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Temporal Disaggregation and the Adjustment of Quarterly National Accounts for Seasonal and Calendar Effects

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The statistical treatment of seasonality and calendar effects in the estimation of quarterly national accounts raises a number of issues that bear important consequences for the assessment of current economic conditions. In many European countries, the quarterly national accounts are constructed by national statistical institutes by disaggregating the original annual measurements using related monthly indicators. In this article we propose and evaluate an alternative approach that hinges upon the estimation of a bivariate basic structural time series model at the monthly frequency, accounting for the presence of seasonality and calendar components. Its main virtue is to enable the adjustment and temporal disaggregation to be carried out simultaneously. The proposed methodology also complies with the recommendations made by the Eurostat – European Central Bank task force on the seasonal adjustment of quarterly national accounts. The overall conclusion is that the identification and consequently the separation of seasonal and calendar effects from aggregate data is highly controversial.

Key words: Structural time series models; calendar effects; Kalman filter and smoother; quarterly national accounts.

1. Introduction

The topic of the article is the temporal disaggregation of economic flow series that are available only at the annual frequency. The resulting quarterly or monthly estimates incorporate the information available from related indicators at the higher frequency. However, since the indicators are affected by seasonal and calendar variation, there arises the problem of adjusting the estimates for those effects.

Seasonality and calendar components explain a relevant part of the fluctuations of economic aggregates. While the former refers to the intra-year movements in economic activity caused by various factors, among which climatic and institutional ones are prominent, calendar effects result from essentially three sources (see Cleveland and Devlin 1980; Bell and Hillmer 1983):

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- Weekly periodicity: the level of economic activity depends on the day of the week. Temporal aggregation of a flow or a stock variable into a monthly or quarterly series is such that the level of the series will be affected by the number of trading days (TD) or working days (WD) in that month or quarter.
- Moving festivals, such as Easter, which change their position in the calendar from year to year.
- The different length of the month or quarter: once TD/WD and seasonal effects have been accounted for, the differential effect is due only to the leap year effect.

Providing quarterly national accounts estimates corrected for seasonality and calendar components satisfies a well-established information need for both business cycle and structural analyses. This is officially acknowledged in Eurostat's Handbook of Quarterly National Accounts (Eurostat 1999). A task force established by Eurostat and the European Central Bank (Eurostat 2002) has also set forth some guidelines for calendar adjustment, some of which motivate this contribution. In particular, the use of regression methods is recommended in place of proportional adjustment, with the regressors constructed so as to take into account the country-specific holidays. When available, adjustment should be performed on monthly series, as calendar effects are more easily identified at that frequency. Essential and up-to-date references on the problem of temporal distribution are Di Fonzo (2003), Guerrero (2003), Dagum and Cholette (2006), to which we refer the reader for more details on regression-based methods and their dynamic generalisations.

The Italian Statistical Institute, ISTAT, started trading day adjustment of quarterly national accounts in June 2003 and has published seasonally adjusted and trading day corrected series since then. See ISTAT (2003) for a description of the methodology. The French methodology is documented in INSEE (2004). Essentially, the current practice involves at least three operations: a separate seasonal and calendar adjustment of the indicator series, and two temporal disaggregations of the annual aggregate using the two versions of the indicator, raw and adjusted. The disaggregation method adopted is based on the technique proposed by Chow and Lin (1971).

We argue that the current practice is unnecessarily complicated. Indeed, the main aim of the article is to show that all these operations can easily be brought under the same umbrella. Within the unifying framework represented by the estimation of a multivariate structural time series model formulated at the higher time frequency, seasonal adjustment of the indicators and the correction for calendar variation are carried out in one step. The multivariate setup also provides a more consistent framework for using the information on related series.

The plan of the article is the following. The next section introduces the disaggregated basic structural model with regression effects which lies at the basis of our approach. Section 3 discusses the effects of temporal aggregation on the seasonal component and considers the consequences regarding modelling and data dissemination policies. The modelling of the calendar component is considered in Section 4. Section 5 illustrates the statistical treatment of the model, whilst Section 6 presents illustrations that are drawn from our experience as members of a research commission set up by ISTAT with the objective of assessing the current practice by which quarterly accounts are built, and of proposing methodological advances. Finally, in Section 7 we compare our proposal with the disaggregation practice currently in use at ISTAT and draw our conclusions (Section 8).

2. The Bivariate Basic Structural Model

The basic structural model (BSM henceforth), proposed by Harvey and Todd (1983) for univariate time series and extended by Harvey (1989) to the multivariate case, postulates an additive decomposition of the series into a trend, a seasonal and an irregular component. Its name stems from the fact that it provides a satisfactory fit to a wide range of seasonal time series, thereby playing a role analogous to the Airline model in an unobserved components framework.

Without loss of generality we focus on a bivariate series $y_t = [y_{1t}, y_{2t}]'$, $t = 1, \ldots, n$, where t is time in months and n is the number of observations; in the sequel y_{1t} will represent the indicator series, whereas y_{2t} is subject to temporal aggregation, being observed only at the annual frequency.

The BSM is such that each of the component series has the representation

$$y_{it} = \mu_{it} + \gamma_{it} + x_{it}'\delta_i + \varepsilon_{it}, \quad i = 1,2; \;\; t = 1, \ldots, n, \;\; \varepsilon_{it} \sim \text{NID}(0, \sigma^2_{i\varepsilon})$$

where the series-specific trend, μ_{it}, is a local linear component

$$\begin{aligned} \mu_{i,t+1} &= \mu_{it} + \beta_{it} + \eta_{it}, \quad \eta_{it} \sim \text{NID}(0, \sigma^2_{i\eta}) \\ \beta_{i,t+1} &= \qquad\quad \beta_{it} + \zeta_{it}, \quad \zeta_{it} \sim \text{NID}(0, \sigma^2_{i\zeta}) \end{aligned} \tag{1}$$

The disturbances η_{it} and ζ_{it} are mutually and serially uncorrelated, but are contemporaneously correlated with the disturbances η_{jt} and ζ_{jt}, respectively, affecting the same equation of the trend for the other series.

