Some Recent Developments and Directions in Seasonal Adjustment

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This article describes recent developments in software, diagnostics and research for seasonal adjustment. These include: software merging X-12-ARIMA and SEATS; improved diagnostics for SEATS derived from finite rather than infinite filters; a diagnostic showing the extent to which the trend delays the current business cycle information more than the seasonal adjustment; four new kinds of models for seasonal adjustment, including frequency-specific Airline models; and, finally, some methodological developments proposed for X-11 method seasonal adjustments. (*JEL* C87, C82, C84).

Key words: Time series; X-12-ARIMA; SEATS; TSW; RunX12; trends; moving holiday effects; sliding spans; spectrum; phase delay; RegComponent models; sampling error; seasonal heteroscedasticity; uncertainty measures.

1. Introduction

This article provides an overview of some recent areas of development in seasonal adjustment. The emphasis is on areas connected to current and planned versions of SEATS and X-12-ARIMA for the combination program being developed at the U.S. Census Bureau with the support of the Bank of Spain (see Monsell, Aston, and Koopman 2003). This program, tentatively named X-13A-S, offers the user the seasonal adjustment methods of both programs with improved and expanded diagnostics for the model-based seasonal adjustments. For this program, Sections 2–4 cover enhancements to regARIMA modeling capabilities, X-12-ARIMA diagnostics for SEATS adjustments, and improved and new diagnostics for SEATS made available by the signal extraction matrix formulas of Bell and Hillmer (1988) and McElroy and Sutcliffe (2005). Section 5 summarizes the features of four new classes of models for seasonal adjustment or trend estimation presented in Bell (2004), Aston, Findley, Wills, and Martin (2004), Proietti (2004) and Wildi (2004), which might influence the future evolution of X-13A-S or other software. Section 6 briefly summarizes recent methodological developments directed toward enhancing the X-11 seasonal adjustment methodology. Some important topics, such as recent research on methods for trend estimation, receive little mention or none because of limitations of the author's experience and expertise.

1.1.   Utility software and documents

Before beginning the discussion of more technical developments in Section 2, we call attention to some rather recent software and documents that will be useful to a broad range of users of the current software as well as users of future versions. We start with software. There is now a fully menu-driven version of TRAMO-SEATS with somewhat reduced capabilities, the TSW program (Caporello and Maravall 2004) distributed by the Bank of Spain, which is completely integrated into the Microsoft Windows operating system environment. For X-12-ARIMA and X-13A-S, the RunX12 program of Feldpausch (2003) distributed by the U.S. Census Bureau is a Windows interface program that enables the user to edit or run existing spc (command) files and metafiles and invoke a (user-specifiable) text editor to display the output files.

   For the user who needs to model complex holiday effects for which the regressors provided by X-12-ARIMA are inadequate, there is an auxiliary program named GenHol (Monsell 2001). This program generates regressor matrices (and the spc file commands needed to use them) for holiday effects that are different in (up to three) separate time intervals surrounding the holiday, as happens, for example, when there is an Easter holiday effect on Easter Sunday and Monday in addition to (and perhaps different from) the holiday's effect in an interval preceding the holiday. The general holiday effect model, presented in Findley and Soukup (2000), follows the simplest model of Bell and Hillmer (1983) in treating the effect of the holiday within an interval as constant. Lin and Liu (2003) describe the steps of the development and successful application within X-12-ARIMA of regressors from GenHol to the modeling and adjustment of three Chinese lunar holidays for a variety of Taiwanese time series. The regressor produced by GenHol for an interval will need to be modified if the daily effect of the holiday in an interval is linearly increasing (or decreasing) or quadratic rather than constant (see Zhang, McLaren, and Leung 2003 for examples).

   On the topic of documentation to help users of the seasonal adjustment programs, Maravall and Sánchez (2000) and Maravall (2005) give examples showing how to use the diagnostics of TRAMO-SEATS to make decisions concerning adjustment options. Findley and Hood (2000) do the same for X-12-ARIMA, and some of their analyses are applicable to SEATS adjustment decisions based on diagnostics that are made available by X-13A-S (see Section 3 below). Finally, the book by Ladiray and Quenneville (2001), French and Spanish versions of which are available from www.census.gov/ts/papers/, provides a very clear, complete, and elaborately illustrated presentation of the X-11 seasonal adjustment methodology of X-12-ARIMA and X-13A-S and of those of its diagnostics that were inherited from the X-11-ARIMA program (Dagum 1980), the foundation on which X-12-ARIMA was built.

2.   RegARIMA Modeling Capabilities of X-13A-S

X-13A-S is an enhanced version of X-12-ARIMA containing the latest version of SEATS and having time series modeling capabilities of TRAMO that were not available in version 2.10 of X-12-ARIMA (see Monsell, Aston, and Koopman 2003). These new capabilities include an automatic regARIMA model selection procedure patterned very closely after the procedure of TRAMO (Gómez and Maravall 2000, with additional details made

available to us by Victor Gómez and summarized in Monsell 2002). There are also additional regressors, such as the seasonal outlier regressors of TRAMO (see Kaiser and Maravall 2003 and Bell 1983). Only the general intervention regressor of TRAMO might not be fully implemented at the time of the release of X-13A-S to statistical offices and central banks for testing and evaluation in late 2005.

In the automatic model identification procedures of X-13A-S patterned after TRAMO's, there are certain differences from TRAMO's procedures in the method of estimation of ARMA parameters, in the criterion for use of the log tranformation, in the thresholds used to include outlier regressors, in the default models for trading day and Easter holiday effects, and in the criteria that determine when trading day, holiday, or user-defined regressors are included in the model. As a result, X-13A-S and TRAMO make identical regARIMA model choices only about one-fourth of the time. In the Appendix, we give more details about these differences and about studies that have been done to compare the automatic modeling procedure of X-13A-S with TRAMO's. For transformation and regressor choice, the studies suggest that the procedure of X-13-ARIMA is modestly better (for U.S. series, at least). Some recent changes were made in the procedure for the selection of the differencing orders and a study will soon be done to determine if it now generally chooses the same differencings as TRAMO's procedure.

3.   Diagnostics of X-13A-S Inherited from X-12-ARIMA

The spectrum is a fundamental diagnostic for detecting the need for, or inadequacy of, seasonal and trading day adjustments, because these effects are basically periodic with known periods. The spectrum can also guide regARIMA modeling decisions in various ways, as we will illustrate. X-13A-S produces log spectral plots of the (usually differenced and log-transformed) original and seasonally adjusted series and of the irregular component (see Findley, Monsell, Bell, Otto, and Chen 1998 and Soukup and Findley 1999; 2001 for details concerning the spectrum estimator and trading day frequencies).

Figure 1 is the spectral plot of the SEATS irregular component of monthly U.S. Exports of Other Agricultural Material (Nonmanufactured) showing a peak at the highest seasonal frequency, six cycles per year, associated with period two months. This suggests that SEATS' seasonal adjustment has not adequately removed all seasonal effects. Further support for this interpretation will be obtained in Subsection 4.2 from a modification of a SEATS diagnostic. For SEATS's adjustments, residual seasonality results from an inadequate model, which in our experience often arises from the use of too long a data span for the estimation of model parameters when the properties of the data are changing.

