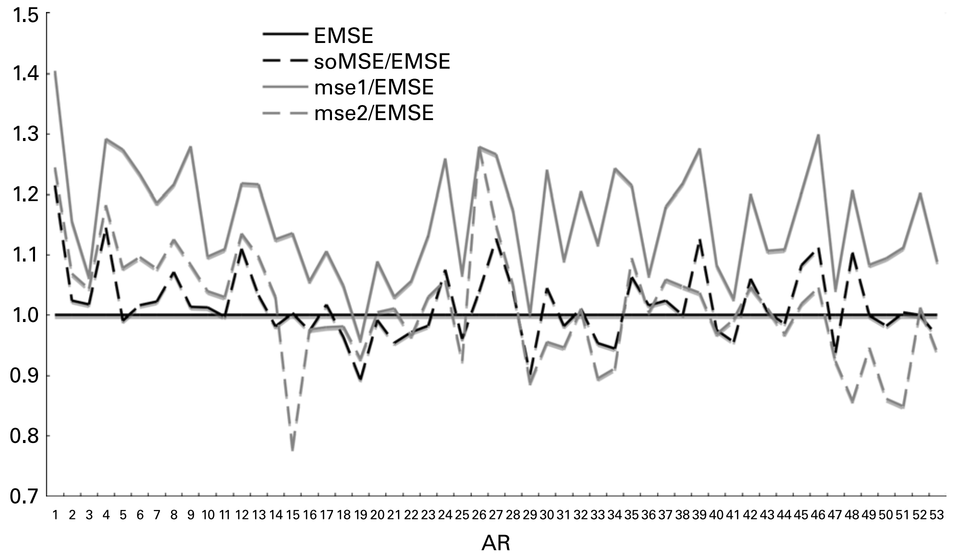

Fig. 3. Ratio of soMSE$_i$, mse$_{1i}$ and mse$_{2i}$ on Empirical MSE$_i$ (EMSE$_i$) of SEBLUP for $m = 53$, $\hat{\sigma}^2_u$, $\rho = -0.75$ and $\hat{\varphi}_i$

Fay–Herriot model. The performance of the BLUP was compared with that of the SBLUP via a simulation study in which the population was generated according to a spatial Fay–Herriot model and a wide range of values, ranging from -0.75 to 0.75, for the spatial correlation were used. In our definition spatial interaction is represented by a spatial interaction matrix **W** whose elements take on nonzero values only for those pairs of small areas, which are contiguous to each other (first-order contiguity). This scheme is common to many real-life situations in applications in the fields of geology, agriculture, and environmental science as well as in certain areas of health studies and epidemiology.

Our main finding is that the SBLUP outperforms the BLUP in terms of efficiency and relative bias in cases of both positive and negative spatial correlation, and this result does not depend on the entity of the sampling variances in different area groups. In other words, the SBLUP is appropriate when spatial dependency is present in the data used for SAE.

Obviously, in real-life situations the parameters of a spatial Fay–Herriot model are not known and must be estimated from survey data. In such a case, attention is devoted to Spatial Empirical BLUP (SEBLUP) and its *MSE*. We basically considered the performance of mse_{1i} and mse_{2i} estimators from Equations (3.3), (3.4) and (A.1.6) of Singh et al. (2005) and referred to the second-order approximation to the *MSE* due to Prasad and Rao (1990).

To summarize, our main findings are:

- in terms of the empirical MSE_i, the SEBLUP is more efficient than the EBLUP when spatial correlation is high ($|\rho| > 0.5$);
- the second-order approximation of the MSE_i due to Prasad and Rao (1990) seems to behave well in the estimation of the empirical MSE_i, i.e., for both the EBLUP and the SEBLUP. This is also obtained when the sampling variability is large;

- optimistic or conservative confidence intervals are respectively obtained by choosing the estimators mse_{2i} or mse_{1i}. These confidence intervals are influenced by the strength of the spatial correlation and by the values of the sampling variances.

Finally, we should remember that the MSE_i estimators used in this article are analytical approximations which usually rely on strong model assumptions and need large numbers of small areas to give good approximations to the true MSE_i values. Resampling techniques are attractive tools that can provide an alternative to the analytical solution (Molina et al. 2008).