The seasonal component, γ_{it}, arises from the combination of six stochastic cycles defined at the seasonal frequencies $\lambda_j = 2\pi j/12$, $j = 1, \ldots, 6$, λ_1 representing the fundamental frequency (corresponding to a period of 12 monthly observations) and the remaining being the five harmonics (corresponding to periods of 6 months, i.e., two cycles in a year, 4 months, i.e., three cycles in a year, 3 months, i.e., four cycles in a year, 2.4, i.e., five cycles in a year, and 2 months)

$$\gamma_{it} = \sum_{j=1}^{6} \gamma_{ijt}, \qquad \begin{bmatrix} \gamma_{ij,t+1} \\ \gamma^*_{ij,t+1} \end{bmatrix} = \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{bmatrix} \gamma_{ij,t} \\ \gamma^*_{ij,t} \end{bmatrix} + \begin{bmatrix} \omega_{ij,t} \\ \omega^*_{ij,t} \end{bmatrix}, \qquad j = 1, \ldots, 5 \tag{2}$$

and $\gamma_{i6,t+1} = -\gamma_{i6t} + \omega_{i6t}$. For the ith series, the disturbances ω_{ijt} and ω^*_{ijt} are normally and independently distributed with common variance $\sigma^2_{i\omega}$ for $j = 1, \ldots, 5$, whilst $\text{Var}(\omega_{i6t}) = \text{Var}(\omega^*_{i6t}) = 0.5\,\sigma^2_{i\omega}$ (see Proietti 2000 for further details).

The symbol ε_{it} denotes the irregular component, which is taken to be idiosyncratic in that it is assumed to be uncorrelated with the irregular component in the other series. This restriction, which is not critical and which can be removed if one wishes to, assigns this source of variation to series-specific measurement error.

The vector x_t contains the values of K regressors accounting for calendar effects, which will be specified in Section 4; δ_i is a vector of unknown regression coefficients for the ith series.

According to the model specification, the indicator variable y_{1t} and the national account flow y_{2t} form a *Seemingly Unrelated Time Series Equations* system (Harvey 1989). There is no cause and effect relationship between them, but they are subject to the same underlying economic environment. In particular, the first series can be viewed as a partial, possibly noisier, measurement of the same underlying phenomenon.

3. The Effects of Temporal Aggregation on the Seasonal Component

The flow series y_{2t} is not observed at the original frequency. If it is assumed that time $t = 1$ represents January, the actual observations pertain to the yearly aggregate series

$$Y_{2\tau} = \sum_{k=0}^{11} y_{2,12\tau-k}, \quad \tau = 1, 2, \ldots, [n/12]$$

where $[a/b]$ denotes the integer part of a/b. If the bivariate BSM holds for the logarithms of the series, rather than the original levels, a nonlinear measurement constraint arises; this situation is dealt with by Proietti and Moauro (2006).

If the original monthly series is seasonal, how will the component γ_{2t} contribute to the dynamics of $Y_{2\tau}$? It is helpful to think of $Y_{2\tau}$ as arising from a systematic sample of every twelfth observation of the filtered process $\sum_{j=0}^{11} y_{t-j}$. As the sum of 12 consecutive values of γ_{2t} is a zero mean invertible moving average process of order equal to 10 months, it immediately follows that the aggregation of γ_{2t}, $\sum_{k=0}^{11} \gamma_{2,12\tau-k}$, yields a white noise process. Indeed, the order of the moving average process is smaller than the sampling step, so that the process $\sum_{k=0}^{11} \gamma_{2,12\tau-k}$, $\tau = 1, 2, \ldots, [n/12]$, has zero mean, constant variance and zero autocorrelation at all annual leads and lags. Without the aid of external information on the indicator series, this white noise process would be indistinguishable from the aggregation of the series specific measurement error, that is $\sum_{k=0}^{11} \varepsilon_{2,12\tau-k}$.

As the seasonal disturbances in y_{2t} are contemporaneously correlated with those driving the seasonal component in the indicator, the bivariate model could in principle identify the component resulting from the aggregation of γ_{2t}, as the white noise source of variation that is independent of $\sum_{k=0}^{11} \varepsilon_{2,12\tau-k}$ and that is due to the interaction with the disturbances ω_{1jt}'s.

However, in the situations typically occurring in practice, where seasonality has a slow and weak evolution and sample sizes are small, this source of variation is negligible to an extent that trying to disentangle it from the measurement error would be asking too much of the available data.

One possibility is to assume it away, as will soon be argued. An alternative feasible strategy is to borrow the seasonal pattern from the indicator as suggested by Moauro and Savio (2005). This is also the strategy adopted by national statistical institutes, who produce disaggregate estimates according to the regression framework

$$\hat{y}_{2t} = b_0 + b_1 y_{1t} + e_t$$

where b_0 and b_1 are the generalised least squares estimates of the regression coefficients based on the Chow and Lin (1971) model, and e_t is the distribution residual.

The estimates $\hat{y}_{2t}$ are referred to as the "raw" estimates; quotation marks are necessary here, since the seasonality of the disaggregated series is a statistical artifact. The underlying assumption is that the seasonal component in the national accounts aggregate is proportional to that in the indicator, the factor of proportionality being the same b_1 which relates the annual series. As a matter of fact, it must be recalled that the Chow–Lin regression coefficients are estimated on the aggregate, e.g., annual, data, which do not contain information about seasonality. Trading day adjusted series are produced by the same scheme, in which y_{1t} is replaced by the series corrected for the TD component.

The conditions under which the seasonal behavior of the aggregate series can be borrowed from y_{1t} via standard generalised regression are indeed rather stringent. Not only are common seasonal features required, but we also need to impose restrictions on the covariance structure of the nonseasonal component, as is illustrated below.

