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Some Markov-Chain Models for Nonresponse in Estimating Gross Labor Force Flows

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Abstract: The longitudinal data bases available from panel surveys may be useful for estimating period-to-period gross flows among survey classifications. One problem in gross flow estimation is how to handle the nonresponse in the data. Typically, we do not believe that such nonresponse occurs at random with respect to the variable of interest. In this paper, I consider a model-based approach to the problem of handling nonresponse in data from a panel survey. I use a Markov-chain model for the underlying gross flow process and propose some Markov-chain models for the process that generates the possibly nonrandom nonresponse. The interacting Markov-chain models are fit using maximum likelihood estimation to data from the Canadian Labour Force Survey.

Key words: Longitudinal data; maximum likelihood estimation; nonrandom nonresponse; panel survey.

1. Introduction

Many large-scale sample surveys, such as the U.S. Current Population Survey and National Crime Survey, use rotating panel designs under which individuals are interviewed several times before rotating out of the sample. Typically, these large-scale surveys are used to produce point-in-time estimates. The rotating panel structure of the survey is a result of the need to reduce costs by keeping the same interviewers and subjects for more than a single interview. Recently, however, there has been increasing interest in using the longitudinal data bases available from such surveys to estimate gross change over time.

In this paper, I consider the problem of estimating period-to-period gross change over time using categorical data from a panel survey where, as one expects in a sample survey, there is nonresponse in the data so that some of the surveyed individuals are completely cross-classified while others are partially cross-classified or completely missing. I use a model-based approach to adjust for possibly nonrandom nonresponse (see, for example, Fay (1986) or Little (1982)). The use of a model-based approach rather than an ad hoc procedure, such as hot deck or raking, allows us to use standard statistical estimation techniques and standard results on the properties of estimators. In particular, I consider some Markov-chain models for the gross flow process along with Markov-chain models for the process

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generating the nonresponse. The models presented here are an improvement over previous models considered for estimating gross flows in the presence of nonrandom nonresponse (see for example Stasny (1983, 1986, and 1988) and Stasny and Fienberg (1985)) because they allow a sampled individual to be missing at one or both of two interview periods.

Section 2 of this paper presents a general two-stage model for panel data with nonresponse. Section 3 describes the Markov-chain models for nonresponse. In Section 4, I fit the models to employment data from the Canadian Labour Force Survey. Section 5 suggests possible extentions of the models.

2. A Two-Stage Model for Panel Data

One possible approach to the problem of estimating gross change over time using panel data is to use only the information from individuals who are respondents in both of two consecutive interview periods. In order to use this approach, we must assume that individuals who do not respond in one or both periods are a random sample of all individuals (Rubin (1976)). However, in most cases, we do not believe that nonresponse occurs at random. For example, using data from the Canadian Labour Force Survey, Paul and Lawes (1982) and Fienberg and Stasny (1983) give evidence that nonresponse is related to labor force classification. Since there is evidence that nonresponse does not typically occur at random, we would like to consider some models for estimating gross flows that allow us to treat nonresponse as related to the survey classifications.

Suppose that the result of each interview is the classification of the respondent into one of K non-overlapping categories. Consider estimating gross flows among these categories using records of surveyed individuals matched over two consecutive interview periods. It is impossible to obtain matches for individuals who were nonrespondents in one or both of the interview periods or who rotated into or out of the survey during the time being considered. Thus, as a result of the matching, we will have a group of individuals for whom we have survey classifications at both interview periods, a group of individuals for whom we have classifications in one but not both periods, and a group of individuals who did not respond to the survey in either period.

The survey classification data for individuals who responded at two consecutive interview times, say t-1 and t, can be summarized in a KxK matrix. The available information for individuals who were nonrespondents for the time t-1 interview but who responded to the time t interview may be summarized in a column supplement. The available information for individuals who were nonrespondents for the time t interview but who responded to the time t-1 interview may be summarized in a row supplement. Individuals who were nonrespondents at both times t-1 and t are counted in a single "missing" cell. Therefore, the observed time t-1 to time t gross flow data can be displayed as in Table 1.

Extending the ideas of Chen and Fienberg (1974) for maximum likelihood estimation in contingency tables with partially cross-classified data, I take the observed gross flow data to be the end result of a two-stage process where, in the unobserved first stage, individuals are allocated to the cells of a KxK matrix according to probabilities from a Markov chain. Let

π_i = initial probability that an individual is in state i at time t-1, where $\Sigma_i \pi_i = 1$, and

p_{ij} = transition probability from state i to state j, where $\Sigma_j p_{ij} = 1$ for all i.

At the second stage of the process each individual in the (i,j) cell of the gross flow matrix may either a) be a nonrespondent at time t-1

Table 1. Observed gross flow data

Time t-1	Time t 1	2	...	K	Row Supplement
1	x_{11}	x_{12}	...	x_{1K}	R_1
2	x_{21}	x_{22}	...	x_{2K}	R_2
.	.	.	.	.	.
.	.	.	.	.	.
K	x_{K1}	x_{K2}	...	x_{KK}	R_K
Column Supplement	C_1	C_2	...	C_K	M

Note: Where x_{ij} = number of sampled individuals with classification i at time t-1 and j at time t,
R_i = number of individuals who were nonrespondents at time t and had classification i at time t-1,
C_j = number of individuals who were nonrespondents at time t-1 and had classification j at time t, and
M = number of individuals who were nonrespondents at both times t-1 and t.

and, hence, lose the row classification, b) be a nonrespondent at time t and lose the column classification, or c) be a nonrespondent at both times and lose both the row and the column classifications. Let

$\xi(i,j)$ = initial probability that an individual in the (i,j) cell of the matrix responds at time t-1,

$\varrho_{RR}(i,j)$ = transition probability from respondent at time t-1 to respondent at time t, and

$\varrho_{MM}(i,j)$ = transition probability from nonrespondent at time t-1 to nonrespondent at time t.