Figure 2 is a spectral plot of the (differenced and log transformed) original series from January, 1992 through September, 2001 of U.S. Shipments of Defense Communications Equipment (Shipments for short) from the U.S. Census Bureau's monthly Manufacturers' Shipments, Inventories, and Orders Survey. There are small-to-moderate peaks at four of the seasonal frequencies and a large seasonal peak at the frequency associated with quarterly movements. These peaks confirm that the series is seasonal but also suggest that its seasonality, being different in character at the quarterly seasonal frequency, is of a type that cannot be well modeled by standard seasonal ARIMA models. A new type of model, called a frequency-specific Airline model, is introduced in Section 5, where its canonical
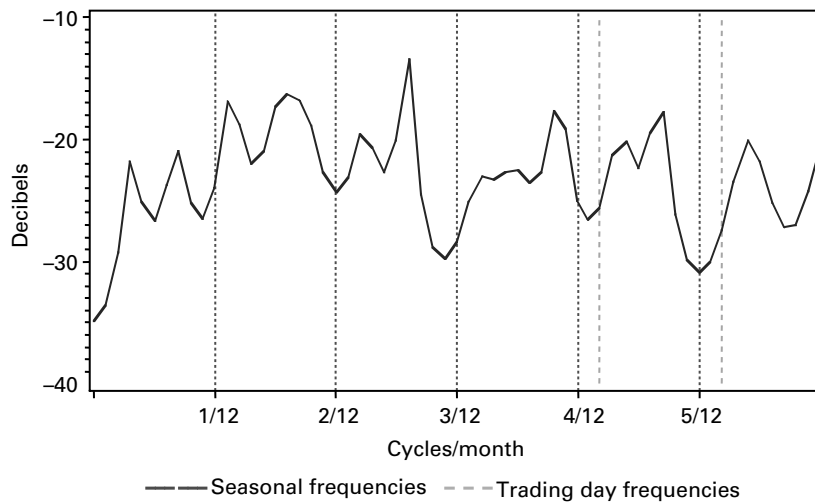
Fig. 1. Log spectrum of the irregular component of U.S. Exports of Agricultural Materials (Nonmanufactured). Note the peak at 6 cycles/year suggesting the presence of a residual component with period two months

seasonal adjustment for this series is compared with that of the Airline model, which models all seasonal frequency components in the same way.

X-13A-S also includes model comparison diagnostics, such as the out-of-sample forecast error diagnostics of X-12-ARIMA discussed in Findley et al. (1998) and Findley (2005), and diagnostics of the stability of seasonal adjustment and trend estimates, such as the revisions history diagnostics and sliding spans diagnostics (see Findley et al. 1998, Hood 2002 and Findley, Monsell, Shulman, and Pugh 1990). For the sliding spans diagnostics of SEATS estimates, a new criterion for choosing the span length is used (see Findley, Wills, Aston, Feldpausch, and Hood 2003). The span length is determined by the ARIMA model's seasonal moving average parameter Θ, which generally determines the effective length of the seasonal adjustment filter. Because SEATS filters can have effective
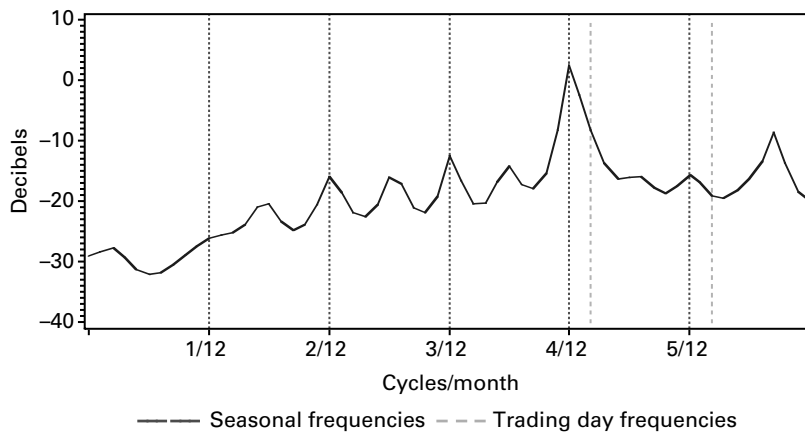

Fig. 2. Log spectrum of (first differences of logs of) Shipments showing a dominant peak at four cycles/year

lengths much greater than X-11 filters, this modified criterion only permits standard sliding spans comparisons to be made when $|\Theta| < 0.685$ with monthly series of length thirteen years, a significant limitation, and research is needed to find more versatile measures of sliding spans' instability for model-based adjustments. However, for a large variety of simulated series of this length whose estimated Θ satisfies $|\Theta| < 0.685$, Feldpausch, Hood, and Wills (2004) show that the sliding spans statistics (and, to a slightly lesser extent, revision histories) are much better than the other diagnostics of TRAMO and SEATS at detecting models that yield quite inaccurate seasonal adjustments.

4.　New or Improved Diagnostics of X-13A-S for SEATS Estimates

It is in the area of diagnostics that the technology of SEATS seems most conspicuously out-of-date and, in some cases, flawed. This is mainly due to the fact that most diagnostics provided are calculated as though an infinite or bi-infinite time series were available. Often the diagnostics obtained this way fail to convey valuable information contained in the corresponding diagnostics associated with the actual length of the time series. Finite-sample analogues of the SEATS diagnostics can be obtained from the signal extraction matrix formulas of Bell and Hillmer (1988) and McElroy and Sutcliffe (2005), which provide the coefficients of the finite filters for frequency domain diagnostics of Subsection 4.1 below and also the covariance matrices needed for the diagnostics and tests for over- or underestimation discussed in Subsection 4.2.

4.1.　*Gain and phase-delay functions of concurrent trend and seasonal adjustment filters*

For seasonal adjustment and trend estimates, the filter diagnostics we now discuss can suggest the extent of the tradeoff between smoothness and the delay or exaggeration of business cycle components in the estimates. Findley and Martin (2005) give examples involving competing seasonal adjustments. Here we will show how the diagnostics can detect when a model's trend might be less informative for recent data than its seasonal adjustment, due to filter properties.

　　Using powers of the backshift operator $BY_t = Y_{t-1}$, a linear filter whose application to a time series Y_t results in an output series $Z_t = \Sigma_j c_j Y_{t-j}$ can be expressed as the function $C(B) = \Sigma_j c_j B^j$, i.e., $Z_t = C(B)Y_t$. For monthly data, the *transfer function* of the filter is the generally complex-valued periodic function defined by $C\left(e^{-i\frac{2\pi}{12}\lambda}\right) = \Sigma_j c_j e^{-i\frac{2\pi}{12}j\lambda}$, $-6 \leq \lambda \leq 6$, when λ is in units of cycles per year. Its amplitude function, $G(\lambda) = \left|C\left(e^{-i\frac{2\pi}{12}\lambda}\right)\right|$, is called the *gain function* of the filter. For example, for the j-step delay filter B^j, the transfer function is $e^{-i\frac{2\pi}{12}j\lambda}$, so the gain function is constant with value 1. For the seasonal sum filter, $\delta_S(B) = \Sigma_{k=0}^{11} B^j$, the transfer function is

$$\delta_S\left(e^{-i\frac{2\pi}{12}\lambda}\right) = \frac{1 - e^{-i\frac{2\pi}{12}12\lambda}}{1 - e^{-i\frac{2\pi}{12}\lambda}} = \left(\frac{\sin \pi\lambda}{\sin \frac{\pi}{12}\lambda}\right) e^{-i\frac{2\pi}{12}5.5\lambda} \tag{1}$$

and the gain function is $\left|\sin \pi\lambda / \sin \frac{\pi}{12}\lambda\right|$ (except at $\lambda = 0$ where their value is 12). For seasonal adjustment and trend filters, $C\left(e^{-i\frac{2\pi}{12}\lambda}\right) = 0$ at the seasonal frequencies $\lambda = \pm 1, \ldots, \pm 5, 6$ (see Appendix C of Findley and Martin 2005), and $C(1) = \Sigma_j c_j = 1$.