Denoting by $z_t = [z_{1t}, z_{2t}]'$ the nonseasonal component, we rewrite the disaggregate bivariate model as $y_{it} = z_{it} + \gamma_{it}$, $i = 1, 2$. Assume now that $\gamma_{2t} = \lambda\gamma_{1t}$ (i.e., the seasonal component of the second series is proportional to that of the first series) and that the nonseasonal component follows a seemingly unrelated system of equations

$$z_t = \frac{\theta(L)}{\phi(L)}\kappa_t, \quad \kappa_t \sim \mathrm{NID}(0, \Sigma_\kappa), \;\; \Sigma_\kappa = \begin{pmatrix} \sigma^2_{1\kappa} & \sigma_{12,\kappa} \\ \sigma_{12,\kappa} & \sigma^2_{2\kappa} \end{pmatrix} \tag{3}$$

where $\theta(L)$ and $\phi(L)$ are suitable scalar lag polynomials. If z_t results from the sum of several orthogonal components, $z_t = \sum_j (\theta_j(L)/\phi_j(L))u_{jt}$, $u_{jt} \sim \mathrm{NID}(0, \Sigma_{ju})$, such as $z_t = \mu_t + \varepsilon_t$, then (3) requires *homogeneity* (see Harvey 1989, Section 8.3), which amounts to having $\Sigma_{ju} = q_j\Sigma_\kappa$, where the q_j's are suitable proportionality factors. If further $\sigma_{12,\kappa} = \lambda\sigma^2_{1\kappa}$, then it is possible to write

$$y_{2t} = \lambda y_{1t} + z^*_{2t}, \quad z^*_{2t} = \frac{\theta(L)}{\phi(L)}\kappa^*_{2t}, \;\; \kappa^*_{2t} \sim \mathrm{NID}(0, \sigma^2_{2\kappa} - \lambda^2\sigma^2_{1\kappa})$$

and thus we can safely attribute the portion λ of the seasonality in the indicator to the aggregate series. The restriction $\sigma_{12,\kappa} = \lambda\sigma^2_{1\kappa}$ implies that for Σ_κ to be positive semidefinite λ has to lie in the interval $\left[-\sigma_{2\kappa}/\sigma_{1\kappa}, \sigma_{2\kappa}/\sigma_{1\kappa}\right]$.

We believe that the strategy of giving up the idea of estimating the seasonality in y_{2t} altogether is more neutral. Thus, in the sequel we shall assume that

$$\sum_{k=0}^{11} \gamma_{2,12\tau-k} = 0 \tag{4}$$

in lieu of $\mathrm{E}\left(\sum_{k=0}^{11}\gamma_{2,12\tau-k}\right) = 0$. Notice that (4) strictly holds when seasonality is deterministic (that is, $\sigma^2_{2\omega} = 0$). In situations where seasonality is not rapidly changing, our assumption seems plausible.

In the light of the previous discussion, the "raw" series are more a statistical artifact than a useful addition to the available menu of official economic statistics. If the primary interest of the investigation were the seasonal fluctuations on their own, it would seem more informative to investigate the monthly indicators from the outset.

A final important point arises as a consequence of (4). The simplification preserves the accounting relationship that the sum of the disaggregated series over 12 months adds up exactly to the annual total, which would not hold otherwise. As for the series corrected for the calendar component, this would sum up to the annual estimate with the calendar effects removed.

In conclusion, the proposed solution has the additional merit of complying with the recommendation of the Eurostat/ECB task force concerning time consistency with annual data (Recommendation 3.c):

> *Time consistency of adjusted data should be maintained for practical reasons. The reference aggregates should be the annual total of quarterly raw data for seasonally adjusted data and annual total of quarterly data corrected for trading day effects for seasonally and trading day adjusted data. Exceptions from the time consistency may be acceptable if the seasonality is rapidly changing.*

4. Calendar Components

Calendar effects have been introduced as regression effects in the model equation for y_{it}. Three sets of regressors are defined to account for each of the three sources of variation mentioned in the introduction.

Trading day (working day) effects occur when the level of activity varies with the day of the week, e.g., it is lower on Saturdays and Sundays. Letting D_{jt} denote the number of days of type j, $j = 1, \ldots, 7$, occurring in month t and assuming that the effect of a particular day is constant, the differential trading day effect for series i is given by

$$\mathrm{TD}_{it} = \sum_{j=1}^{6} \delta_{ij}(D_{jt} - D_{7t})$$

The regressors are the differential number of days of type j, $j = 1, \ldots, 6$, compared to the number of Sundays, to which Type 7 is conventionally assigned. The Sunday effect on the ith series is then obtained as $\left(-\sum_{j=1}^{6} \delta_{ij}\right)$. This expedient ensures that the TD effect is zero over a period corresponding to multiples of the weekly cycle.

The regressors are then corrected to take into account the national calendars. For instance, if Christmas falls on a Monday and for a particular application a holiday can be assimilated to a Sunday, one unit should be deducted from D_{1t} and reassigned to D_{7t}. This type of correction is recommended by Eurostat and is adopted in this article, giving

$$\mathrm{TD}_{it} = \sum_{j=1}^{6} \delta_{ij}^{*}(D_{jt}^{*} - D_{7t}^{*})$$

It is often found that the effect of the working day from Monday to Friday is not significantly different and that it helps to avoid collinearity among the regressors to assume that $\delta_{ij}^{*} = \delta_{i}^{*}$ for $j = 1, \ldots, 5$. In such case a single regressor can validly be employed, writing

$$\mathrm{TD}_{it} = \delta_{i}^{*} D_{t}^{*}, \quad D_{t}^{*} = \sum_{j=1}^{5} D_{jt}^{*} - \frac{5}{2}(D_{6t}^{*} + D_{7t}^{*})$$

As far as moving festivals are concerned, the only occurrence for Italy is Easter. Its effects are modelled as $E_t = \delta h_t$, where h_t is the proportion of the 7 days before Easter that fall in month t. Subtracting the long run average, computed over the first 400 years of the Gregorian calendar (1583–1982), from h_t yields the regressor $h_t^* = h_t - \bar{h}_t$, where $\bar{h}_t$ takes the values 0.354 and 0.646 respectively in March and April, and zero otherwise. Finally, the length of month (LOM) regressor results from subtracting from the number of days in each month, $\sum_j D_{jt}$, its long-run average, which is 365.25/12.

What are the consequences of temporal aggregation from the monthly frequency to the annual one? The holiday effect becomes constant ($h_\tau = 1$, $h_\tau^* = 0$), whilst the LOM regressor takes the value 3/4 in leap years and $-1/4$ in normal years, describing a four-year cycle, which is an identifiable, though not necessarily significant, effect. Moreover, as shown by Cleveland and Devlin (1980), the presence of trading day effects in a monthly time series induces a peak in the spectrum at the frequency $0.348 \times 2\pi$ in radians, and a secondary peak at $0.432 \times 2\pi$. For yearly data the relevant frequencies are $0.179 \times 2\pi$ and $0.357 \times 2\pi$, corresponding to a period of 5.6 years and 2.8 years, respectively. In conclusion, the presence of a calendar component in yearly data produces peaks at the frequencies 0.358π (TD), 0.5π (leap year), 0.714π (TD) and π (leap year).