The data are observed after this second stage. From the observed data, we want to make inferences about both the probabilities of the Markov chain generating the gross flow data and the probabilities of the Markov chain generating nonresponse. In the context of this two-stage model, the underlying probabilities for the observed gross flow matrix are as given in Table 2.

Table 2. Probabilities for observed gross flow data

Time t-1	Time t 1 2 ... K	Row Supplement
1, 2, . . . K	$\{\xi(i,j)\varrho_{RR}(i,j)\pi_i p_{ij}\}$	$\{\Sigma_j \xi(i,j)[1-\varrho_{RR}(i,j)]\pi_i p_{ij}\}$
Column Supplement	$\{\Sigma_i[1-\xi(i,j)][1-\varrho_{MM}(i,j)]\pi_i p_{ij}\}$	$\Sigma_i\Sigma_j[1-\xi(i,j)]\varrho_{MM}(i,j)\pi_i p_{ij}$

3. Markov-Chain Models for Nonresponse

3.1. The models

The likelihood function for the observed data under the two-stage model described in Section 2 is proportional to:

$$\begin{aligned} &\{\Pi_i\Pi_j[\xi(i,j)\varrho_{RR}(i,j)\pi_i p_{ij}]^{X_{ij}}\} \\ \times\ &\{\Pi_i[\Sigma_j\xi(i,j)[1-\varrho_{RR}(i,j)]\pi_i p_{ij}]^{R_i}\} \\ \times\ &\{\Pi_j[\Sigma_i[1-\xi(i,j)][1-\varrho_{MM}(i,j)]\pi_i p_{ij}]^{C_j}\} \\ \times\ &\{[\Sigma_i\Sigma_j[1-\xi(i,j)]\varrho_{MM}(i,j)\pi_i p_{ij}]^{M}\}, \end{aligned}$$

where i and j take on the values from 1 to K. We wish to use maximum likelihood estimation to obtain estimates of the parameters in this model. There are, however, $4K^2+K$ parameters with $K+1$ constraints on the parameters in the above likelihood function and only $(K+1)^2$ cells of observed counts with the single constraint that the observed counts sum to the total sample size. Thus, we must reduce the number of parameters to be estimated. We will do this by considering four models for the ξ and ϱ parameters, the parameters pertaining to nonresponse. The models are as follows:

Model A. $\xi(i,j)=\xi$, $\varrho_{RR}(i,j)=\varrho_{RR}$, $\varrho_{MM}(i,j)=\varrho_{MM}$.

Model B. $\xi(i,j)=\xi(i)$, $\varrho_{RR}(i,j)=\varrho_{RR}$, $\varrho_{MM}(i,j)=\varrho_{MM}$.

Model C. $\xi(i,j)=\xi$, $\varrho_{RR}(i,j)=\varrho_{RR}(i)$, $\varrho_{MM}(i,j)=\varrho_{MM}(i)$.

Model D. $\xi(i,j)=\xi$, $\varrho_{RR}(i,j)=\varrho_{RR}(j)$, $\varrho_{MM}(i,j)=\varrho_{MM}(j)$.

Models A and B have $2K$-2 and K-1 associated degrees of freedom respectively while there are no remaining degrees of freedom associated with Models C and D. The π and p probabilities for the gross flow process remain as defined in Section 2.

Under Model A, the initial probability that an individual responds at time t-1 is the same for all survey classifications. The transition probabilities from respondent to respondent or from nonrespondent to nonrespondent also do not depend on the survey classification. Under Model B, the initial probability that an individual responds at time t-1 depends on the individual's classification at time t-1 while the transition probabilities from respondent to respondent or from nonrespondent to nonrespondent do not depend on classification. Under Models C and D, the initial probability that an individual responds at time t-1 is the same for all survey classifications. The transition probabilities from respondent to respondent or from nonrespondent to nonrespondent under Model C depend on the survey classification at time t-1 while under Model D they depend on the classification at time t. Note that Model A is a special case of each of the other three models. The methods used to fit each of these models are described below.

3.2. Parameter estimates

The likelihood function for the observed data under Model A can be written as the product of two factors: one involving only the π and p parameters, the other involving the ξ and ϱ parameters. Thus, estimates of the π and p parameters can be found separately from the estimates of the ξ and ϱ parameters. In addition, this means that, under Model A, the Markov chains for the gross flow process and for nonresponse are two separate, independent Markov chains.

Under Model B, there is a single Markov chain for the gross flow process and there are three Markov chains for nonresponse. The Markov chains for nonresponse are tied to the Markov chain for the gross flow process through the dependence of ξ, the initial probability of being a respondent, on the survey classification at time t-1. The likelihood function for the observed data under Model B can be written as the product of two factors: one involving only the π, p, and ξ parameters, the other involving only the ϱ parameters. Thus, estimates of the π, p, and ξ parameters can be found separately from the estimates of the ϱ parameters.