A real-valued function $\phi(\lambda)$ that is defined whenever $C\left(e^{-i\frac{2\pi}{12}\lambda}\right) \neq 0$ and has the property that $C\left(e^{-i\frac{2\pi}{12}\lambda}\right) = \pm G(\lambda)e^{i\frac{2\pi}{12}\phi(\lambda)}$ holds for such λ is a *phase function* of the filter. Here $\pm G(\lambda)$ denotes a function that is real-valued, even (i.e., such that $\pm G(-\lambda) = \pm G(\lambda)$), and nonnegative at $\lambda = 0$, and whose absolute value is $G(\lambda)$ for all λ. Some values can be negative. An example is $\sin \pi\lambda / \sin \frac{\pi}{12}\lambda$ in (1), which changes sign at $\lambda = \pm 1, \ldots, \pm 5$. Thus, from (1), $\phi_{\delta_S}(\lambda) = -5.5\lambda$ is a phase function for $\delta_S(B) = \Sigma_{k=0}^{11}B^j$ which is a continuous function of λ.

In general, when $\phi(\lambda)$ is a continuous phase function for a filter with transfer function $C\left(e^{-i\frac{2\pi}{12}\lambda}\right)$ such that $C(1) > 0$ and $\phi(0) = 0$, then for every $\lambda \neq 0$ for which $C\left(e^{-i\frac{2\pi}{12}\lambda}\right) \neq 0$, the value of the function $\tau(\lambda) = -\phi(\lambda)/\lambda$ is interpreted as the (time) *delay*, or the *advance* if $\tau(\lambda)$ is negative, induced by the filter on the frequency component of Y_t with frequency λ. (Subsection A.5 of Findley and Martin (2005) justifies this interpretation of $\tau(\lambda)$ for stationary Z_t. Some numerical experiments like those of Findley (2001) for the phase function are needed to clarify the properties of $\tau(\lambda)$ for nonstationary data.) We call $\tau(\lambda)$ the *phase delay* function. (Rabiner and Gold (1975, p. 80) use this term for $-\tau(\lambda)$, which Wildi (2004, p. 50) calls the time shift.) For example, the filter B^j has a constant phase delay of j months, and $\delta_S(B) = \Sigma_{k=0}^{11}B^j$ has the constant phase delay $\tau_{\delta_S}(\lambda) = 5.5$. Delay properties are the most useful information conveyed by the phase function, and the phase delay function reveals delay properties more directly than the phase.

Figure 3, whose phase delays were obtained by the method of Appendix D of Findley and Martin (2005), shows the squared gain and phase delay functions of the filters that produce the SEATS concurrent seasonal adjustment and trend from the Airline model

$$(1 - B)(1 - B^{12})Y_t = (1 - \theta B)(1 - \Theta B)a_t \tag{2}$$

with $\theta = 0.8$ and $\Theta = 0.4$ for a series of length 109. Note that the phase delay of the trend filter is roughly twice that of the seasonal adjustment filter and is between two and four months over most of $0 < \lambda \leq 0.5$ cycles/month, the frequency interval associated with trend and business cycle movements with periods two years or more. Consequently the trend could have substantial timing distortions of many business cycle components.

Also, the squared gains of both filters are greater than one over much of this interval, indicating that any business cycle components at such frequencies will be exaggerated in the filter output (see Appendix A of Findley and Martin 2005). Where both exceed one, the trend filter squared gain is usually substantially greater than the seasonal adjustment filter squared gain, indicating correspondingly greater exaggeration. This phenomenon and the greater phase delay are generally connected to the greater smoothing of the trend filter (i.e., the greater suppression, as indicated by smaller values of the squared gain, at higher frequencies).

The squared gains and phase delays (not shown) of the infinite concurrent filters of the same model (calculated using formulas of Bell and Martin 2004a) are indistinguishable from those of Figure 3, because of the rapid decay of the filter coefficients. Findley and Martin (2005) provide a variety of examples in which the squared gains and phase delays of the series-length filters implicitly used by SEATS differ in important ways from the corresponding function of infinite length series, and they discuss sources of such differences. They also argue that phase delays should only be plotted over [0,1) for
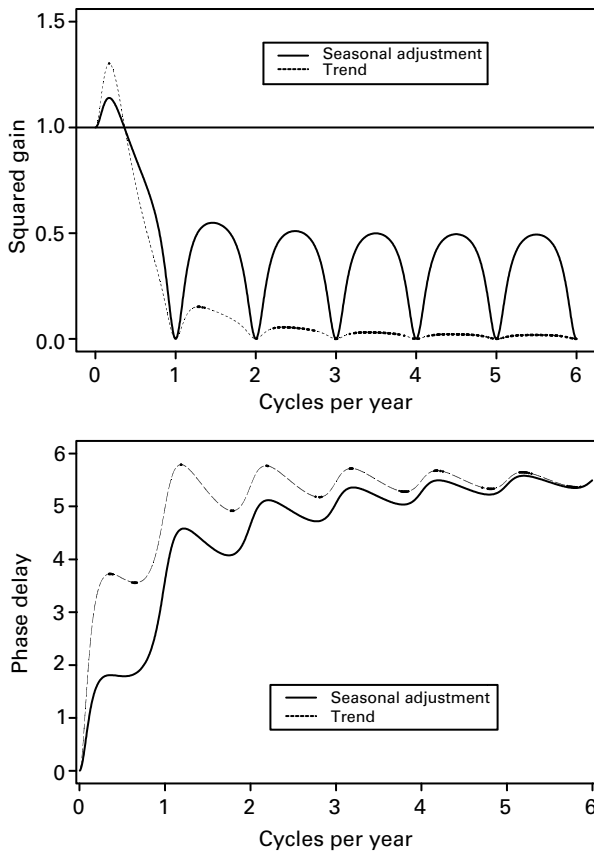
Fig. 3. *Squared gains and phase delays of concurrent SEATS seasonal adjustment and trend filters of length 109 for the Airline model with $\theta = 0.8$ and $\Theta = 0.4$. Note that the phase delay of the trend filter is roughly twice that of the seasonal adjustment filter and between two and four months over most of the frequency interval [0, 1]. Consequently the trend could have large timing distortions for many business cycle components, substantially larger than those of the seasonal adjustment*

seasonal adjustment and trend filters, because delays of higher frequency components are not of interest for business cycle analyses. That is, five-sixths of the phase-delay plot of Figure 3 is visual "noise" whose presence results in a lack of resolution over [0, 1). (They show in Appendix D that concurrent seasonal adjustment and trend filter phase delays tend to 5.5 at the frequency $\lambda = 6$ because the filters contain $\delta_S(B)$.)

Matrix filter formulas from Bell and Hillmer (1988) such as (3) below and from McElroy and Sutcliffe (2005) are currently being programmed into X-13A-S so that the coefficients of series-length concurrent and central filters, their graphs, and the graphs of the values of their squared gains and phase delays can be produced. The analogous calculations for X-11 filters with forecast extension are more cumbersome to produce (see e.g., Appendix B of Findley and Martin 2005) and various strategies for providing them are being considered.

