Journal of Official Statistics
Vol. 7. No. 2, 1991, pp. 181–194
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The Autocorrelation of Residuals from the X11ARIMA Method

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Abstract: The problem of significant autocorrelation in the residuals of X11ARIMA and the U.S. Bureau of the Census X11 variant as well has generated much controversy.

This paper shows that the presence of significant autocorrelations at certain lags is not necessarily an indication of inadequacy of the methods. It is recognized that residuals from reasonable decompositions can contain some non-zero autocorrelations given the effect of the linear filters on white noise irregulars. In many cases, however, significant autocorrelation can generally be corrected by using different trend-cycle and seasonal filters, removing trading-day variations and Easter effects, and by using an additive decomposition model. The elimination of significant autocorrelation, however, is not recommended as a goal in itself. It should not be done without regard to the quality of the seasonally adjusted values and a priori information concerning the generating structure of the series under investigation.

Key words: Trend-cycle filters; seasonal filters; trading-day variations.

1. Introduction

The presence of autocorrelation in the residuals from time series decomposition methods has preoccupied researchers and practitioners for a long time and has been the topic of much controversy. Among several authors, Granger (1978), Nerlove (1964) and Pierce (1978) pointed out the presence of negative autocorrelated values of the residuals at the seasonal lags or equivalently, dips in the seasonal frequency bands of the seasonally adjusted series using either the U.S. Bureau of Labor Statistics or the U.S. Bureau of the Census X11 Method. These authors then concluded that the two methods overadjust for seasonality. On the other hand, commenting on Granger's paper, Sims (1978) agreed that it was reasonable to have dips given the low signal-to-noise ratio in the neighbourhood of the seasonal frequencies. Similarly, Tukey (1978) and Wecker (1978) maintained that the presence of dips in the spectrum did not indicate inadequacy of the method. Unless the white noise component has zero power at the seasonal frequencies the spectrum of the seasonally adjusted series will have dips at the seasonal frequencies. In fact, almost a decade earlier, Grether and Nerlove (1970) showed that all minimum mean squared

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Acknowledgement: We would like to thank two anonymous referees for their valuable comments to an earlier version of this paper.

error (MMSE) signal extraction estimators will produce dips at the seasonal frequencies.

In a comparative study of various seasonal adjustment methods for labor series, Bordignon (1988) found significant autocorrelated values of the residuals at lags 1 and 4 (for quarterly series) when using the X11ARIMA (Dagum 1980), BAYSEA (Akaike and Ishiguro 1980) and to lesser extent, SIGEX (Burman 1980). Similarly, Daddi and D'Esposito (1985) in a comparison between X11ARIMA and S.A.B.L. (Cleveland, Dunn, and Terpenning 1978) pointed out that both procedures left significant autocorrelated residuals at the trend and seasonal lags. Bell and Hillmer (1984) also found dips in the spectrum *implied* by the linear filters of X11 when applied to seasonally adjusted Employed Non-Agriculture Males aged 20 and over series. Commenting on Bell and Hillmer (1984), Ansley and Wecker (1984) showed how it is possible to construct an MMSE seasonal adjustment method that will not have dips in the spectrum but it will no longer be optimal. Burman (1980) showed that a reasonable decomposition can be done including a low order moving average model for the irregulars.

This investigation aims to analyse the autocorrelation of the residuals from the X11ARIMA/88 version (Dagum 1988) when the *standard option* (default) is applied; and to show how, for most cases, the presence of significant autocorrelation at certain lags can often be eliminated by choosing longer filter options or removing trading-day and Easter effects, while maintaining favourable diagnostics measures such as, the X11ARIMA Q, the months (quarters) for cyclical dominance MCD (QCD), and the average duration of run ADR.

Section 2 shows how the X11ARIMA residuals under white noise irregulars will be autocorrelated because of the linear filters applied by the method. It also reviews the test for autocorrelation of the residuals based on Bartlett's approximation of standard errors. This test is applied at each lag and the results obtained for a sample of monthly and quarterly series are given. Section 3 shows how significant autocorrelation at lag 1 can be corrected; Section 4 shows how significant autocorrelation at the seasonal lag can be corrected; Section 5 discusses the effect of trading-day variations in the autocorrelation of the residuals. Finally, Section 6 contains conclusions.