Under Model C, there is a single Markov chain for the gross flow process and there are three Markov chains for nonresponse. The Markov chains for nonresponse are tied to the Markov chain for the gross flow process through the dependence of the ϱ parameters, the transition probabilities associated with the nonresponse process, on the survey classification at time t-1. The likelihood function for the observed data under Model C can be written as the product of two factors: one involving only the π, p, and ϱ_{MM} parameters, the other involving the ξ and ϱ_{RR} parameters. Thus, estimates of the π, p, and ϱ_{MM} parameters can be found separately from the estimates of the ξ and ϱ_{RR} parameters.

Under Model D, there is a single Markov chain for the gross flow process and there are three Markov chains for nonresponse. The Markov chains for nonresponse are tied to the Markov chain for the gross flow process through the dependence of the ϱ parameters, the transition probabilities associated with the nonresponse process, on the survey classification at time t. The likelihood function for the observed data under Model D can be written as the product of two factors: one involving only the π, p, and ϱ parameters, the other involving only the ξ parameter. Thus, estimates of the π, p, and ϱ parameters can be found separately from the estimate of the ξ parameter.

The parts of the likelihood functions under Models A, B, C, and D involving the π and p parameters are maximized using Lagrange multipliers to impose the constraints that $\Sigma_i\pi_i=1$ and $\Sigma_j p_{ij}=1$ for all i. In general, an iterative procedure must be used to provide estimates of some of the parameters under each model. The iterative steps for obtaining the parameter estimates under Models A, B, C, and D are given in Appendix I. In addition, at least one parameter under each model has a closed-form solution. These are also provided in Appendix I.

3.3. *Fitting the models*

For each of the models, the steps of the iterative procedures given in Appendix I are repeated for $\nu = 0, 1, 2, \ldots$ until the parameter estimates converge to the desired degree of accuracy. The formulas given for $\pi_i^{(0)}$, $p_{ij}^{(0)}$, $\xi(i)^{(0)}$, $\varrho_{RR}(i)^{(0)}$, and $\varrho_{MM}(i)^{(0)}$ are merely suggested initial estimates. Any values between 0 and 1 satisfying $\Sigma_i\pi_i=1$ and $\Sigma_j p_{ij}=1$ for all i may be used. In the data analysis reported in Section 4, a number of different starting values were used. In each case, the final estimates were the same.

After any one of the above models has been fit to the data, the cell probabilities underlying the observed data may be estimated following the formulas given in Table 2. These estimated probabilities are then multiplied by the total sample size to obtain the expected cell counts. Either the Pearson X^2 or the likelihood ratio statistic, G^2, can be compared to a χ^2 distribution with the appropriate degrees of freedom to help assess the fit of the model.

4. Example From the Canadian Labour Force Survey

4.1. *The Labour Force Survey*

The Canadian Labour Force Survey (LFS) is based on monthly interviews with respondents in approximately 56 000 households. Sampled households are retained in the sample for six months before being rotated out of the sample. Under this LFS scheme, the month-to-month overlap of sampled housing locations is 83 %. A detailed description of the LFS can be found in *Methodology of the Canadian Labour Force Survey 1976*, Statistics Canada (1977).

Month-to-month gross flows in labor force participation show how persons with each labor force classification in one month are classified in the following month providing, for example, estimates of the numbers of persons who were employed in one month and unemployed in the next, unemployed in one month and employed in the next, employed in both months, and so forth.

A single panel of micro-data from the LFS was available for my use. The data set contains responses for a subset of the survey questions for all individuals from the panel that rotated into the sample in August 1979 and remained in the sample through January 1980. Information is available for each individual in that panel who responded at least once during the six month period. Since the data is from a single LFS panel, there is no nonresponse due to rotation into or out of the sample included in this data set. Unweighted cell counts for a gross flow matrix can be obtained from this micro-data. The models described in this paper are suitable for unweighted data from a simple random sample. The LFS uses a multistage cluster sample. Thus, the models proposed in this paper are not ideally suited to describe the data. We will, however, fit the models to the data for illustrative purposes and as a first attempt at modeling the nonresponse in the data.

Persons interviewed for the LFS in a given month are classified as employed, unemployed, not in the labor force, or outside the population of interest. Naturally, persons outside the population of interest are not intentionally included in the LFS sample. The relatively few persons classified as outside the population of interest who do appear in the sample are included by accident rather than by design. Thus, the out-of-population cells based on the available panel of data are mostly empty and I do not include them in the analysis given here. In addition, I removed from the data nonrespondents who were classified as being outside the population of interest in a month when they did respond to the survey. The vast majority of such cases were children who were too young to be included in the survey and who should not, therefore, be counted as nonrespondents. I will consider estimating gross flows among the three labor force classifications using records of individuals matched over two consecutive months.

The observed gross flow data is given in Appendix II.

4.2. *The fits of the models*

All four models described in Section 3 were fit to the five possible month-to-month gross flow matrices constructed from the available panel of data. Since there are $K=3$ possible survey classifications, Model A has 4 associated degrees of freedom and Model B has 2 degrees of freedom. Models C and D, which have no associated degrees of freedom, will both fit the data exactly (within rounding error) although they need not produce the same parameter estimates.

The criterion for stopping the iterative procedures necessary for obtaining some of the parameter estimates was that the maximum difference between estimates at two consecutive steps was less than 0.0005. The iterative procedure for fitting Model A converged quickly, requiring only two steps to converge for each of the five observed gross flow matrices. The iterative procedure for fitting Model B converged relatively slowly requiring between 24 and 58 iterations. The iterative procedures for fitting Models C and D converged in between 16 and 25 iterations, and between 9 and 13 iterations respectively. The parameter estimates for all models and the X^2 and G^2 values for Models A and B are given in Appendix III.