In conclusion, the calendar component has detectable effects on the annually aggregated time series $Y_{2\tau}$, with the notable exception of moving festivals. As a consequence, one possibility is to estimate its effects by including in the time series equation for the second variable the component $x_{2t}'\delta_2^*$ among the fixed effects. It is understood that x_t will be aggregated to a yearly series. An alternative parsimonious strategy is to assume that $\delta_2^* = \kappa\delta_1^*$ for a scalar κ, which amounts to assuming that the calendar effects on the second series are proportional to those affecting the first. This would require the estimation of a single coefficient. The difference with the approach of estimating the vector δ_2^* without the above restriction is that the disaggregated time series including the calendar component would feature the Easter effect. The latter, which would otherwise be absent, as it is nonidentifiable without imposing that particular restriction, is proportional to the Easter effect on the series y_{1t}, κ being the proportionality factor.

5. Statistical Treatment

The state space methodology provides the necessary inferences, starting from the estimation of unknown parameters, such as the variances of the disturbances driving the components, the regression coefficients, the estimation of the disaggregated values y_{2t} and the assessment of their reliability. Moreover, diagnostic checking can be carried out on the model's innovations, so as to detect any departure from the stated assumptions and possibly carry out the corrective action against it.

As a first step, the monthly bivariate model, with temporal aggregation concerning solely the second variable, is cast in the state space form using an approach due to Harvey (1989, Section 6.3), which translates the aggregation problem into a missing value problem. According to this approach, the following cumulator variable is defined for the second variable:

$$y^c_{2t} = \psi_t y^c_{2,t-1} + y_{2t}, \quad \psi_t = \begin{cases} 0, & t = 12(\tau - 1) + 1, \quad \tau = 1, \ldots, [n/12] \\ 1, & \text{otherwise} \end{cases} \tag{5}$$

In the case of monthly flows whose annual total is observed,

$$\begin{array}{llll} y^c_{21} = y_{21}, & y^c_{22} = y_{21} + y_{22}, & \cdots & y^c_{2,12} = y_{21} + \cdots + y_{2,12}, \\ y^c_{2,13} = y_{2,13}, & y^c_{2,14} = y_{2,13} + y_{2,14}, & \cdots & y^c_{2,24} = y_{2,13} + \cdots + y_{2,24}, \\ \vdots & \vdots & \ldots & \vdots \end{array}$$

Only a systematic sample of every 12th value of y^c_{2t} process is observed, $y^c_{2,12\tau}$, $\tau = 1, \ldots, [n/12]$, so that all the remaining values are missing.

The cumulator is included in the state vector and the state space representation if formed. The associated algorithms, and in particular the Kalman filter and smoother, are used for likelihood evaluation and for the estimation of the missing observations and thus of the disaggregated values of the series. The smoothed estimates of the monthly series are then aggregated to the quarterly frequency. All the computations concerning the illustrations presented in the next section were carried out in Ox.[3] The statistical treatment of the model was performed using the augmented Kalman filter and smoother due to de Jong (1991), (see also de Jong and Chu-Chun-Lin 1994), suitably modified to take into account the presence of missing values, which is accomplished by skipping certain updating operations. The unknown parameters are estimated by maximum likelihood, where the likelihood is evaluated with the support of the augmented Kalman filter. More technical details, which we purposively omit for the sake of brevity, and computer programs are available from the authors.

6. Illustrations

Recently, ISTAT has set up a Commission for the assessment of current temporal disaggregation practices and the proposal of methodological advances in this important area. A set of representative national accounts series were selected and submitted to the Commission as case studies. The applications that we present in this section refer to this set.

Our first illustration deals with the problem of disaggregating the annual production resulting from the National Accounts (NA) for the electrical and optical equipment industry (subsection DL of the Nace Rev.1 economic activity classification), hereinafter denoted $Y_{2\tau}$, using the related information available from the monthly industrial production (IP) index of the same industry, denoted y_{1t}. The NA annual aggregate is measured at constant prices (1995 = 100), and covers the years from 1977 to 2003. The monthly IP index (base year 2000 = 100) is available in seasonally unadjusted form for

[3] Ox is a matrix programming language developed by J.A. Doornik (2001).

the sample period January 1977–December 2003. The original series are plotted in Figure 1 (the NA production series has been rescaled to fit in the same graph).

The bivariate basic structural time series model discussed in Section 2, subject to the temporal aggregation constraint for the NA series, was estimated by maximum likelihood. The variance matrix of the slope disturbances ζ_{it} was estimated equal to a zero matrix and thus the dynamic representation of the trend component is a random walk with constant drift. The estimates of the parameters are

$$\hat{\sigma}_{1\eta} = 1.334 \quad \hat{\sigma}_{2\eta} = 6.137 \quad \hat{\rho}_{\eta} = 0.737$$
$$\hat{\sigma}_{1\varepsilon} = 1.916 \quad \hat{\sigma}_{2\varepsilon} = 0.035$$
$$\hat{\sigma}_{1\omega} = 0.224$$

ρ_η denotes the correlation between the trend disturbances η_{1t} and η_{2t}. The suffix 1 is used for the IP index, whilst 2 refers to the NA series. The trend disturbances are positively correlated, although the correlation is not perfect, which suggests that the series are not cointegrated. Seasonality is present only in the time series equation for the monthly indicator. The nonzero value for the seasonal variance parameter $\sigma^2_{1\omega}$ indicates that the seasonal pattern changes in the sample period. Anyway, the size of the parameter estimates hints that the variation in the seasonal disturbances is not very large, relative to the other disturbances. As argued before, the assumption that seasonality is absent from the second series appears to us more suitable in this framework.

Figure 2 plots the estimated components of the IP monthly series resulting from the application of the Kalman filter and smoother to the relevant state space model using the maximum likelihood estimates. The seasonal pattern extracted for the monthly industrial production index is plotted in the top right-hand panel of Figure 2, whilst the seasonally adjusted series is displayed in the top left panel.