Remark. Strictly interpreted, the phase delay of the concurrent filter provides information about the delays in a series consisting of concurrent estimates, which is not the kind of series produced by statistical offices. However, using $T_{N|M}$ to denote the trend estimate at time N

obtained from data $Y_t, 1 \leq t \leq M$, some limited analyses kindly provided by Donald Martin for filters from the Airline model used for Figure 3 indicate that if the phase delay is approximately four at a business cycle frequency for the filter that produces the concurrent trend $T_{N|N}$, then it is approximately three at this frequency for the filter that produces $T_{N|N+1}$, two for the filter that produces $T_{N|N+2}$, . . . , and less than one and tending to zero for the filters that produce $T_{N|M}$ for $M \geq N + 4$, with a similar pattern of decrease holding for other business cycle phase delay values. These results suggest that concurrent phase delays are usefully predictive. Somewhat analogously, the squared gains of the filters that produce $T_{N|M}$ for $M > N$ evolve only slowly away from the shape of the squared gain of the concurrent filter as M increases, suggesting that this function is also predictive.

4.2. Bias-correcting modifications of SEATS's diagnostic for over- or underestimation and associated test statistics

We start by presenting a precise concept for an example of a diagnostic type of SEATS for detecting over- or underestimation of the irregular component, and of the stationary transforms of the seasonal, trend and seasonally adjusted series, from a decomposition $Y_t = S_t + T_t + I_t, \; 1 \leq t \leq N$. This diagnostic suite is one of the few in SEATS for the component estimates that does not assume that the estimated regARIMA model for Y_t is correct. The concept we present leads to a finite-sample-based diagnostic that does not suffer from the strong bias (toward indicating underestimation) that can occur with SEATS's diagnostics with series of moderate lengths. We also present an associated test statistic of Findley, McElroy, and Wills (2005) for testing whether overestimation or underestimation is statistically significant.

Here we consider only the diagnostic for the irregular component I_t modeled as white noise, because its formulas are simpler. McElroy and Sutcliffe (2005) provide the basic formulas required for the stationary transforms of the other components.

First we have to address a scaling issue. The calculations that derive the canonical models for the seasonal, trend and irregular components from the ARIMA model for Y_t produce models whose innovations variances are given in units of this model's innovation variance σ_a^2 (i.e., as though this variance were one). We use σ_I^2/σ_a^2 to denote the variance specified for I_t in this manner. With $\delta(B)$ denoting the differencing operator of the ARIMA model for Y_t, e.g., $(1 - B)(1 - B^{12})$ in (2), let Σ_W denote the autocovariance matrix of order $N - d$ of the differenced series $W_t = \delta(B)Y_t, d + 1 \leq t \leq N$ specified by this ARIMA model but calculated as though σ_a^2 were equal to one. Finally, let Δ denote the $N - d \times N$ matrix which transforms $[Y_1, \ldots, Y_N]'$ to $W = [W_{d+1}, \ldots, W_N]'$. With this notation, the vector $\hat{I} = [\hat{I}_1, \ldots, \hat{I}_N]'$ of SEATS estimates of the irregulars has the formula

$$\hat{I} = \frac{\sigma_I^2}{\sigma_a^2} \Delta' \Sigma_W^{-1} W \tag{3}$$

(see (4.3) of Bell and Hillmer 1988).

Now we can state what we take to be the basic concept of SEATS's diagnostic: investigate the sign (and, in an unspecified way, also the size of) the difference between the sample mean square $\overline{\hat{I}^2} = N^{-1}\Sigma_{t=1}^N \hat{I}_t^2$ and its expected value $N^{-1}\Sigma_{t=1}^N E\hat{I}_t^2$. Here E denotes expectation calculated for quadratic forms of the $W_t = \delta(B)Y_t, d + 1 \leq t \leq N$ as

if the autocovariances of the W_t specified by the estimated ARIMA model for the series Y_t were correct.

Formula (3) immediately yields a formula for $\Sigma_{\hat{I}/\sigma_a}$, the covariance matrix of $\hat{I}$ in units of σ_a^2, namely $\Sigma_{\hat{I}/\sigma_a} = (\sigma_I^2/\sigma_a^2)^2 \Delta' \Sigma_W^{-1} \Delta$. For $1 \le t \le N$, we denote the t-th diagonal entry of this matrix by σ_t^2/σ_a^2. To obtain $E\hat{I}_t^2$, we multiply σ_t^2/σ_a^2 by the bias-corrected maximum likelihood estimate of σ_a^2 due to Ansley and Newbold (1981), $\hat{\sigma}_a^2 = c_N(N-d)^{-1}W'\Sigma_W^{-1}W$, with bias correction factor $c_N = (N-d)/(N-d-n_{coeffs})$, where n_{coeffs} denotes the number of estimated ARMA coefficients, estimated via exact maximum likelihood estimation. Thus, with tr denoting matrix trace, i.e., the sum of the diagonal entries, $N^{-1}\Sigma_{t=1}^N E\hat{I}_t^2 = \hat{\sigma}_a^2 \left(N^{-1}tr\Sigma_{\hat{I}/\sigma_a}\right)$. Our basic diagnostic is therefore

$$\tau = \overline{\hat{I}^2} - \hat{\sigma}_a^2 \left(N^{-1}tr\Sigma_{\hat{I}/\sigma_a}\right) \tag{4}$$

By analogy with SEATS and Maravall (2003), *underestimation* is indicated when $\tau < 0$, and *overestimation* is indicated when $\tau > 0$, where, reformulating the description in Marvall (2003), underestimation means inadequate suppression, and overestimation means excessive suppression, of the complementary component to the one being estimated, e.g., inadequate or excessive suppression of $S_t + T_t$ when estimating I_t.

The diagnostic provided by SEATS is effectively $\tau^{SEATS} = \overline{\hat{I}^2} - \hat{\sigma}_a^2 \left(\sigma_{WK}^2/\sigma_a^2\right)$, where σ_{WK}^2/σ_a^2 denotes the variance of the Wiener-Kolmogorov estimator of I_t from bi-infinite data from the ARIMA model for Y_t in units of σ_a^2. In Proposition 1 of Findley, McElroy, and Wills (2005), it is shown that $\sigma_{WK}^2/\sigma_a^2 > \sigma_t^2/\sigma_a^2$ holds for all t when the model has a moving average component. Consequently, SEATS's diagnostic has a bias toward indicating underestimation. For series lengths $N = 72, 144$, the bias is shown in Tables 1 and 3 of Findley, McElroy, and Wills (2005) to strongly compromise the performance of SEATS's diagnostic. This is particularly the case relative to the performance of a modification $\tau^{(2)}$ of (4) obtained by using averages of $\hat{I}_t^2$ and $E\hat{I}_t^2$ over the more restricted range $13 \le t \le N - 12$ (see Subsection 3.2 and Subsection A.3 of this reference where approximate standard deviations for τ and $\tau^{(2)}$ are derived for near-Gaussian data, e.g., $\hat{\sigma}_\tau = \frac{\sqrt{2}\hat{\sigma}_a^2}{N}\left\{tr\Sigma_{\hat{I}/\sigma_a}^2 - \frac{2c_N - c_N^2}{N-d}\left(tr\Sigma_{\hat{I}/\sigma_a}\right)^2\right\}^{1/2}$ for τ). These yield test statistics, $\overline{\tau} = \tau/\sigma_\tau$ and the analogously defined $\overline{\tau}^{(2)}$, which can be compared to the standard normal distribution for one-sided tests of the significance of over- or underestimation. For example, for the Export series whose spectrum in Figure 1 has a peak at the seasonal frequency six cycles year, indicating residual seasonality in the irregular component, the value of $\overline{\tau}^{(2)}$ is -2.177, which has a p-value of .013 indicating significant underestimation of the irregular component. (The value of $\overline{\tau}$ is -0.546). Note that calculation of the trace of $\Sigma_{\hat{I}/\sigma_a}^2$, and therefore of $\overline{\tau}$, requires the full autocovariance matrix $\Sigma_{\hat{I}/\sigma_a}$, and $\overline{\tau}^{(2)}$ requires all but its first and last twelve rows and columns, more autocovariances than can be obtained from standard state space algorithms.