The fits of Models A and B to the August to September data are similar. For the remaining four gross flow matrices, Model B provides a better fit to the data. (Note that, given the large cell counts in the observed gross flow matrices, we find the fits of Model B reasonable even though the X^2 and G^2 values are larger than the value of $\chi^2_{.99}(2)=9.21$.) Recall that under Model A the probabilities of nonresponse are the same for individuals in all employment classifications while under Model B the initial probability of being a nonrespondent in month t-1 depends on the employment classification in that month. Thus, since Model B provides a better fit than Model A, we have some evidence that nonresponse does depend on employment status.

In part B of Appendix III, we see that the estimated initial probabilities of falling in each labor force classification are similar under Models A, C, D, and, in August to September, under Model B. Under Model B in all other months, however, the estimated initial probability that a person is unemployed is higher than under other models. For example, in October to November, the percentage of persons initially unemployed is estimated to be about 3.8% under Models A, C, and D while it is about 5.0% under Model B.

Part C of Appendix III shows that the estimated transition probabilities among the various employment classifications do not vary greatly from model to model. Note, however, that the estimated transition probabilities do appear to change over time. These changes may be due to actual changes in the labor force over time, particularly the changes between the August to September and September to October estimates, but they may also be due in part to the effects of rotation group bias (see, for example, Bailar (1975 and 1979)).

Parts D, E, F, and G of Appendix III give the estimates of the ξ, ϱ_{RR}, and ϱ_{MM} parameters under Models A, B, C, and D respectively. As is shown in Appendix I, the estimate of ξ, the initial probability of being a respondent for any individual regardless of labor force status, are identical under Models A, C, and D. Under Model B, however, the initial probability of being a respondent depends on the labor force classification. Note that in all months, these initial probabilities of being a respondent are quite similar for persons who are employed or not in the labor force. In the matrices other than the August to September matrix, the initial probability of being a respondent is estimated to be about 0.8 for persons who are classified as employed or not in the labor force. The corresponding estimated values of ξ for unemployed persons are much lower, ranging from about 0.5 to 0.6. For the August to September data this difference in estimates of ξ (0.75 for persons classified as employed and not in the labor force compared to 0.71 for unemployeds) is not very large. This is expected since the fits of Models A and B are similar for the August to September data. This comparison of the estimates of initial probabilities of being a nonrespondent illustrates that response rates do appear to differ by labor force classification.

The estimates of ϱ_{RR} and ϱ_{MM}, the probabilities of transitions from respondent to respondent and nonrespondent to nonrespondent respectively, are identical under Models A and B and do not depend on labor force classification. They do depend on the classification under Models C and D. Note that the estimates of the $\varrho_{RR}(E)$, $\varrho_{RR}(N)$, $\varrho_{MM}(E)$, and $\varrho_{MM}(N)$ do not differ much from Model C to Model D although, except for $\hat{\varrho}_{RR}(N)$ in September to October, the $\hat{\varrho}_{RR}(E)$ and $\hat{\varrho}_{RR}(N)$ are larger under Model D than under Model C and the estimates of the $\varrho_{MM}(E)$ and $\varrho_{MM}(N)$ are always smaller under Model D than under Model C. The difference in these estimates range from about 0.001 to about 0.007 between probabilities that are estimated to be from about 0.71 to about 0.97. The differences between estimates of the $\varrho_{RR}(U)$ and $\varrho_{MM}(U)$ under Models C and D are somewhat larger, ranging from about 0.01 to about 0.1 on estimates that range from about 0.55 to about 0.95. The estimated probabilities of an unemployed person remaining a respondent, $\hat{\varrho}_{RR}(U)$, are always larger under Model C than under Model D while the estimated probabilities of an unemployed person remaining a nonrespondent, $\hat{\varrho}_{MM}(U)$, are always smaller under Model C than under Model D.

Thus we see that while both Models C and D provide exact fits to the data, they do not result in the same parameter estimates and, hence, they do not give the same estimated expected cell counts for the 3x3 gross flow matrices. This can be seen in Part A of Appendix III. Note, however, that the estimated expected cell counts after the first stage do not differ much under any of the Models A, C, and D in any of the matrices. The Model B estimates are somewhat different from the estimates under Models A, C, and D except in August to September. In particular, note that the expected cell counts in the row for persons classified as unemployed in month t-1 are larger under Model B.

From this analysis, it is not completely clear which model we would prefer for modeling nonresponse in this labor force data. Because Models C and D fit the data exactly, we cannot choose between those two models based on the fits for the models. Additional information is needed in order to determine which model is more appropriate. Certainly, however, we prefer Model B to Model A except in August to September. Since nonresponse is related to labor force classification under Model B, we have evidence that nonresponse does not occur at random. In addition, the parameter estimates under Models B, C, and D provide evidence that unemployed persons are more likely to be nonrespondents than are persons who are employed or not in the labor force. Since response rates appear to be fairly similar for persons who are employed or not in the labor force, it may be worthwhile to consider variations of the models fit above where the probabilities associated with those classifications are the same. In general, the results described above suggest that procedures to adjust for nonresponse which do not allow for differential nonresponse rates may not be appropriate.

5. Extensions of the Models

An advantage of the Markov-chain models for nonresponse proposed here is that there is a natural way to think about extending the models to allow us to use more than two periods of data in estimating gross flows. This is not the case for the discrete-time models described by Stasny and Fienberg (1985) and by Stasny (1983, 1986, and 1988). Therefore, an important generalization of this work will be to extend the models to handle gross flows over more than two periods.