2. Test for the Autocorrelation of the Residuals

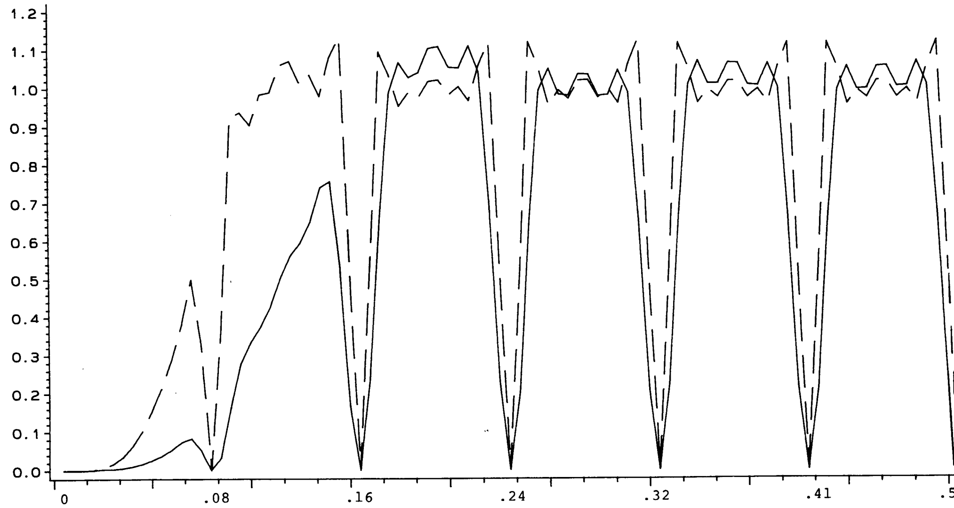
Similarly to the X11ARIMA/80, the X11ARIMA/88 computer package prints the sample autocorrelations of the residuals in table F2.G from lag 1 to lag 14 for monthly data; and from 1 to 6 for quarterly data. The estimate of the kth lag autocorrelation ρ_k is given by

$$r_k = 1/N \sum_{t=1}^{N-k} (I_t - \bar{I})(I_{t-k} - \bar{I})/S_I^2;$$

$$k = 0, 1, \ldots, 6 \text{ or } 14 \qquad (2.1)$$

where I_t, $\bar{I}$ and S_I^2 denote the tth residual, its mean and variance, respectively, and N is the series length. The residuals used to calculate r_k are given in table D13 of X11ARIMA/88. These residuals include outliers and all effects that have not been well estimated and removed from the original series estimated as seasonality, trend-cycle, trading-day, and Easter holiday variations.

Under the assumptions that the series is additively decomposed into trend-cycle plus seasonality plus irregulars and that no treatment of extreme values is done, the residuals will be autocorrelated because the X11ARIMA method is based on moving average techniques. The type of autocorrelation followed by the residuals depends on



—— standard (default) option.
- - - 3 × 9 seasonal moving average and 23-term Henderson.

Fig. 1. Gain functions of X11ARIMA symmetric filters for the irregulars

the kind of seasonal and trend-cycle filters applied.

For the symmetric filter of the default option of X11ARIMA/80, (3 × 3 m.a. and 3 × 5 m.a. for the seasonal factors and 2 × 12 m.a. and 13-term Henderson for the trend-cycle), the autocorrelation of the residuals (assuming the irregulars affecting the series are white noise) are as follows:

$$\rho_1 = -0.34 \quad \rho_9 = 0.02$$
$$\rho_2 = -0.21 \quad \rho_{10} = 0.07$$
$$\rho_3 = -0.06 \quad \rho_{11} = 0.11$$
$$\rho_4 = 0.05 \quad \rho_{12} = -0.32$$
$$\rho_5 = 0.08 \quad \rho_{13} = 0.11$$
$$\rho_6 = -0.03$$
$$\rho_7 = -0.05$$
$$\rho_8 = -0.03$$

and the variance $\sigma_I^2 = 0.55$.

Figure 1 shows the gain functions of the X11ARIMA symmetric filters for the irregular component corresponding to the standard (default) option and the combination of the 3 × 9 seasonal moving average with the 23-term Henderson filter. As shown, the area under the latter symmetric filter is larger, indicating that the variance σ_I^2 is larger than for the default option.