SEATS does not have a test statistic for τ^{SEATS} and its analogues. Maravall (2003) illustrates how such a test statistic could be obtained, but the consequences of the bias of τ^{SEATS} shown in Findley, McElroy, and Wills (2005) indicate that such tests will often be unreliable. These results, those of Findley and Martin (2005), and the availability of the needed finite-filter formulas like those just presented have led us to conclude that all of the

diagnostics of SEATS that depend on Wiener-Kolmogorov (i.e., infinite-data-based) coefficients or autocovariances should be replaced by quantities appropriate to the length of the time series to which SEATS procedures are being applied. This is a goal the U.S. Census Bureau is pursuing for X-13A-S.

5. New Time Series Models for Seasonal Adjustment

5.1. The RegComponent models of Bell

Bell (2004) describes methods and associated software for estimating the various model parameters and unobserved components when observed data $Y_t, 1 \leq t \leq N$ are assumed to have the form

$$Y_t = x_t'\beta + \sum_{i=1}^{m} h_{it}z_{it}$$

where x_t' is a row vector of known regression variables, β is the vector of constant regression coefficients, h_{it} for $i = 1, \ldots, m$ are series of known constants called "scale factors" (often $h_{it} = 1$), and z_{it} for $i = 1, \ldots, m$ are independent unobserved component series following ARIMA models, $\phi_i(B)\delta_i(B)z_{it} = \theta_i(B)\varepsilon_{it}$. (Sometimes, as in the first example below, a regARIMA model is used for an unobserved component.) Such models for Y_t are called *RegComponent models*, or simply (ARIMA) *Component models* when no regressors occur. RegARIMA models are the special case with $m = 1$, $h_{1t} = 1$. Bell (2004) shows how the various types of seasonal "structural" models of Harvey (1989) can be formulated as RegComponent models and also gives other important examples, two of which we present below. Bell and Martin (2004b) show how the time-varying trading-day effects model of Harvey (1989) can be formulated as a RegComponent model and they compare its performance to that of the time-varying trading-day model considered in Bell (2004).

5.1.1. Seasonal adjustment of a time series with sampling error
For the U.S. Monthly Retail Trade Survey series of (unbenchmarked estimates) of Sales of Drinking Places from September, 1977 through October, 1989, let Y_t denote the logarithm of the survey estimates, y_t the corresponding true population quantities, and e_t the sampling error in Y_t as an estimate of y_t. The RegComponent model for Y_t considered in Bell (2004) is, in our notation,

$$Y_t = y_t + e_t; \quad (1 - B)(1 - B^{12})(y_t - x_t'\hat{\beta})$$

$$= (1 - 0.23B)(1 - 0.88B^{12})b_t \quad \sigma_b^2 = 3.97 \times 10^{-4}$$

$$(1 - 0.75B)(1 - 0.66B^3)(1 - 0.71B^{12})e_t = (1 + 0.13B)c_t \quad \sigma_c^2 = 0.93 \times 10^{-4}$$

where x_t' consists of trading day regressors. The model for e_t was developed given knowledge of the survey design, and its parameters were estimated using estimates of sampling error autocovariances. With these parameters fixed, the estimated parameters of the regARIMA model for y_t were obtained from Bell's REGCMPNT program (which will be made available soon). The value of $\hat{\beta}$ is not of concern here. From this model, a

canonical seasonal decomposition of the model for $y_t - x_t'\hat{\beta}$ yields a RegComponent model $Y_t = x_t'\hat{\beta} + S_t^{(1)} + N_t^{(1)} + e_t$. From this, the state space smoothing algorithm of the REGCMPNT program produces an estimate $\hat{N}_t^{(1)}$ of the nonseasonal component of the unobserved sampling-error-free series y_t. This can be compared to the nonseasonal component estimate $\hat{N}_t^{(2)}$ obtained from the alternative RegComponent model $Y_t = x_t'\beta + S_t^{(2)} + N_t^{(2)}$ arising from the canonical decomposition of the directly estimated regARIMA model for Y_t,

$$(1 - B)(1 - B^{12})(Y_t - x_t'\beta) = (1 - 0.29B)(1 - 0.56B^{12})a_t \quad \sigma_a^2 = 6.37 \times 10^{-4}.$$

Figure 4 presents the graphs of the resulting nonseasonal component estimates $\exp\left(\hat{N}_t^{(1)}\right)$ and $\exp\left(\hat{N}_t^{(2)}\right)$ of the original data. The series $\exp\left(\hat{N}_t^{(1)}\right)$ is smoother than $\exp\left(\hat{N}_t^{(2)}\right)$. Figure 12.1 of Bell (2004, p. 273) shows that its signal extraction standard errors are larger than those of $\exp\left(\hat{N}_t^{(2)}\right)$ at all times t, as one would expect, because $\hat{N}_t^{(1)}$ attempts to suppress both $S_t^{(1)}$ and e_t when estimating $N_t^{(1)}$.

5.1.2. Modeling seasonal heteroscedasticity

For the series of logarithms of U.S. Northeast region monthly total housing starts from August, 1972 to March, 1989. Bell (2004) notes that standard ARIMA modeling procedures without outlier identification leads to the ARIMA model $(1 - B) \times (1 - B^{12})Y_t = (1 - .64B)(1 - .58B^{12})a_t$ with $\hat{\sigma}_a^2 = .050$. However, automatic outlier detection in X-12-ARIMA with a critical value of 3.0 leads to a regARIMA model, $(1 - B)(1 - B^{12})(Y_t - x_t'\tilde{\beta}) = (1 - .55B)(1 - 1.00B^{12})a_t$ with $\hat{\sigma}_a^2 = .024$, whose regresssors model four additive outliers for Januarys and five for Februarys, due presumably to the effects of unusually bad or unusually good winter weather on construction activity in these months. The value 1.00 of the seasonal moving average parameter in the second model indicates that the seasonal pattern of the data is fixed, whereas the coefficient .58 of the first model suggests that it is rather variable.
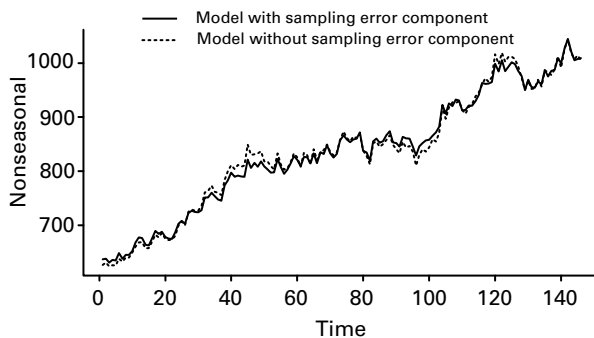

Fig. 4.   The graph of the nonseasonal component estimate of the unobserved sampling-error-free series, denoted $\exp(\hat{N}_t^{(1)})$ in the text, is noticeably smoother than the graph of nonseasonal component estimate, denoted $\exp(\hat{N}_t^{(2)})$ in the text, from the observed data model without a sampling error component

Bell (2004) then presents a Component model without regressors that includes an observation error component for Januarys and Februarys and requires only one more parameter than the initial Airline model instead of nine more,

$$Y_t = z_{1t} + h_{2t}z_{2t}$$

$$(1 - B)(1 - B^{12})z_{1t} = (1 - .54B)(1 - 1.00B^{12})b_t$$

$$z_{2t} = c_t, \sigma_c^2 = .12, \sigma_b^2 = .023$$

where b_t and c_t are mutually independent white noise processes, and where

$$h_{2t} = \begin{cases} 1, & t \sim \text{January or February} \\ 0, & \text{all other months.} \end{cases}$$

The model for z_{1t} is almost identical to the model obtained for the outlier corrected data and leads to quite similar fixed seasonal factors and seasonal adjustments with moderate exceptions for January and February. This third model is more appealing, because it avoids the instabilities associated with the use of a fixed and somewhat arbitrarily chosen critical value for outlier detection (not to mention instability associated with the estimation of nine regression coefficients in this case). It also leads to seasonal factor standard errors that are significantly larger for January and February than for the other months, as seems appropriate, something the other models do not show.