The generalization to more than two periods is not trivial, however, because we are not willing to make many simplifying assumptions. We do not believe that the employment process is stationary over time. In addition, we do not believe that nonresponse probabilities are unchanging over the life of a panel. Without any simplifying assumptions, the model for more than two periods quickly becomes difficult to manage. A possible solution may be to consider the higher-order Markov-chain models of Raftery (1985) which add only a single parameter to model each extra period. Work on this problem is still in progress.

A somewhat simpler extension of the model would be to allow for different types of nonresponse. For example, nonresponse due to panel rotation is designed nonresponse and we may be willing to assume that it occurs at random. Although the nature of the available LFS data for this example did not allow us to consider different types of nonresponse, in general this would be an issue for panel surveys.

Another extension of the model would be to allow the Markov-chain model for the gross flow process to be a continuous-time Markov chain. After the π_i and p_{ij} have been estimated under one of the above models using one of the procedures described in the previous sections, the estimates may then be used to estimate the intensity matrix for a continuous-time Markov chain for the gross flow process. Descriptions of estimating continuous-time Markov chains from data collected at discrete intervals are given by Singer and Spilerman (1976) and Stasny (1983).

Appendix I

Equations for Parameter Estimation

The following may be used as initial estimates when needed for the iterative procedures given in this appendix. The dot notation in a subscript indicates summation over that subscript. Note that these initial estimates correspond to the closed-form estimates of the parameters appropriate under some of the models.

$$\pi_i^{(0)} = x_{i\cdot}/x_{\cdot\cdot}$$

$$p_{ij}^{(0)} = x_{ij}/x_{i\cdot}$$

$$\xi(i)^{(0)} = [x_{\cdot\cdot} + R_\cdot]/[x_{\cdot\cdot} + R_\cdot + C_\cdot + M]$$

$$\varrho_{RR}(i)^{(0)} = x_{\cdot\cdot}/[x_{\cdot\cdot} + R_\cdot]$$

$$\varrho_{MM}(i)^{(0)} = M/[C_\cdot + M]$$

Model A

Closed-form estimators:

$$\hat{\xi} = [x_{\cdot\cdot} + R_\cdot]/[x_{\cdot\cdot} + R_\cdot + C_\cdot + M]$$

$$\hat{\varrho}_{RR} = x_{\cdot\cdot}/[x_{\cdot\cdot} + R_\cdot]$$

$$\hat{\varrho}_{MM} = M/[C_\cdot + M]$$

Iterative procedure:

$$\pi_i^{(v+1)} = \{x_{i\cdot} + R_i + \Sigma_j[C_j\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k\pi_k^{(v)}p_{kj}^{(v)}]\} \times \{x_{\cdot\cdot} + R_\cdot + C_\cdot\}^{-1}$$

$$p_{ij}^{(v+1)} = \{x_{ij} + [C_j\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k\pi_k^{(v)}p_{kj}^{(v)}]\} \times \{x_{i\cdot} + \Sigma_j[C_j\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k\pi_k^{(v)}p_{kj}^{(v)}]\}^{-1}$$

Model B

Closed-form estimators:

$$\hat{\varrho}_{RR} = x_{\cdot\cdot}/[x_{\cdot\cdot} + R_\cdot]$$

$$\hat{\varrho}_{MM} = M/[C_\cdot + M]$$

Iterative procedure:

$$\pi_i^{(v+1)} = \{x_{i\cdot} + R_i + \Sigma_j[C_j[1-\xi(i)^{(v)}]\pi_i^{(v)}p_{ij}^{(v)}/ \Sigma_k[1-\xi(k)^{(v)}]\pi_k^{(v)}p_{kj}^{(v)}] + [M[1-\xi(i)^{(v)}]\pi_i^{(v)}/\Sigma_k[1-\xi(k)^{(v)}]\pi_k^{(v)}]\} \times \{x_{\cdot\cdot} + R. + C. + M\}^{-1}$$

$$p_{ij}^{(v+1)} = \{x_{ij} + [C_j[1-\xi(i)^{(v)}]\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k[1-\xi(k)^{(v)}]\pi_k^{(v)}p_{kj}^{(v)}]\} \times \{x_{i\cdot} + \Sigma_j[C_j[1-\xi(i)^{(v)}]\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k[1-\xi(k)^{(v)}]\pi_k^{(v)}p_{kj}^{(v)}]\}^{-1}$$

$$\xi(i)^{(v+1)} = \{x_{i\cdot} + R_i\} \times \{x_{i\cdot} + R_i + \Sigma_j[C_j[1-\xi(i)^{(v)}]\pi_i^{(v)}p_{ij}^{(v)}/ \Sigma_k[1-\xi(k)^{(v)}]\pi_k^{(v)}p_{kj}^{(v)}] + [M[1-\xi(i)^{(v)}]\pi_i^{(v)}/\Sigma_k[1-\xi(k)^{(v)}]\pi_k^{(v)}]\}^{-1}$$