In practice, however, the autocorrelation of the residuals may be very different from those generated by the symmetric linear filter. The main reasons for this are: (1) the first and last three and a half years of residuals are produced using asymmetric filters, the autocorrelations of which are different for each point; (2) the use of multiplicative models; (3) treatment of extreme values; (4) estimation of trading-day variations; (5) estimation of Easter effects; and (6) often data are generated from periodic surveys where the sampling errors are autocorrelated. Consequently, to test whether the sample autocorrelated values r_k are significant, we use Bartlett's approximations (Box and Jenkins 1970 p. 34–35) according to which, the variance of the estimated autocorrelation r_k, at lags k greater than some value q beyond which the theoretical autocorrelation function may be deemed to have "died out", is given by

$$\text{var } r_k \simeq 1/N\left(1 + 2\sum_{v=1}^{q} \rho_v\right), \; k > q. \tag{2.2}$$

On the assumption that a series is completely random, we have that $q = 0$. Then, for all lags (2.2) gives

$$\text{var } r_k \simeq 1/N. \tag{2.3}$$

If only ρ_1 is expected to be significant, then $q = 1$ and $\text{var } r_k \simeq 1(1 + 2\rho_1)/N$, $k > q$, and so on. In practical applications ρ_k is replaced by its estimated value r_k.

We use Bartlett's approximations to test for the presence of significant autocorrelation of the residuals on a sample of 50 monthly and 20 quarterly series for the period 1979–88. The level of significance is fixed at 5% for each lag k, from 1 to 14 for the monthly series; and from 1 to 6 for the quarterly series. All the series are seasonally adjusted using the *standard option* (default) of the X11ARIMA/88 version, that is, multiplicative decomposition (unless zeroes or negative values are present, in which case it applies the additive model), variable selection of the Henderson trend-cycle filters, and variable selection of the seasonal filters. We also remove trading-day variations if significant. The results are shown in Table 1 for those series where at least one significant autocorrelated value is found. It is worthwhile to note here that even when the residuals are white noise, if the individual tests were independent, the probability of obtaining one or more significant r_k's out of 14 would slightly exceed 50%.

For the monthly series, significant autocorrelation is mainly found at lags 1 and 12 for 21 out of the 50 series tested. For the quarterly series significant autocorrelation is shown mainly at lag 1 for 14 out of the 20 series tested. In all cases the values are negative. The standard (default) option always chose the 13-term Henderson filter with the exception of series D767386 and for all the quarterly series, the 5-term Henderson filter. The standard (default) variable seasonal routine always chose the 3 × 5 term moving average.

3. Correcting for Significant Autocorrelation at Lag One

For all the series of Table 1 where r_1 is significantly different from zero, the estimated value is negative. This has often been interpreted as an overadjustment and may be due to the length of the automatically chosen Henderson trend-cycle filter. The steps involved in the selection of the variable trend-cycle filter by X11ARIMA/80 and X11ARIMA/88 are as follows:

1. As a preliminary estimate of the trend-cycle C, a 13-term Henderson moving average of the seasonally adjusted series is computed.
2. As a preliminary estimate of the irregulars I, the 13-term moving average is divided (*subtracted*) into (*from*) the seasonally adjusted series.
3. The average month-to-month percent change (*difference*) without regard to sign of the preliminary C and I is calculated to obtain an I/C ratio.
4. If $0 \leqslant I/C \leqslant 0.99$, the program selects the 9-term Henderson. If $1 \leqslant I/C \leqslant 3.49$, the program selects the 13-term Henderson. If $I/C \geqslant 3.50$, the program selects the 23-term Henderson.

For quarterly series, the 5-term and 7-term Henderson filters are available, and applied as follows:

If $0 \leqslant I/C \leqslant 3.49$, the program selects the 5-term Henderson.