Trimbur (2005) will provide further results for Bell's approach to seasonal heteroscedasticity together with the results of his study of a different, structural model-based approach of Proietti (2004), some initial results from which we present next.

Whereas X-11 based software has always had an approach to dealing with seasonal heteroscedasticity that is often effective (see Findley and Hood (2000) for some examples), no versatile approach has been established for model-based seasonal adjustment.

5.2. Proietti's seasonal specific structural models

Proietti (2004) has developed a structural-model-based approach to modeling seasonal heteroscedasticity in which a season-specific noise process, analogous to Bell's $h_{2t}z_{2t}$, is added to the process governing the level changes from one period to the next. Another such noise process can be added to the process governing the trend slope changes. We refer the reader to Proietti's paper for details. Thomas Trimbur kindly provided Figure 5 and the results we now summarize for eleven U.S. construction series published by the U.S. Census Bureau. These results are taken from a preliminary version of Trimbur (2005). These series are ones whose X-12-ARIMA seasonal adjustments are obtained by specifying different seasonal filter lengths for some calendar months, the X-11 procedure for dealing with calendar month heteroscedasticity.

The X-11 procedure addresses heteroscedasticity in the detrended series (the "SI" ratios) whereas Proietti models heteroscedasticity in the levels. The log likelihood ratios comparing Proietti's heteroscedastic and homoscedastic local linear trend models suggest that only three of the eleven series are heteroscedastic in Proietti's sense. The properties of

the trend estimates of Proietti's models that were observed for these three series are most strongly seen in the series with the most statistically significant seasonal hetero-scedasticity, the series of U.S. Midwest Region Total Building Permits (1992-2003) whose model-based trends from 1995 on are shown in Figure 5, along with the X-12-ARIMA final trend (Table D 12) obtained with a longer seasonal filter for the months December through March than for the other months. The trend from the heteroscedastic model with extra noise sources for December through March (and from a modification of the mean correction of Section 4 of Proietti 2004) is notably smoother than the trend from the homoscedastic model in most winter months, but usually somewhat less smooth than the X-12-ARIMA trend. The trends from Proetti's models have smaller values on average than the X-12-ARIMA trend. This is likely a deficiency related to the fact that twelve month sums of log seasonal factors are not required to have mean zero for either model.

5.3. Frequency-specific generalized Airline models

Aston, Findley, Wills, and Martin (2004) consider special cases of the model

$$(1 - B)^2 \left(\sum_{j=0}^{11} B^j \right) Y_t = (1 - B)^2 \left[(1 + B) \prod_{j=1}^{5} \left(1 - 2 \cos\left(\frac{2\pi j}{12}\right) B + B^2 \right) \right] Y_t$$

$$= (1 - \theta B)(1 - b_0 B) \left[(1 + b_6 B) \prod_{j=1}^{5} \left(1 - 2b_j \cos\left(\frac{2\pi j}{12}\right) B + b_j^2 B^2 \right) \right] \varepsilon_t \qquad (5)$$
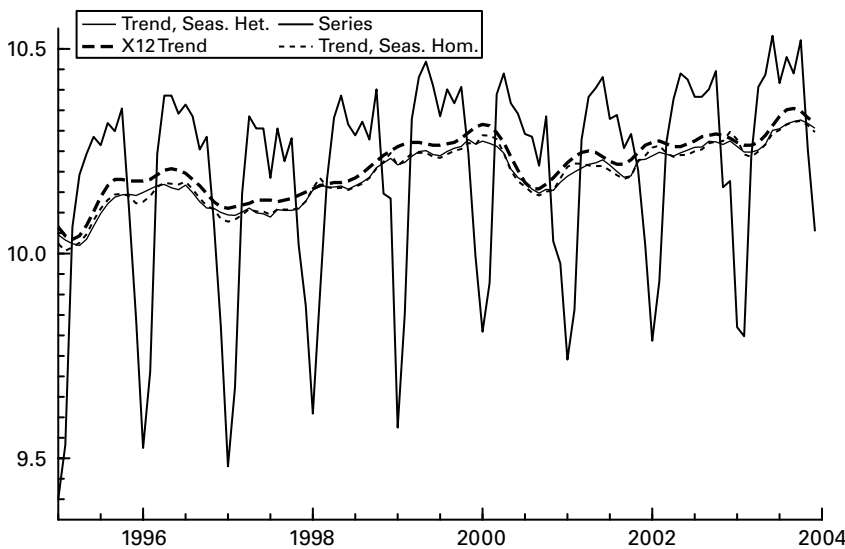

Fig. 5. *Logs of the original series (thin line) and three trends from 1995-2004 for U.S. Midwest-Region Total Building Permits. The X-12-ARIMA trend (long dashes), from seasonal adjustments with longer filters for December through January, is greater, on average, and slightly smoother than the trend (thick line) from Proietti's heteroscedastic model with extra noise sources for these months, which is notably smoother in these months than the trend from Proietti's homoscedastic model (dashes)*

This eight-coefficient model is the most general form of what we call a *frequency-specific Airline model*. It has an individual coefficient for each moving average factor that corresponds to a seasonal unit root factor of the differencing operator on the left. We call these six moving average factors *seasonal frequency factors*. Box-Jenkins Airline models (2) with $\Theta \geq 0$ are the special case $b_j = \Theta^{1/12}$, $0 \leq j \leq 6$. Aston et al. (2004) conclude that the generalized Airline models of the form (5) which are most useful for seasonal adjustment are those in which only two of the coefficients $b_j, 0 \leq j \leq 6$ are distinct. The other models have too great a tendency to have estimated coefficients equal to one in situations in which the evidence for such a value is not strong.

Here we single out six three-coefficient frequency-specific models, denoted 5-1(3) models, in which five seasonal frequency factors have one coefficient and the remaining seasonal frequency factor has its individual coefficient, for example

$$(1 - B)^2 \left(\sum_{j=0}^{11} B^j \right) Y_t = (1 - aB)(1 - c_1 B)$$

$$\times \left\{ (1 + c_2 B) \prod_{j=1}^{5} \left(1 - 2c_1 \cos\left(\frac{2\pi j}{12}\right) B + c_1^2 B^2 \right) \right\} a_t$$

The situation in which a series (perhaps after taking logs and a first difference) has a spectrum with a strong seasonal peak only at one of the seasonal frequencies, as in Figure 2 for the Shipments series, is the most easily identified one for which a 5-1 (3) frequency-specific model is a natural alternative to the Airline model, and this situation is not particularly rare, as Wildi (2004) also observes. Conventional seasonal ARIMA models like the Airline model, which use the same coefficient $\Theta^{1/12}$ for each frequency, are ill-suited to such series.