Model C

Closed-form estimators:

$$\hat{\xi} = [x_{\cdot\cdot} + R.]/[x_{\cdot\cdot} + R. + C. + M]$$

$$\hat{\varrho}_{RR}(i) = x_{i\cdot}/(x_{i\cdot} + R_i)$$

Iterative procedure:

$$\pi_i^{(v+1)} = \{x_{i\cdot} + R_i + \Sigma_j[C_j[1-\varrho_{MM}(i)^{(v)}]\pi_i^{(v)}\, p_{ij}^{(v)}/\Sigma_k[1-\varrho_{MM}(k)^{(v)}]\, \pi_k^{(v)}p_{kj}^{(v)}] + [M\varrho_{MM}(i)^{(v)}\pi_i^{(v)}/\Sigma_k[\varrho_{MM}(k)^{(v)}]\pi_k^{(v)}]\} \times \{x_{\cdot\cdot} + R. + C. + M\}^{-1}$$

$$p_{ij}^{(v+1)} = \{x_{ij} + [C_j[1-\varrho_{MM}(i)^{(v)}]\pi_i^{(v)}p_{ij}^{(v)}/ \Sigma_k[1-\varrho_{MM}(k)^{(v)}]\pi_k^{(v)}p_{kj}^{(v)}]\} \times \{x_{i\cdot} + \Sigma_j[C_j[1-\varrho_{MM}(i)^{(v)}]\pi_i^{(v)}p_{ij}^{(v)}/ \Sigma_k[1-\varrho_{MM}(k)^{(v)}]\pi_k^{(v)}p_{kj}^{(v)}]\}^{-1}$$

$$\varrho_{MM}(i)^{(v+1)} = \{\Sigma_j[C_j[1-\varrho_{MM}(i)^{(v)}]\pi_i^{(v)}p_{ij}{}^{(v)}/ \Sigma_k[1-\varrho_{MM}(k)^{(v)}]\pi_k^{(v)}p_{kj}^{(v)}] + M\varrho_{MM}(i)^{(v)}\pi_i^{(v)}/\Sigma_k[\varrho_{MM}(k)^{(v)}\pi_k^{(v)}]\}^{-1} \times \{M\varrho_{MM}(i)^{(v)}\pi_i^{(v)}/\Sigma_k[\varrho_{MM}(k)^{(v)}\pi_k^{(v)}]\}$$

Model D

Closed-form estimators:

$$\hat{\xi} = [x_{\cdot\cdot} + R.]/[x_{\cdot\cdot} + R. + C. + M]$$

Iterative procedure:

$$\pi_i^{(v+1)} = \{x_{i\cdot} + R_i + \Sigma_j[C_j\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k\pi_k^{(v)}p_{kj}^{(v)}] + \Sigma_j[M\varrho_{MM}(j)^{(v)}\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k\Sigma_h[\varrho_{MM}(h)^{(v)}]\pi_k^{(v)}p_{kh}^{(v)}]\} \times \{x_{\cdot\cdot} + R_{\cdot} + C_{\cdot} + M\}^{-1}$$

$$p_{ij}^{(v+1)} = \{x_{ij} + [R_i[1-\varrho_{RR}(j)^{(v)}]p_{ij}/\Sigma_k[1-\varrho_{RR}(k)^{(v)}]p_{ik}] + [C_j\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k\pi_k^{(v)}p_{kj}^{(v)}] + [M\varrho_{MM}(j)^{(v)}\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k\Sigma_h[\varrho_{MM}(h)^{(v)}\pi_k^{(v)}p_{kh}^{(v)}]\} \times \{x_{\cdot j} + R_i + \Sigma_j[C_j\pi(i)^{(v)}p_{ij}^{(v)}/\Sigma_k\pi(k)^{(v)}]p_{kj}^{(v)}] + [M\Sigma_j\varrho_{MM}(j)^{(v)}\pi_i^{(v)}p_{ij}^{(v)}/\Sigma_k\Sigma_h[\varrho_{MM}(h)^{(v)}]\pi_k^{(v)}p_{kh}^{(v)}]\}^{-1}$$

$$\varrho_{RR}(j)^{(v+1)} = x_{\cdot j} \times \{x_{\cdot j} + \Sigma_i[R_i[1-\varrho_{RR}(j)^{(v)}]\, p_{ij}^{(v)}/\Sigma_k[1-\varrho_{RR}(k)^{(v)}]p_{ik}^{(v)}]\}^{-1}$$

$$\varrho_{MM}(j)^{(v+1)} = 1 - \{C_j\Sigma_k\Sigma_h\varrho_{MM}(h)^{(v)}\pi_k^{(v)}p_{kh}^{(v)}/M\Sigma_k\pi_k^{(v)}p_{kj}^{(v)}\}$$

Appendix II

Observed LFS Data

Sept. 1979

Aug. 1979	*E*	*U*	*N*	Row Supplement
E	9 222	128	662	473
U	221	322	151	59
N	256	164	5 941	292
Column Supplement	996	69	676	4 353

Oct. 1979

Sept. 1979	*E*	*U*	*N*	Row Supplement
E	9 697	169	355	474
U	177	317	143	46
N	326	159	6 522	423
Column Supplement	554	59	339	4 225

Nov. 1979

Oct. 1979	*E*	*U*	*N*	Row Supplement
E	9 778	178	392	406
U	159	362	130	53
N	212	145	6 728	274
Column Supplement	419	54	269	4 426

Dec. 1979

Nov. 1979	*E*	*U*	*N*	Row Supplement
E	9 683	202	304	379
U	129	405	157	48
N	204	155	6 928	232
Column Supplement	291	60	218	4 590

Jan. 1980

Dec. 1979	*E*	*U*	*N*	Row Supplement
E	9 411	191	366	339
U	162	450	168	42
N	187	180	7 004	236
Column Supplement	252	50	186	4 761