Table 1. Autocorrelations of residuals r_k and summary measures from X11ARIMA/88 using standard options and longer filters

Autocorrelations r_k significant at the 5% level				Summary Measures					
Monthly Series	Standard Option	Longer Filters		Standard Option			Longer Filters		
		Trend Cycle	Seas.	Q	MCD/QCD	ADR	Q	MCD/QCD	ADR
Retail Trade									
All Stores: Chain (D650000)	$r_1 r_2$	–	n.a.	0.12	2	1.40	0.08	2	1.51
Combination Stores (D650001)	$r_1 r_{13}$	–	–	0.27	3	1.47	0.28	3	1.48
Grocery-Confect. Sundries (D650002)	$r_1 r_2$ $r_3 r_{12}$	–	n.a.	0.17	2	1.51	0.32	2	1.53
General Merch. Stores Chain (D7650005)	r_1	–	n.a.	0.52	6	1.34	0.52	7	1.42
All Stores: Canada (D650058)	$r_1 r_{13}$	–	n.a.	0.14	2	1.37	0.11	2	1.47
Nova Scotia (D650350)	r_1	–	n.a.	0.11	2	1.49	0.14	3	1.53
Ontario (D650702)	r_{10}	n.a.	n.a.				n.a.	n.a.	n.a.
Alberta (D651142)	$r_1 r_8$ r_{12}	–	n.a.	0.12	2	1.45	0.14	2	1.45
Dept. Stores (D650062)	r_1	–	n.a.	0.23	4	1.42	0.24	4	1.42
Service Stations (D650068)	$r_2 r_5 r_7$ r_{10}	n.a.	n.a.	0.23	2	1.51	n.a.	n.a.	n.a.
Florist – All stores (D650082)	$r_1 r_{11}$ r_{12}	$r_1 r_{11}$ r_{12}	$r_1^{(1)}$	0.22	4	1.51	0.28	4	1.51

Table 1. (Continued)

Autocorrelations r_k significant at the 5% level				Summary Measures					
Monthly Series	Standard Option	Longer Filters		Standard Option			Longer Filters		
		Trend Cycle	Seas.	Q	MCD/QCD	ADR	Q	MCD/QCD	ADR
Imports – Balance of Payments									
Fresh Vegetables (D397775)	$r_2 r_{10}$	n.a.	n.a.	0.46	6	1.63	n.a.	n.a.	n.a.
Non-Metal Minerals (D397809)	$r_6 r_{12}$	n.a.	–(1)	0.81	4	1.53	0.78	4	1.53
Agri. Machines Tractors (D397817)	r_3	n.a.	n.a.	0.52	4	1.45	n.a.	n.a.	n.a.
Furniture, Utensils and Goods (D397831)	$r_2 r_{11}$	n.a.	n.a.	0.77	4	1.35	n.a.	n.a.	n.a.
Labour Force									
Employment – Men 25 yrs. & over (D767386)	r_1	–	n.a.	0.13	1	1.40	0.13	1	1.45
Unemployment – Women 25 yrs. & over (D767519)	$r_{12} r_8$	n.a.	–(2)	0.58	3	1.76	0.53	3	1.70
Employment Total – PEI (D767976)	r_{12}	n.a.	–(2)	0.35	4	1.34	0.30	4	1.41
Unemployment Total – PEI (D767977)	$r_{12} r_{14}$	n.a.	–(2)	0.99	12	1.57	0.96	12	1.49
Employed Men – Quebec (D768421)	r_{12}	n.a.	–	0.21	3	1.55	0.18	3	1.47
Unemployed Women – BC (D769184)	$r_{12} r_{14}$	n.a.	–	0.98	4	1.59	0.84	4	1.59

Quarterly Series	Standard Option	Longer Filters		Standard Option			Longer Filters		
		Trend Cycle	Seas.	Q	MCD/QCD	ADR	Q	MCD/QCD	ADR
Wages and Labour Income (D10002)	r_4	n.a.	–	0.12	1	1.39	0.15	1	1.39
Company Profits (D10003)	r_1	–	n.a.	0.47	1	1.30	0.51	1	1.39
Farm Net Income (D10005)	r_1	–	n.a.	0.80	4	1.34	0.77	4	1.40
Non-Farm Net Income (D10006)	r_1	–	n.a.	0.24	1	1.30	0.24	1	1.35
Indirect Taxes (D10008)	r_1	–	n.a.	0.63	1	1.18	0.59	1	1.44
Capital Cons. Allowance (D10009)	r_1	r_1	n.a.	0.41	1	1.26	0.42	1	1.26
Personal Expenditures (D10012)	r_1	r_1	n.a.	0.15	1	1.39	0.09	1	1.39
Government Expenditures (D10013)	r_1	r_1	n.a.	0.21	1	1.30	0.20	1	1.50
Business Fixed Capital (D10036)	r_1	–	n.a.	0.15	1	1.24	0.15	1	1.24
Imports-Food-Alcohol (D397844)	r_1	–	n.a.	0.39	1	1.18	0.36	2	1.44