For the Shipments series, the quarterly-effect frequency, four cycles/year, is the one associated with the c_2 coefficient because this frequency has the dominant spectral peak in Figure 2. The c_1 coefficient estimate is 0.9718, close to the twelfth root of the estimated Airline model seasonal coefficient Θ ($\sqrt[12]{0.6667} = 0.9668$). However, the c_2 coefficient estimate is 0.8086 ($= \sqrt[12]{0.0781}$). This smaller value results in the concurrent squared gains of the 5-1 (3) model in Figure 6 having wider troughs at the four cycles/year frequency than the Airline model's filters and similarly for the squared gains of the symmetric filter (see Findley, McElroy, and Wills 2005). The wider troughs indicate more suppression of variability around this quarterly frequency. As a result, for this series, the canonical seasonal adjustment of the 5-1(3) model is generally smoother than that of the Airline (see Figure 7).

There are six 5-1(3) models corresponding to the available choices of season frequency factors for c_2. Analogously, there are fifteen so-called 4-2(3) models in which c_2 is assigned to two seasonal frequency factors and c_1 to the other four. Aston et al. (2004) find this to be type of three-coefficient models most frequently preferred over the Airline model by their generalization of Akaike's AIC that seeks to account for the increase in uncertainty in selecting among numerous frequency-specific models and the Airline model. Finally, there are twenty so-called 3-3(3) models in which the c_1 and c_2 are each
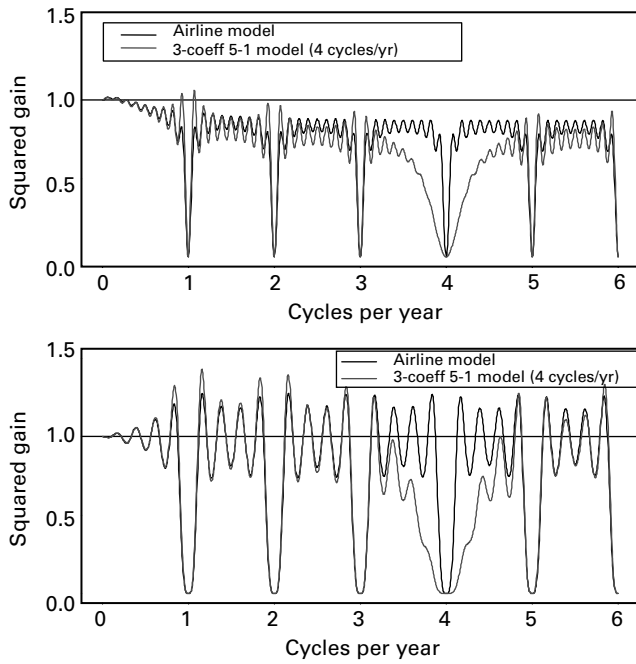
Fig. 6. *Squared gains of the concurrent (above) and central (below) model-based filters of length 117 for Shipments from the Airline model and the frequency-specific model for Shipments. Note the latter model's substantially greater suppression of near quarterly components*

assigned to three seasonal moving average factors. This type does not appear to be as useful as the other two.

Estimation of the frequency-specific Airline models and the calculation of their canonical ARIMA model-based seasonal adjustments were done with a version of the state space modeling software discussed in Aston and Koopman (2003). This software is being further developed to have the capabilities of Bell's REGCMPNT program.
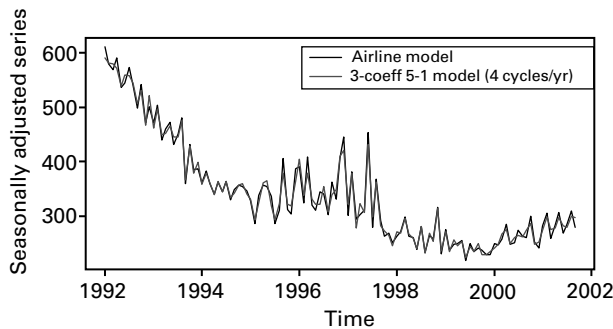

Fig. 7. *Airline and 5-1(3) model-based seasonal adjustment of Shipments. The 5-1(3) model has frequency four cycles/year associated with the c_2 coefficient and its filters provide a mostly smoother seasonal adjustment due to greater suppression of the strong frequency components close to this frequency (see Figure 4)*

5.4. Wildi's ZPC-QMP filters

The monograph by Wildi (2004) provides a radically different approach to model-based trend estimation (and, secondarily, seasonal adjustment) which is very stimulating, also because some new and substantial auxiliary theoretical results on frequency domain techniques for nonstationary series are obtained. It can be argued that the long-run behavior of any "true" ARIMA process (meaning cases like the Airline model (2) with $\theta = 1$ or $\Theta = 1$ are excluded) is unlike that of any persistently seasonal economic time series because each calendar month's seasonal factors will change direction over time, but Wildi emphasizes how especially ill-suited such models seem to be for the long index series he considers, whose values are constrained to satisfy $-100 \leq Y_t \leq 100$. He proposes a new class of "zero-pole-combination quasi-minimum-phase" (abbreviated ZPC-QMP) filters with ARMA transfer functions.

The case of primary interest in Wildi (2004) is that of one-sided ZPC-QMP filters for concurrent trend estimates. Their seventeen parameters are estimated by minimizing a frequency domain expression, calculated from the periodogram of the time series of interest or its differences, which estimates the mean squared difference (or revision error) between the concurrent filter's output and the output of a prescribed symmetric trend filter for the given series. Constraints on the magnitude of the phase-delay (or time-shift) of the filter can also be imposed on the estimation criterion.

If the prescribed symmetric filter is appropriate, this would seem to be more application-appropriate criterion than maximum likelihood estimation of an ARIMA model, which is related to minimizing the model's mean squared one-step-ahead forecast error of the original seasonal time series. Estimation of Wildi's filters turns out too often to be difficult and delicate, but for the set of series he considers, they generally yield better trends, e.g., for turning point estimation, than the maximum likelihood based trend estimates he investigates.

Wildi's approach seems quite impractical for statistical offices with a large number of time series, or with short series, but it may give indications of important limitations of current methods and indications of how some new methods that will be useful for statistical offices could be found.

6. Recent Developments for X-11 Filter Seasonal Adjustments

Here we call attention to two noteworthy recent research directions related to seasonal adjustments from X-11 filters.

6.1. Shrinkage methods for X-11 seasonal factors

We start with the discussion paper of Miller and Williams (2004), which summarizes a study of the application to X-11 seasonal factors of a "global" and a "local" shrinkage procedure developed by the authors. It shows, through a forecasting study with a large test set of series frequently used by academic forecasters, that the shrinkage methods lead to seasonal adjustments from which better forecasts are obtained from certain methods that forecast seasonal time series by (i) seasonally adjusting, (ii) forecasting the seasonally adjusted series, and (iii) reseasonalizing these forecasts.

Findley, Wills, and Monsell (2004) investigate whether seasonal adjustment after application of the shrinkage methods to the seasonal factors improves some of the X-12-ARIMA Q2, M7, M8, M10, and M11 quality statistics (see Ladiray and Quenneville 2001). They find that the "local" shrinkage method of Miller and Williams, whose statistical motivation is less clear than that of their "global" method, increases the number of multiplicatively seasonal series with acceptable values of these statistics (defined for this study as being any value less than 1.10) from a little over 200 to almost 250. The shrinkage smoothes the evolution from year to year of the seasonal factors for many of the calendar months. However, for the few series examined in detail, the resulting seasonally adjusted series had seasonal peaks in their spectra that did not occur without shrinkage. This suggests that shrinkage of the seasonal factor by this method often leads to residual seasonality, a significant problem for official seasonal adjusters if not for forecasters. The global shrinkage method provided no useful improvement to the set of diagnostics. Thus, it remains to be seen whether shrinkage methods can be used to improve X-11 filter-based seasonal adjustments in a way that is useful to producers of official statistics.