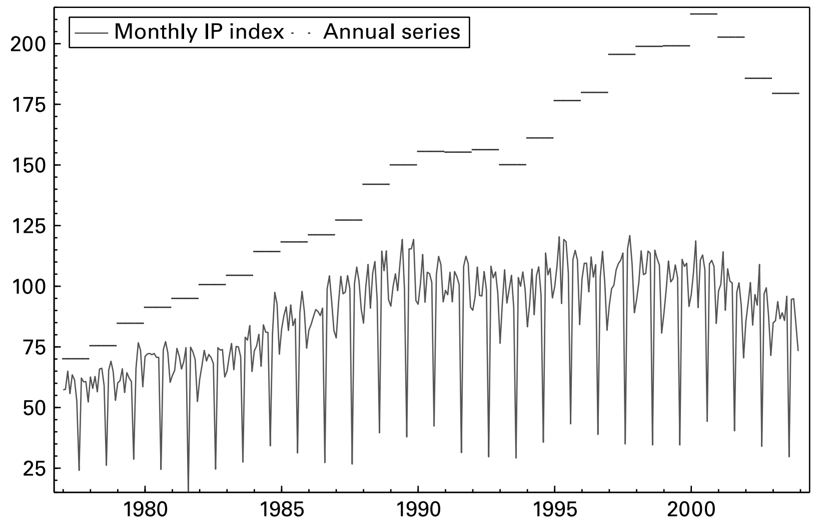

Fig. 1. Annual production (rescaled by 10^{-1}) and monthly industrial production index for electrical and optical equipment

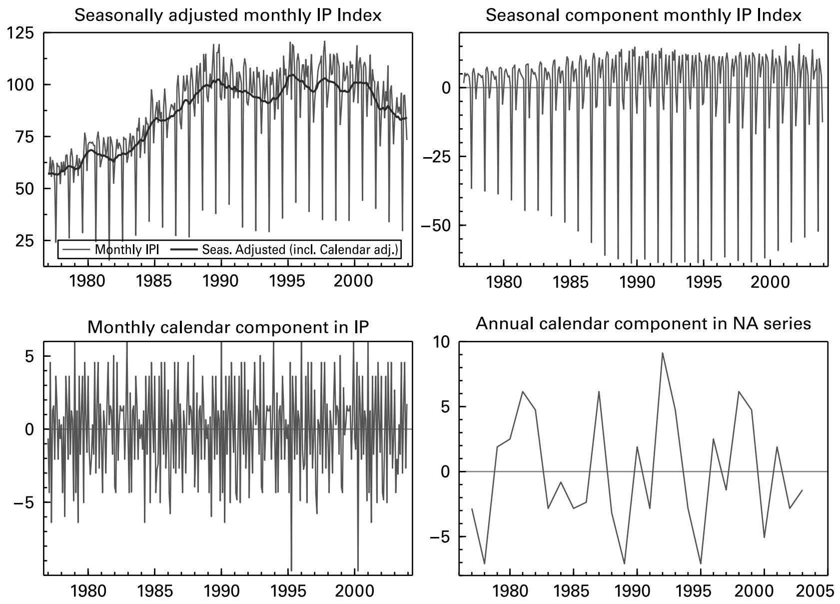

Fig. 2. Bivariate BSM model for the industrial production index (monthly) and the NA production series (annual) for electrical and optical equipment. Seasonally adjusted series and seasonal component of monthly indicator (upper panels). Monthly and quarterly calendar effects (lower panels)

The model specification also includes 3 regressors representing the calendar effects: the single trading days regressor D_t^*, the Easter variable h_t^* using seven days before Easter, and the length of the month (LOM) variable. The trading day variable accounts for the Italian specific holidays (e.g., New Year's Day, Easter Monday, First of May, 8th of December, Christmas), which are regarded as Sundays. The estimated coefficients for the industrial production index, denoted respectively by $\hat{\delta}_1^*$, $\hat{\delta}_1^{\text{Easter}}$ and $\hat{\delta}_1^{\text{LOM}}$, are

$$\underset{(0.060)}{\hat{\delta}_1^* = 0.946}, \qquad \underset{(0.781)}{\hat{\delta}_1^{\text{Easter}} = -2.774}, \qquad \underset{(1.252)}{\hat{\delta}_1^{\text{LOM}} = 2.026}$$

where in parenthesis are reported the standard errors. The estimated calendar effect is shown in the bottom panels of Figure 2. All the parameters are significant, with the exception of LOM, and have the expected signs.

For the second series the calendar effects have been restricted to be proportional to the coefficients of the indicators. The scale factor κ was significant with estimated value $\hat{\kappa} = 2.955$ and standard error 0.238. We may thus conclude that the calendar effect is significant for the NA aggregate. The calendar component for the NA production series is obtained by multiplying by 2.955 the estimates reported in the bottom right panel of Figure 2.

One of the advantages of the state space approach outlined in this article is the availability of a set of diagnostics that are immediately available from the output of the Kalman filter. It is important to stress that such diagnostics based on the innovations are not immediately available in the regression implementation of the Chow–Lin approach. Figure 3 plots the Kalman filter innovations for the IP series and the NA aggregate, along with 2 standard deviation error bounds and the density of standardised innovations. Diagnostics based on these values suggest that the fit is satisfactory for both the equations. In particular, the Box–Ljung statistic based on 15 and 6 autocorrelations for the two series is equal to 10.078 and 0.183, respectively. The Bowman–Shenton normality test gives 8.054 for the IP index and 0.915 for the annual NA aggregate.

The smoothed estimates of the disaggregate NA production series are available at both the monthly and quarterly observation frequency. They are presented in unadjusted form in Figure 4, along with their 95% upper and lower confidence limits. The size of the confidence interval embodies the uncertainty surrounding the estimation of the calendar effects (but not that ascribed to the estimation of the hyperparameters – namely the variance parameters).

The quarterly estimates, adjusted for calendar effects, are presented and compared to the raw ones in the last two panels of Figure 4. The last plot refers to the estimated growth rates on an annual basis and highlights not only that the adjusted series is smoother, but also the adjustment influences the location and sharpness of turning points.