Table 1. (Continued)

Quarterly Series	Standard Option	Longer Filters		Standard Option			Longer Filters		
		Trend Cycle	Seas.	Q	MCD/QCD	ADR	Q	MCD/QCD	ADR
Imports – Balance of Payments									
Transportation (D397900)	r_1	–	n.a.	0.98	2	1.39	0.96	3	1.40
Printed Matter (D397904)	r_1	–	n.a.	0.36	1	1.30	0.44	2	1.39
End Products (D397907)	r_1	–	n.a.	0.87	1	1.34	0.90	1	1.34
Corporate Claims (D151986)	$r_1 r_4$	$r_1 r_4$	r_1	0.99	2	1.22	0.98	2	1.30

(1) Means also Easter effect removed
(2) means additive Model used
"–" means no significant autocorrelations
"n.a." means longer filter not applied.

If $I/C \geqslant 3.50$, the program selects the 7-term Henderson.

We then applied a longer Henderson trend-cycle filter to all those series where r_1 was significant and negative. As shown in Table 1, increasing the length of the Henderson filter eliminates the significant autocorrelation at lag 1 and sometimes at other lags for all the monthly series except one. For the quarterly series, only 6 out of the 14 series still have significant autocorrelation at a 5% level of significance. The longer filters used for the monthly and quarterly series are the 23-term and 7-term Henderson moving averages, except for Employment Men 25 years old and over (D767386), where the default option chose the 9-term Henderson and so the longer filter is the 13-term Henderson.

Table 1 also shows measures of the quality of the seasonal adjustment before and after the elimination of the significant autocorrelations. These measures are: (1) the overall assessment Q measure; (2) the months (quarters) for cyclical dominance MCD (QCD) and (3) the average duration of run ADR. As can be seen, for those series where only the autocorrelation associated with the trend-cycle filter has been eliminated, the Q statistic changed very little with no systematic increase or decrease; the MCD remains the same except in four cases where it increased by one month (quarter); and the ADR improved systematically as expected.

4. Correcting for Significant Autocorrelation at Seasonal Lags

The presence of negative autocorrelation at lags 12 and 4 for monthly and quarterly series, respectively, has been interpreted as an overadjustment. These negative values may be caused by several reasons but the most common are:

1. the application of too short moving averages or
2. the use of the wrong decomposition model.

To estimate the seasonal factors, the X11ARIMA/88 is the first version since the X10 variant to incorporate a variable seasonal moving average routine as part of the default option.

The use of different seasonal filters for each month (quarter) was originally suggested by Marris (1960) based on what he called moving seasonality ratios (MSR) or I/S (irregular-seasonal) ratios, where S is an unweighted 7-term average of the D8 and D9 SI ratios (differences) and I is obtained by dividing S into the ratios (subtracting from the differences).

The seasonal moving averages that were selected automatically by the X10 variant on the basis of the monthly (quarterly) I/S are given below:

MSR(I/S)	Moving Averages
0 to 1.49	3-term
1.50 to 2.49	3 × 3-term
2.50 to 4.49	3 × 5-term
4.50 to 6.49	3 × 9-term
6.50 to 8.49	3 × 15-term
8.50 and over	n-term (stable seasonal)

The MSR (I/S) ratios for each month (quarter) are also printed in the D9 table of the X11 variant but are not automatically implemented. They are used as an *indication* of the amount of moving seasonality present in a particular month and, *only if the user asks for it*, different seasonal filters can be applied for different months. The default (standard) option of the X11 variant does

Table 2. Autocorrelations of residuals r_k and summary measures with and without trading-day variations