Appendix III

Parameter Estimates

A. Estimates of the expected cell counts after the first stage

	Model A			Model B			Model C			Model D		
	E	*U*	*N*	*E*	*U*	*N*	*E*	*U*	*N*	*E*	*U*	*N*
Aug.–Sept. 1979												
E	12 955	181	928	12 955	179	930	12 947	180	929	12 937	194	926
U	321	472	219	339	492	231	321	469	220	310	488	211
N	359	232	8 317	357	228	8 274	359	230	8 330	359	248	8 312
Sept.–Oct. 1979												
E	12 955	233	471	12 927	225	473	12 939	226	474	12 924	235	480
U	242	449	194	315	565	253	242	434	196	236	441	194
N	441	223	8 776	430	210	8 587	441	215	8 819	434	221	8 820
Oct.–Nov. 1979												
E	12 956	244	517	12 900	235	517	12 952	236	519	12 937	253	518
U	219	516	178	295	673	241	219	499	179	210	515	172
N	281	199	8 875	273	187	8 664	281	192	8 907	280	206	8 894
Nov.–Dec. 1979												
E	12 767	279	401	12 497	261	392	12 795	267	402	12 784	281	399
U	175	576	213	276	871	336	176	552	214	170	565	206
N	268	213	9 093	262	199	8 889	268	204	9 107	269	216	9 094
Dec. 1979–Jan. 1980												
E	12 444	261	483	12 249	249	476	12 457	253	484	12 450	261	483
U	218	627	226	321	896	333	219	607	227	214	616	222
N	247	246	9 234	240	231	8 989	247	238	9 253	247	246	9 245

B. Estimates of the initial probabilities of being in each employment classification

	Model A	Model B	Model C	Model D
Aug.–Sept. 1979				
$\hat{\pi}_E$	.5864	.5864	.5860	.5860
$\hat{\pi}_U$	.0422	.0443	.0421	.0421
$\hat{\pi}_N$	.3714	.3693	.3719	.3719
Sept.–Oct. 1979				
$\hat{\pi}_E$	.5695	.5681	.5686	.5686
$\hat{\pi}_U$	.0369	.0473	.0363	.0363
$\hat{\pi}_N$	.3936	.3847	.3950	.3950
Oct.–Nov. 1979				
$\hat{\pi}_E$	.5719	.5692	.5715	.5715
$\hat{\pi}_U$	.0381	.0504	.0374	.0374
$\hat{\pi}_N$	.3900	.3804	.3911	.3911
Nov.–Dec. 1979				
$\hat{\pi}_E$	.5607	.5483	.5613	.5614
$\hat{\pi}_U$	.0402	.0619	.0393	.0393
$\hat{\pi}_N$	.3991	.3898	.3994	.3994
Dec. 1979–Jan. 1980				
$\hat{\pi}_E$	.5499	.5409	.5501	.5501
$\hat{\pi}_U$	.0446	.0646	.0439	.0439
$\hat{\pi}_N$	.4055	.3944	.4060	.4060

C. Estimates of the transition probabilities, p_{ij}

	Model A			Model B			Model C			Model D		
	E	*U*	*N*	*E*	*U*	*N*	*E*	*U*	*N*	*E*	*U*	*N*
Aug.–Sept. 1979												
E	.9211	.0129	.0660	.9211	.0128	.0661	.9211	.0128	.0661	.9203	.0138	.0659
U	.3175	.4660	.2164	.3188	.4634	.2178	.3184	.4641	.2175	.3073	.4833	.2094
N	.0403	.0260	.9337	.0402	.0257	.9340	.0402	.0258	.9340	.0403	.0278	.9319
Sept.–Oct. 1979												
E	.9484	.0171	.0345	.9488	.0165	.0347	.9487	.0165	.0347	.9476	.0172	.0352
U	.2735	.5069	.2196	.2779	.4985	.2236	.2778	.4978	.2244	.2710	.5067	.2222
N	.0468	.0236	.9296	.0466	.0227	.9307	.0465	.0227	.9308	.0459	.0233	.9308
Oct.–Nov. 1979												
E	.9445	.0178	.0377	.9449	.0172	.0379	.9449	.0172	.0379	.9438	.0184	.0378
U	.2397	.5652	.1951	.2439	.5569	.1992	.2442	.5562	.1996	.2345	.5739	.1916
N	.0300	.0212	.9487	.0299	.0205	.9496	.0299	.0205	.9496	.0299	.0220	.9481
Nov.–Dec. 1979												
E	.9494	.0207	.0298	.9503	.0198	.0298	.9503	.0198	.0298	.9495	.0209	.0296
U	.1815	.5975	.2210	.1863	.5872	.2266	.1866	.5862	.2272	.1810	.6000	.2190
N	.0280	.0222	.9498	.0280	.0213	.9507	.0280	.0213	.9507	.0281	.0225	.9494
Dec. 1979–Jan. 1980												
E	.9435	.0198	.0367	.9441	.0192	.0367	.9441	.0192	.0367	.9436	.0198	.0366
U	.2037	.5853	.2110	.2073	.5779	.2148	.2077	.5770	.2153	.2038	.5853	.2109
N	.0254	.0252	.9494	.0254	.0244	.9502	.0254	.0244	.9502	.0254	.0253	.9493

D. ξ, ϱ_{RR}, *and* ϱ_{MM} *parameter estimates under Model A*

	$\hat{\xi}$	$\hat{\varrho}_{RR}$	$\hat{\varrho}_{MM}$	X^2	G^2
Aug.–Sept. 1979	.7459	.9539	.7143	19	16
Sept.–Oct. 1979	.7842	.9499	.8161	39	36
Oct.–Nov. 1979	.7845	.9610	.8564	49	40
Nov.–Dec. 1979	.7849	.9650	.8897	74	58
Dec. 1979–Jan. 1980	.7812	.9671	.9070	43	34

Note: $\chi^2_{.99}(4) = 13.28$.