In this first illustrative example we were able to identify and estimate the calendar component. An important empirical issue is whether this finding is general and can be extended to a wider range of case studies. For this purpose we extended our application to the remaining 14 case studies making up the ISTAT database, which refers to manufacturing. For each NA annual series, the monthly IP index for the same variable is available. The bivariate BSM was fitted to all the industries, such that the calendar component for the monthly series is $C_{1t} = \delta_1^* D_t^* + \delta_1^{\text{Easter}} h_t^* + \delta_1^{\text{LOM}}\text{LOM}$, whilst for the aggregate series we assume that $C_{2t} = \kappa C_{1t}$. The estimation results, reported in Table 1, display the maximum likelihood estimates of the coefficients associated to the calendar

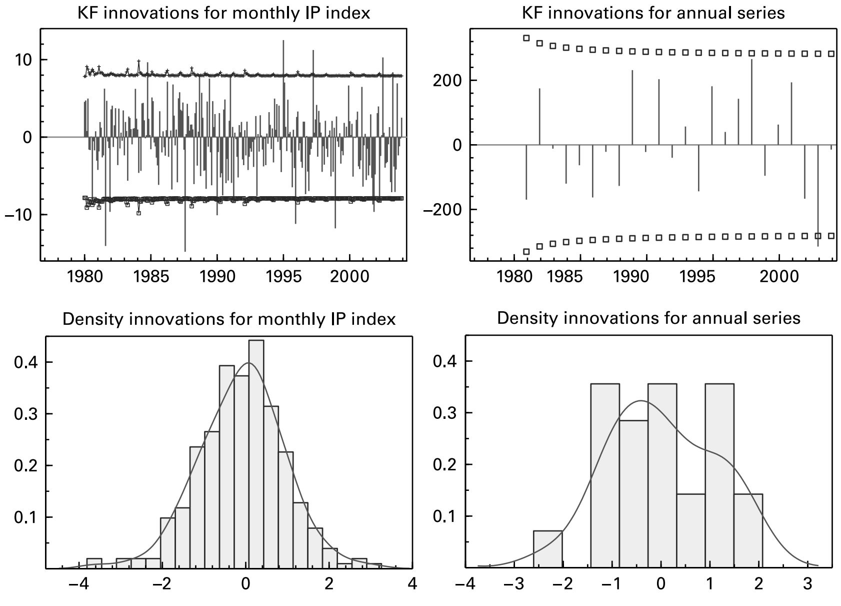

Fig. 3. Kalman filter innovations of the model for production of electrical and optical equipment

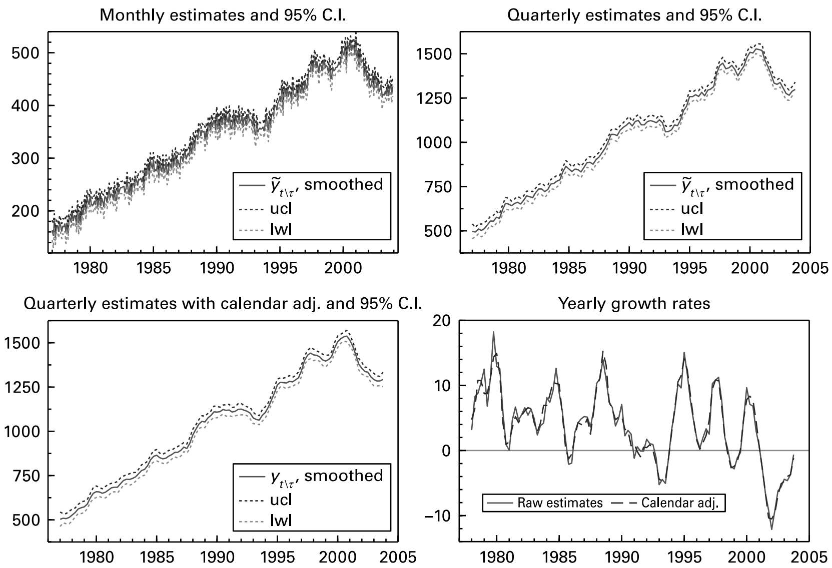

Fig. 4. Temporal disaggregation of annual production electrical and optical equipment by a bivariate BSM formulated at the monthly frequency. Monthly and quarterly disaggregated production series

regressors for the monthly series and of the proportionality factor κ. The symbol $(-)$ is used to signify that the estimate is not significant.

The evidence is clear-cut. The calendar components are always significant for the monthly indicator, whereas as far as the annual aggregate is concerned, in 9 cases $\hat{\kappa}$ is not significantly different from zero, and in three cases the estimated coefficient is positive but unreliable. The reliability was assessed by a rolling estimation experiment, according to which the coefficient κ is reestimated adding one observation at a time. If the estimates are unstable then we consider them unreliable. This is the case for *Chemical products* and *Mining and quarrying*. In the remaining applications the estimates were both significant and stable.

In conclusion, what emerges from the experiment is that moving from a finer timing interval towards a larger one reduces the accuracy of the estimates of calendar components to the extent that these are no longer detectable under annual temporal aggregation. Despite the fact that from a theoretical standpoint the calendar components are observable over the annual period, their estimation is not an easy matter. In these situations it may be advisable to give up the estimation of a disaggregated series adjusted for the calendar component.

7. Comparison with Current Practice

In this section we compare our proposal (which will be referred to as the BSM method) with the methodology currently adopted by some European National Statistical Institutes for estimating the quarterly national accounts. As hinted in the introduction, the latter is a multi-step procedure requiring a separate seasonal and calendar adjustment of the monthly indicator and two distinct distributions of the annual aggregate to the quarters, typically by