6.2. Uncertainty measures

The second research direction of note is the development of methods to provide standard errors for X-11 filter based seasonal adjustments, statistics that have always been available for model-based method adjustments but not for X-11 adjustments. We are referring to the method of Pfeffermann (1994) and its further development described in Scott, Sverchkov, and Pfeffermann (2004) and to the quite different method of Bell and Kramer (1999). These methods can take account of sampling error in the data through the use of time series models of the sampling error like the model of Subsection 5.1.1 and the models of Nguyen, Bell, and Gomish (2002) and Scott et al. (2004).

The Pfeffermann approach described in Scott et al. (2004) accounts for time series uncertainty beyond sampling error, which the Bell and Kramer approach does not, by modeling the irregular component of the unobserved sampling-error-free series with a moving average model of order $0 \leq q \leq 3$, assuming at least one such model can be found that is not incompatible with the sampling-error autocovariances and the autocovariances of the X-11 irregulars.

Going beyond uncertainty due to sampling error, the approach of Bell and Kramer (1999) extends the approach of Wolter and Monsour (1981) to address uncertainty associated with ARIMA forecast extension before application of X-11 filters in a way that the Pfeffermann approach does not. When sampling error is negligible or not modeled, the Bell and Kramer mean squared errors are estimates of the square of the seasonal adjustment revisions that will occur when future data become available, to make possible the application of more symmetric filters. Such revision estimates (and the standard error estimates mentioned previously and also those of ARIMA model-based procedures) do not account for the effect of revisions to the unadjusted data, for example revisions due to late reporting or benchmarking.

As an alternative or complement to the procedures mentioned above, simple descriptive statistics calculated from several years of published revisions over time to originally

published adjusted values have several useful properties. (1) They reflect the effects of revisions to the unadjusted data as well as the effects of applying different asymmetric filters. (2) They describe an intelligible basic type of empirical uncertainty, namely the kinds of changes a data user will see over time. (3) They have the very important property that they can be provided for indirectly seasonally adjusted series, e.g., sums of seasonally adjustment component series. (Univariate time series model-based methods like that of SEATS cannot provide plausible standard errors for indirect adjustments, nor can the univariate methods of Pfeffermann and Bell and Kramer.) Finally, (4) they are technically easy to produce (after the historical data are assembled).

Of course, such descriptive statistics suffer from two fundamental limitations that must be made clear to users: they ignore both the uncertainty due to sampling error (which is small in many high level aggregates) and the uncertainty due to the signal extraction error that remains even when enough future data become available that further revisions are negligible. For these reasons, they cannot be used to establish the statistical significance of, say, an increase in the seasonally adjusted series from one month to the next. (But the value of such significance tests is debatable for several reasons. For example, it has been noted that the null hypothesis of no change is seldom plausible for economic time series. See Smith 1978 for other inadequacies.) With these limitations, empirical revision statistics may provide useful indications of basic uncertainty for seasonal adjustments where currently none are provided, and, in the case of indirectly adjusted aggregates, where more satisfactory measures have yet to be established.

Some branches of the U.S. Census Bureau are undertaking the preparations necessary to enable production of robust descriptive statistics on an ongoing basis for the revisions of some of their published (direct and indirect) seasonal adjustments or seasonally adjusted month-to-month changes. This is being done in a spirit of experimentation to investigate how such statistics could be used by consumers of seasonally adjusted data.

7.   Some Concluding Remarks

Most of this article has been related to model-based seasonal adjustment of one kind or another. This is a relatively young area in which there is much interest because of the seeming simplicity and the analyzability of models and also because some of the progress of model development in other areas of statistics (e.g., Bayesian methods) can be expected to stimulate progress in model-based seasonal adjustment. This is therefore the area of seasonal adjustment in which the most significant progress can be expected to occur in the next years, in the development of better direct or indirect diagnostics for model inadequacies, in the development and use of new models and new model fitting criteria, and also in more effective uses of existing model types. For example, for the problem of data spans too long for modeling with a single model or coefficient vector, the approach of BAYSEA (Akaike 1980) could be explored. This involves estimating a model of specified type or sometimes a set of models (see Akaike and Ishiguro 1983) over a shorter data window which is advanced by steps of length one year through the available data. It can also involve a method of averaging the results from different windows or different models like that illustrated in Section 6 of Akaike and Ishiguro (1983). The Australian Bureau of

Statistics is investigating a somewhat analogous approach to estimating time-varying trading-day effects.

Finally, the important issue of staff training for model-based seasonal adjustment must be mentioned. The greater variety of seasonal adjustment filters available from SEATS than from X-12-ARIMA can lead to worse results from an inadequately trained user of the software who makes poor option choices and does not recognize this fact. (It is not enough to check that the *p*-value of a model's Ljung-Box statistic at lag 24 is greater than 0.05. For example, models that omit significant trading day and outlier effects can have this property.) The X-13A-S program is an important development, not only because of its stronger suite of diagnostics but also because the user can always compare a SEATS adjustment with an X-11 adjustment. Conversely, with this software a user of the X-12-ARIMA approach can gradually become familiar with the alternatives made available by model-based adjustment. Thus X-13A-S will be an important training tool, but inadequate by itself. We expect that statistical agencies and central banks who wish to make broad use of model-based seasonal adjustments will find it necessary to make a considerable and sustained effort, even more than with X-12-ARIMA, to insure that enough sufficiently trained staff is available to maintain adjustment quality when already trained staff moves on.

Appendix: Differences between the Automatic Modeling Procedure of X-13A-S and TRAMO

The most important automatic regARIMA model selection procedure differences from TRAMO are that X-13A-S uses: (1) exact maximum likelihood coefficient estimates where TRAMO uses Hannan-Risannen estimates or approximate maximum likelihood estimates; (2) larger critical values for inclusion of outlier regressors (based on theoretical results of Ljung 1993); (3) a theoretical value derived in Findley et al. (1998) rather than an estimated coefficient for the length-of-month/leap-year effect component of multiplicative trading-day models; (4) candidate intervals of lengths 1, 8, and 15 for the Easter effect regressor instead of an interval of length 6. Also, (5) Akaike's AIC is used instead of individual coefficient *t*-statistics for inclusion of trading day and holiday regressors.

McDonald-Johnson and Hood (2001) investigate the automatic outlier procedure of X-13A-S (and X-12-ARIMA) and consider the effects of lower critical values like those used by TRAMO but find them disadvantageous more often than they are advantageous. McDonald-Johnson, Nguyen, Hood, and Monsell (2002) study other aspects of the automatic regARIMA modeling procedure and discuss differences from TRAMO's procedure. The results of these and other unpublished studies done at the U.S. Census Bureau mildly favor the automatic transformation and trading day and holiday effect decision procedures of X-13A-S and show that TRAMO selects differencings and seasonal differencings slightly more often than X-13A-S, whereas X-13A-S has a mild tendency to choose ARIMA models with more coefficients. Some improvements have been made recently to the way in which X-13A-S decides on differencing operators but a large-scale study to see if the decisions are now the same as TRAMO's more often has not yet been done.

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