Appendix A

The $MSE\left[\tilde{\vartheta}_i^S\left(\sigma_u^2,\rho\right)\right]$, depending on two parameters $\left(\sigma_u^2,\rho\right)$, can be expressed as:

$$MSE\left[\tilde{\vartheta}_i^S\left(\sigma_u^2,\rho\right)\right] = g_{1i}\left(\sigma_u^2,\rho\right) + g_{2i}\left(\sigma_u^2,\rho\right) \tag{A.1.1}$$

with

$$g_{1i}\left(\sigma_u^2,\rho\right) = \mathbf{b}_i^T\left[\mathbf{G} - \mathbf{G}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{Z}\mathbf{G}\right]\mathbf{b}_i \tag{A.1.2}$$

and

$$g_{2i}\left(\sigma_u^2,\rho\right) = \left[\mathbf{x}_i - \mathbf{b}_i^T\mathbf{G}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{X}\right]\left[\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X}\right]^{-1}\left[\mathbf{x}_i - \mathbf{b}_i^T\mathbf{G}\mathbf{Z}^T\mathbf{V}^{-1}\mathbf{X}\right]^T \tag{A.1.3}$$

For the Spatial EBLUP, given the normality of random effects, the $MSE\left[\tilde{\vartheta}_i^S(\hat{\sigma}_u^2,\hat{\rho})\right]$ is:

$$MSE\left[\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2,\hat{\rho}\right)\right] \approx g_{1i}\left(\sigma_u^2,\rho\right) + g_{2i}\left(\sigma_u^2,\rho\right) + g_{3i}\left(\sigma_u^2,\rho\right) \tag{A.1.4}$$

Following the results of Kackar and Harville (1984), Prasad and Rao (1990), and Datta and Lahiri (2000) and considering $\mathbf{C} = \left[\left(\mathbf{I} - \rho\mathbf{W}^T\right)\left(\mathbf{I} - \rho\mathbf{W}\right)\right]$ and $\mathbf{A} = \sigma_u^2\left[-\mathbf{C}^{-1}\left(2\rho\mathbf{W}^T\mathbf{W} - \mathbf{W} - \mathbf{W}^T\right)\mathbf{C}^{-1}\right]$, the $g_{3i}\left(\sigma_u^2,\rho\right)$ expression is:

$$g_{3i}\left(\sigma_u^2,\rho\right) = tr\left\{\begin{array}{l}\left[\begin{array}{l}\mathbf{b}_i^T\left(\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^1\left(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\right)\right)\\ \mathbf{b}_i^T\left(\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T\left(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1}\right)\right)\end{array}\right]\mathbf{V}\\ \left[\begin{array}{l}\mathbf{b}_i^T\left(\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T\left(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{C}^{-1}\mathbf{Z}^T\mathbf{V}^{-1}\right)\right)\\ \mathbf{b}_i^T\left(\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1} + \sigma_u^2\mathbf{C}^{-1}\mathbf{Z}^T\left(-\mathbf{V}^{-1}\mathbf{Z}\mathbf{A}\mathbf{Z}^T\mathbf{V}^{-1}\right)\right)\end{array}\right]\bar{\mathbf{V}}\left(\sigma_u^2,\rho\right)\end{array}\right\} \tag{A.1.5}$$

where $\bar{\mathbf{V}}\left(\sigma_u^2,\rho\right)$ is the asymptotic covariance matrix of the estimators $\hat{\sigma}_u^2$ and $\hat{\rho}$ (Singh et al. 2005; Pratesi and Salvati 2008).

For practical applications an estimator of $MSE\left[\tilde{\vartheta}_i^S\left(\hat{\sigma}_u^2,\hat{\rho}\right)\right]$ is shown in Expression (3.3), where $\left(\hat{\sigma}_u^2,\hat{\rho}\right)$ are REML estimators, and in Expression (3.4), if $\left(\hat{\sigma}_u^2,\hat{\rho}\right)$ are ML estimators. Singh et al. (2005) derived the estimator of the *MSE* for large m neglecting all terms of order $O(m^{-1})$. Their estimator differs from (3.3) and (3.4) for the subtraction of a term $g_{5i}\left(\hat{\sigma}_u^2, \hat{\rho}\right)$. This component is in matrix form:

$$g_5\left(\hat{\sigma}_u^2, \hat{\rho}\right) = \frac{1}{2} \underset{m}{tr}\left\{ \left[\mathbf{I}_2 \otimes \left(\mathbf{R}\mathbf{V}^{-1}\right)\right] \frac{\partial^2 \mathbf{V}}{\partial\left(\sigma_u^2, \rho\right) \partial\left(\sigma_u^2, \rho\right)^T} \left[I^{-1}\left(\sigma_u^2, \rho\right) \otimes \left(\mathbf{V}^{-1}\mathbf{R}\right)\right] \right\} \tag{A.1.6}$$

where $\mathbf{I}_2$ is a 2×2 identity matrix, $\partial^2 \mathbf{V}/\partial\left(\sigma_u^2, \rho\right)\partial\left(\sigma_u^2, \rho\right)^T$ is a partitioned matrix of order $2m \times 2m$, $I\left(\sigma_u^2, \rho\right)$ is the information matrix and $\otimes$ represents Kronecker product.

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