Table 1. Estimation of calendar components

Industry	$\hat{\delta}_1^*$	$\hat{\delta}_1^{\text{Easter}}$	$\hat{\delta}_1^{\text{LOM}}$	$\hat{\kappa}$
Mining and quarrying	0.552 (0.094)	–	–	1.714 (0.038)
Food, beverages and tobacco	0.767 (0.048)	−2.264 (0.685)	–	–
Textiles	0.924 (0.063)	−1.721 (0.922)	4.433 (1.464)	–
Leather products	1.071 (0.078)	−1.931 (1.068)	5.002 (1.714)	–
Wood products	0.762 (0.065)	−1.717 (0.948)	4.703 (1.507)	–
Paper, publishing and printing	0.430 (0.047)	−2.265 (0.712)	–	–
Chemical products	0.403 (0.042)	−1.292 (0.632)	–	23.278 (0.613)
Rubber and plastic products	0.769 (0.045)	−2.255 (0.647)	2.126 (1.026)	3.732 (0.154)
Nonmetallic mineral products	0.561 (0.039)	−0.983 (0.561)	2.548 (0.910)	–
Metal products	0.881 (0.060)	−1.864 (0.923)	2.267 (1.457)	–
Machinery	0.713 (0.052)	−2.075 (0.723)	2.317 (1.161)	7.431 (0.537)
Electrical and optical equipment	0.947 (0.060)	−2.774 (0.800)	2.024 (1.283)	2.955 (0.238)
Cars	0.969 (0.090)	−1.935 (1.398)	2.396 (2.232)	1.060 (0.237)
Other transport equipment	0.966 (0.090)	−1.930 (1.326)	4.249 (2.135)	–
Other manufacturing products	0.733 (0.080)	–	2.106 (1.883)	–

performing a Chow–Lin regression on the quarterly raw and adjusted indicator. Henceforth it will be referred to as the MS (multi-step) method. For the comparison we turn back to our original case study concerning the disaggregation of annual production of the electrical and optical equipment industry, using the monthly industrial production index as an indicator.

The quality of the two disaggregation strategies is assessed by looking at their revision histories. The revision histories are generated as follows. Starting from the year 1997 we perform a rolling nowcast experiment such that at the beginning of each year τ nowcasts are made for its four quarters, using the annual information available up to the year $\tau - 1$ for the annual aggregate and the information on the monthly indicator up to and including the quarter of interest. At the same time the estimates concerning the four quarters of the previous year, $\tau - 1$, are revised. This exercise assumes that the annual aggregate for the year $\tau - 1$ accrues between the end of the fourth quarter of the year $\tau - 1$ and the first quarter of the year τ. Let us denote by $\hat{Y}_{\tau,j}, j = 1, 2, 3, 4$, the estimates of quarterly production conditional on the annual total of year $\tau - 1$, and by $\tilde{Y}_{\tau,j}$, $j = 1, 2, 3, 4$, those conditional also on the annual total of the year τ; their difference, $\hat{Y}_{\tau,j} - \tilde{Y}_{\tau,j}$, denotes the revision due to the accrual of the information on the annual total.

We shall base the comparison on the seasonally adjusted production levels, not corrected for the calendar effects. This choice is motivated by the fact that the estimates $\tilde{Y}_{\tau,j}$ arising from the two methods add up to the same annual total. The quarterly nowcasts from the BSM method result from the aggregation of the three monthly nowcasts. For the MS method, seasonal adjustment of the indicator is carried out in real time by the TRAMO-SEATS procedure (see Gómez and Máravall 1996). The latter automatically identifies the Airline model on the monthly levels of the indicator and uses exactly the same calendar regressors as the BSM method. For the temporal disaggregation step of the MS method we consider three variants of the Chow–Lin procedure, which differ solely for the regressors set, which includes, along with the quarterly seasonally adjusted indicator, a constant and a linear trend.

The models are reestimated by maximum likelihood in real time, as a new annual observation becomes available. At the end of the experiment seven sets of revision errors are available for four horizons (one quarter to four quarters). These are employed to construct the root mean squared revision error (RMSRE)

$$\mathrm{RMSRE}_j = \left[\frac{1}{7}\sum_{\tau=1}^{7}(\hat{Y}_{\tau,j} - \tilde{Y}_{\tau,j})^2\right]^{1/2}$$

We also compute the root mean squared revision error for the estimated quarterly and yearly growth rates

$$\mathrm{RMSRE}_j^{(k)} = \left[\frac{1}{7}\sum_{\tau=1}^{7}\left(\hat{g}_{\tau,j}^{(k)} - \tilde{g}_{\tau,j}^{(k)}\right)^2\right]^{1/2}, \quad k = q, y$$

where $\hat{g}_{\tau,j}^{(q)}$ is the quarterly growth rate estimated without using the year τ annual total, where, for instance, $\hat{g}_{\tau,j}^{(q)} = (\hat{Y}_{\tau,j} - \hat{Y}_{\tau,j-1})/\hat{Y}_{\tau,j-1}$, $j > 1$, and $\hat{g}_{\tau,1}^{(q)} = (\hat{Y}_{\tau,1} - \hat{Y}_{\tau-1,4})/\hat{Y}_{\tau-1,4}$,

$\hat{g}^{(y)}_{\tau,j}$ denotes the yearly growth rate, and $\tilde{g}^{(q)}_{\tau,j}$ and $\tilde{g}^{(y)}_{\tau,j}$ denote, respectively, the quarterly and yearly growth rates computed on the series which uses the annual total for year τ.

The RMSRE measures are presented in Table 2 for the four quarterly horizons. The last column, labelled "Annual," refers to the estimation of the annual aggregate level and evaluates the mean squared revision $\sum_{j=1}^{4} \hat{Y}_{\tau,j} - Y_{\tau}$, where the subtrahend is the observed annual total. Dividing by $Y_{\tau-1}$ we obtain the RMSRE concerning the annual growth rates, also reported in the last column. The overall evidence is that our proposed one-step disaggregation methodology based on the bivariate BSM outperforms the multi-step methodology, since it provides more accurate estimates in that the revisions concerning the levels and the growth rates display on average smaller size. It should also be noted that for the Chow–Lin disaggregation the inclusion of a constant or a linear trend does not improve upon the specification with no deterministic variable.