E. ξ, ϱ_{RR}, *and* ϱ_{MM} *parameter estimates under Model B*

	$\hat{\xi}(E)$	$\hat{\xi}(U)$	$\hat{\xi}(N)$	$\hat{\varrho}_{RR}$	$\hat{\varrho}_{MM}$	X^2	G^2
Aug.–Sept. 1979	.7455	.7090	.7510	.9539	.7143	19	16
Sept.–Oct. 1979	.7850	.6027	.8053	.9499	.8161	20	19
Oct.–Nov. 1979	.7877	.5827	.8065	.9610	.8564	26	21
Nov.–Dec. 1979	.8036	.4979	.8041	.9650	.8897	24	20
Dec. 1979–Jan. 1980	.7944	.5302	.8041	.9671	.9070	9	9

Note: $\chi^2_{.99}(2) = 9.21$.

F. ξ, ϱ_{RR}, *and* ϱ_{MM} *parameter estimates under Model C*

	$\hat{\xi}$	$\hat{\varrho}_{RR}(E)$	$\hat{\varrho}_{RR}(U)$	$\hat{\varrho}_{RR}(N)$	$\hat{\varrho}_{MM}(E)$	$\hat{\varrho}_{MM}(U)$	$\hat{\varrho}_{MM}(N)$
Aug.–Sept. 1979	.7459	.9549	.9216	.9561	.7131	.6697	.7212
Sept.–Oct. 1979	.7842	.9557	.9327	.9431	.8155	.5462	.8418
Oct.–Nov. 1979	.7845	.9622	.9247	.9628	.8591	.6137	.8758
Nov.–Dec. 1979	.7849	.9641	.9350	.9691	.9021	.5790	.9029
Dec. 1979–Jan. 1980	.7812	.9671	.9489	.9690	.9145	.6915	.9202

G. ξ, ϱ_{RR}, *and* ϱ_{MM} *parameter estimates under Model D*

	$\hat{\xi}$	$\hat{\varrho}_{RR}(E)$	$\hat{\varrho}_{RR}(U)$	$\hat{\varrho}_{RR}(N)$	$\hat{\varrho}_{MM}(E)$	$\hat{\varrho}_{MM}(U)$	$\hat{\varrho}_{MM}(N)$
Aug.–Sept. 1979	.7459	.9557	.8852	.9582	.7119	.7077	.7184
Sept.–Oct. 1979	.7842	.9568	.9165	.9430	.8112	.6952	.8346
Oct.–Nov. 1979	.7845	.9634	.8965	.9643	.8552	.7425	.8697
Nov.–Dec. 1979	.7849	.9650	.9138	.9706	.8977	.7372	.8955
Dec. 1979–Jan. 1980	.7812	.9676	.9356	.9699	.9108	.7965	.9146

6. References

Bailar, B.A. (1975): The Effects of Rotation Group Bias on Estimates for Panel Surveys. Journal of the American Statistical Association, 70, pp. 23–30.

Bailar, B.A. (1979): Rotation Sample Biases and Their Effects on Estimates of Change. Proceedings of the Conference on Recent Developments in Statistical Methods and Applications, pp. 13–29.

Chen, T. and Fienberg, S.E. (1974): Two-Dimensional Contingency Tables With Both Completely and Partially Cross-Classified Data. Biometrics, 30, pp. 629–642.

Fay, R.E. (1986): Causal Models for Patterns of Nonresponse. Journal of the American Statistical Association, 81, pp. 354–365.

Fienberg, S.E. and Stasny, E.A. (1983): Estimating Monthly Gross Flows in Labour Force Participation. Survey Methodology, 9, pp. 77–102.

Little, R.J.A. (1982): Models for Nonresponse in Sample Surveys. Journal of the American Statistical Association, 77, pp. 237–250.

Paul, E.C. and Lawes, M. (1982): Characteristics of Respondent and Nonrespondent Households in the Canadian Labour Force Survey. Survey Methodology, 8, pp. 48–85.

Raftery, A.E. (1985). A Model for High-order Markov Chains. Journal of the Royal Statistical Society, Series B, 47, pp. 528–539.

Rubin, D.B. (1976): Inference and Missing Data. Biometrika, 63, pp. 581–592.

Singer, B. and Spilerman, S. (1976): Some Methodological Issues in the Analysis of Longitudinal Surveys. Annals of Economic and Social Measurement, 5, pp. 447–474.

Stasny, E.A. (1983): Estimating Gross Flows in Labor Force Participation Using Data From the Canadian Labour Force Survey. Unpublished Ph.D. dissertation, Department of Statistics, Carnegie-Mellon University.

Stasny, E.A. (1986): Estimating Gross Flows Using Panel Data With Nonresponse: An Example From the Canadian Labour Force Survey. Journal of the American Statistical Association, 81, pp. 42–47.

Stasny, E.A. (1988): Modeling Non-ignorable Nonresponse in Categorical Panel Data: An Example in Estimating Gross Labor-Force Flows. Journal of Business and Economic Statistics, in press.

Stasny, E.A. and Fienberg, S.E. (1985): Some Stochastic Models for Estimating Gross Flows in the Presence of Nonrandom Nonresponse. Proceedings of the Conference on Gross Flows in Labor Force Statistics, U.S. Department of Commerce and U.S. Department of Labor, pp. 25–39.

Statistics Canada (1977): Methodology of the Canadian Labour Force Survey 1976, Ottawa, Canada.

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