Autocorrelations r_k significant at the 5% level			Summary Measures					
Monthly series	Trading-day Variations Present	Trading-day Variation removed	Trading-day Variation present			Trading-day Variation removed		
			Q	MCD	ADR	Q	MCD	ADR
Retail Trade								
All Stores:								
Chain (D650000)	$r_1 r_3 r_4 r_6 r_7 r_9 r_{10} r_{11} r_{13} r_{14}$	$r_1 r_2$	0.47	5	1.37	0.12	2	1.40
Combination Stores (D650001)	$r_1 r_3 r_4 r_6 r_7 r_9 r_{10} r_{11} r_{13} r_{14}$	$r_1 r_{13}$	1.59	11	1.47	0.27	3	1.47
Grocery – Confect. Sundries (D650002)	$r_1 r_3 r_4 r_6 r_7 r_9 r_{10} r_{11} r_{12} r_{13} r_{14}$	$r_1 r_2 r_3 r_{12}$	0.52	3	1.37	0.17	2	1.51
General Merchandise Stores – Chain (D650005)	$r_1 r_3 r_4 r_{14}$	r_1	0.73	9	1.42	0.52	6	1.34
All Stores								
Canada (650058)	$r_1 r_3 r_4 r_6 r_{7\,9\,10} r_{11} r_{13} r_{14}$	$r_1 r_{13}$	0.36	3	1.40	0.14	2	1.37
Nova Scotia (D650350)	$r_1 r_3 r_4 r_6 r_7 r_9 r_{10} r_{11}$	r_1	0.42	5	1.42	0.11	2	1.49
Ontario (D650702)	$r_1 r_3 r_4 r_6 r_7 r_{10} r_{11} r_{13} r_{14}$	r_{10}	0.34	3	1.40	0.11	2	1.47
Alberta (D651142)	$r_1 r_3 r_4 r_7 r_{10} r_{11} r_{12} r_{13} r_{14}$	$r_1 r_8 r_{12}$	0.37	3	1.38	0.12	2	1.45
Department Stores (D650062)	$r_1 r_3 r_6 r_7 r_{10} r_{11} r_{13} r_{14}$	r_1	0.49	6	1.32	0.23	4	1.42
Service Stations (D650068)	$r_2 r_3 r_5 r_6 r_{13} r_{14}$	$r_2 r_6 r_7 r_{10}$	0.27	2	1.51	0.23	2	1.51

Imports – Balance of Payments								
Fresh Vegetables (D397775)	$r_2 r_4 r_{13}$	$r_2 r_{10}$	0.52	8	1.59	0.46	6	1.63
Non-Metallic Minerals (D397809)	$r_6 r_7 r_{11} r_{13}$	$r_6 r_{11} r_{12}$	1.09	5	1.47	0.81	4	1.53
Agriculture Machine – Tractors (D397817)	$r_4 r_6$	r_3	0.65	5	1.43	0.52	4	1.45
Furniture, Utensils and Goods (D397831)	$r_3 r_4 r_7 r_{10} r_{13} r_{14}$	$r_2 r_{11}$	1.16	5	1.40	0.77	4	1.35

not select a different seasonal filter for each month, but uses the 3 × 3 m.a. in the first iteration and the 3 × 5 m.a. in the second iteration for all the months.

In the X11ARIMA/80 the default option is the same as in the X11 variant but the program also enables the user to apply an option that *automatically* selects the length of the moving average for each month (quarter) as follows (Lothian 1984):

MSR(I/S)	Moving Average
0 to 1.49	3 × 1
1.5 to 2.49	3 × 3
2.5 to 7.0	3 × 5
> 7.0	3 × 9

The rationale for broadening the interval for the 3 × 5 moving average was to avoid unnecessary revisions because the I/S ratios for a given month (quarter) can easily change from year to year.

The steps involved in the current automatic variable seasonal routine of X11ARIMA/88 are as follows:

1. A global I/S ratio is calculated using complete years for the entire series, that is, up to and including year N where N must be greater than five. The global I/S ratio is the average of the 12(4) I/S ratios printed in table D9. The selection of the seasonal filter is based on the value of the I/S_N ratio as follows:

   (a) if $I/S_N \leqslant 2.5$ select the 3 × 3 m.a.
   if $3.5 \leqslant I/S_N \leqslant 5.5$ select the 3 × 5 m.a.
   if $I/S_N \geqslant 6.5$ select the 3 × 9 m.a.

   (b) if $2.5 < I/S_N < 3.5$ or
   if $5.5 < I/S_N < 6.5$
   redo (a) using the I/S_{N-1} ratio.

If none of the conditions in (a) are satisfied when using I/S_{N-1}, redo (a) using I/S_{N-2} and so on. If none of the I/S ratios satisfy the conditions in (a), then the program uses the 3 × 5 m.a.