The results of this comparison are by no means conclusive, as they refer to a particular case study, although they extend to the series considered in the previous section. They nevertheless suggest that the proposed methodology stands up to the comparison with the traditional methodology, which is much more elaborate. We also recall that the estimation error variance of the estimates, which incorporates the uncertainty arising from using the seasonally adjusted estimates, is not available in the MS method. We end our discussion by comparing in Figure 5 the estimated disaggregated series arising from the BSM and the MS. The remarkable feature emerging from this comparison is that the former is smoother than the latter. This is the likely consequence of the different ways in which the calendar effects are treated by the two methods. In the BSM case these were restricted to be proportional to the indicators, and the estimated scale factor κ turned out to be significant. Thus to a certain extent the calendar effects are treated separately from the other components. For the MS method, on the contrary, the Chow–Lin regression is conducted on the indicator adjusted for seasonality but not for the calendar component. As a result the

Table 2. Comparison of the root mean squared revision error in the estimation of the levels and the growth rates of quarterly production for electrical and optical equipment. The rolling nowcast experiment was carried out over the years 1997–2003

	1 step	2 steps	3 steps	4 steps	Annual
Levels: $RMSRE_j$					
BSM	36.7185	47.2979	56.2062	64.4361	51.0829
Chow–Lin	33.8068	50.7653	55.0599	66.4720	51.2611
Chow–Lin (constant)	33.7470	50.7120	56.2294	66.2994	51.5216
Chow–Lin (trend)	41.0146	58.9855	65.7329	73.6705	59.7332
Quarterly growth rates: $RMSRE^{(q)}_j$					
BSM	1.9263	1.6960	1.9392	0.9210	
Chow–Lin	2.2247	1.3367	2.6722	1.2528	
Chow–Lin (constant)	2.2550	1.3470	2.5923	1.1221	
Chow–Lin (trend)	2.7474	1.5951	2.5727	0.8363	
Yearly growth rates: $RMSRE^{(y)}_j$					
BSM	3.4218	4.0437	4.3221	3.6585	0.9088
Chow–Lin	3.4585	4.2064	4.7636	3.7258	0.9481
Chow–Lin (constant)	3.4554	4.2611	4.7791	3.7622	0.9517
Chow–Lin (trend)	4.3576	4.8933	5.3069	3.9270	1.0783

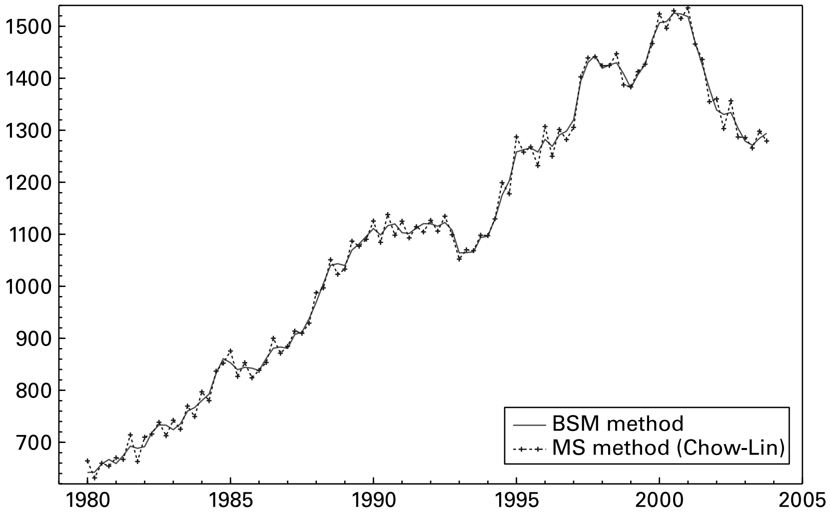

Fig. 5. Comparison between the quarterly seasonally adjusted estimates (no calendar adjustment) of production for electrical and optical equipment, using the direct BSM method and the multi-step (MS) methodology currently in use at ISTAT

calendar effects that are present in the indicator are fully transferred to the estimated quarterly NA series along with the other components and are not treated as a separate component.

8. Conclusions

This article has proposed a disaggregation strategy for the estimation of quarterly national account series that has several advantages over current practice. The strategy is a novel application of the ideas contained in Harvey (1989) and Harvey and Chung (2000).

The estimates of the quarterly national accounts aggregates originate from fitting a multivariate structural time series model formulated at the monthly interval; the model relates the annual national account series to the corresponding monthly indicator. The monthly frequency allows more accurate estimation of the calendar effects. Maximum likelihood estimation of the unknown parameters, the estimation of the disaggregated observations and their reliability, diagnostic checking and the assessment of goodness of fit are achieved through the state space methodology.

The approach proposed in this article has several advantages over the practice followed by national statistical institutes for the estimation of quarterly national accounts variables. First and foremost it yields simultaneously "raw" and adjusted (for seasonality and calendar components) estimates without the need to iterate the disaggregation procedure. The measurement model is transparent, as it takes into account the presence of components of interest, whereas the treatment of seasonality and calendar components is not explicit in the iterations of the Chow–Lin method, upon which the current practice is based. Moreover, from a more philosophical standpoint, the approach has the merit of moving away from the exogeneity assumption underlying the disaggregation methods based on a regression framework, such as Chow and Lin (1971), according to which the indicator is considered as an explanatory variable. Finally, although we have illustrated the bivariate case, which is nevertheless the leading case of interest for statistical agencies, the approach can be extended to higher-dimensional systems and other frequencies of observations.

Our study has also cast some light on the limitations that are faced in the problem of temporal disaggregation from the annual frequency. The limitations are related to the statistical identifiability of a particular component. There are two facets that need to be considered.

As far as seasonality is concerned, our discussion has spelled out the kind of restrictions that one has to impose on the bivariate model of the aggregate series and its indicator, for the optimality of the current practice of producing quarterly seasonal estimates, inappropriately termed "raw," borrowing the seasonal pattern from an indicator series. Once the restrictions are understood, it is reasonable to conclude that they are indeed very stringent and, perhaps, implausible. Therefore we have proposed forgetting about estimating the seasonal component for the aggregate time series altogether, and concentrating on the estimation of a quarterly (or monthly if one wishes) nonseasonal series. The idea that the contribution of seasonality to the variability of an annual aggregate is negligible appears more neutral and inevitable, perhaps, since that contribution is not easily identifiable.

Secondly, the calendar effects associated with trading days and the leap years are measurable. Nevertheless, extensive experimentation has shown that either the estimates of these effects are not very accurate or they are insignificant for the aggregate series. In these situations, our preferred strategy is to assume that they are absent from the latter. Be that as it may, the bivariate structural model will select from the monthly indicator the information, devoid of the seasonal and calendar components, that is needed for the disaggregation of the annual series.

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