For the series analysed, the automatic option of X11ARIMA/88 always selected the 3 × 5 m.a. and used the multiplicative decomposition model. Table 1 shows how the significant autocorrelated values are corrected. For three series, the use of an additive decomposition is preferable to increasing the length of the seasonal moving average whereas for the remainder, the application of the 3 × 9 m.a. is adequate. Furthermore, for two series, namely, (1) Import Balance of Payments, Non Metal Minerals and (2) Retail Trade, Florists All Stores, adjustment for Easter effects are necessary. Finally, for two series, namely (1) Retail Trade, All Stores Alberta and (2) Retail Trade, Grocery, Confectionary and Sundries, the use of a longer Henderson filter corrected the significant autocorrelated values at lag 12.

The summary measures for the series where only significant autocorrelation was found at the seasonal lag and removed by either applying a longer seasonal filter or an additive decomposition model indicate a systematic improvement in both the Q measure and the ADR without deterioration of the MCD (QCD). Although not shown, we looked at the F-values for stable, moving and residual seasonality for all the series. In general, the F-values for stable seasonality decreased, whereas there was little or no systematic changes for the F-values corresponding to moving and residual seasonality.

5. Autocorrelation of the Residuals Due to the Presence of Trading-day Variations

Among the monthly series, a sample of 14 was selected to analyse the autocorrelation of the residuals before and after removal of trading-day variations. Table 2 shows that when trading-day variations are present, the autocorrelation of the residuals is significant for a large number of lags. Positive autocorrelation is often found at lags 3, 6, 9, 11, and 14. Negative autocorrelation is found at lags 4, 7, 10, and 13. The removal of trading-day variations corrects for most of the significant autocorrelated values.

As expected, the summary measures before and after removal of trading-day variations change significantly. As shown in Table 2, there is a systematic improvement in Q, MCD, and ADR.

6. Conclusions

This study has shown that the significant autocorrelation of the residuals produced by X11ARIMA for certain lags are not necessarily an indication of the inadequacy of the method. It is recognized that residuals from reasonable decompositions can contain some non-zero autocorrelations given the effect of linear filters on white noise irregulars. In many cases, however, significant autocorrelation can be corrected by choosing different filters, different decomposition models, removing trading-day and Easter effects depending on the time lags at which significant autocorrelations are present.

Negative autocorrelation at lag one, usually interpreted as an overadjustment of the trend-cycle, can be corrected using a longer Henderson filter. Similarly, negative correlation at the seasonal lag can be corrected either with a longer seasonal moving average or changing to an additive model. Finally, the presence of positive autocorrelation at lags 3, 6, 9, and 14 with negative autocorrelation at lags 4, 7, 10, and 13 are mainly due to the presence of trading-day variations and are corrected once these variations are removed.

Since the residuals of X11ARIMA are also affected by extreme values, the occurrence of two or more extreme values in the same month or in the same year can cause significant autocorrelated values at certain lags. These significant autocorrelated values can be corrected only if the extreme values are a priori permanently modified.

We have also observed that significant negative autocorrelation at lag 12 or 4 which could not be eliminated by changing the decomposition model and using longer seasonal filters, disappears after the removal of Easter effects if present in the series. The summary measures that can be more affected by the nature of the irregulars, namely Q, MCD (QCD), and ADR were also analysed before and after removal of the significant autocorrelation at certain lags. We observed a systematic improvement when trading-day variations and Easter effects were removed and to a lesser extent, when autocorrelation at the seasonal lag was eliminated using a longer filter or an additive model. On the other hand, the Q and MCD (QCD) measures changed very little when the autocorrelation was observed at lag one and removed using a longer Henderson trend-cycle filter. The values for stable seasonality decreased systematically whereas minor changes were observed for the F-values corresponding to moving and residual seasonalities.

Finally, we would like to point out that we do not recommend the elimination of significant autocorrelation in the residuals as a goal in itself. It should be done with consideration taken to its effect on the quality of the seasonally adjusted values and regarding a priori information on the generating structure of the series under investigation.

7. References

Akaike, H. and Ishiguro, M. (1980). BAYSEA, A Bayesian Seasonal Adjustment Program. Computer Science Monograph No. 13, The Institute of Statistical Mathematics, Tokyo, Japan.

Ansley, C.F and Wecker, W.E. (1984). Comment. Journal of Business and Economic Statistics, 2, 323-324.

Bell, W.R. and Hillmer, S.C. (1984). Issues Involved with the Seasonal Adjustment of Economic Time Series. Journal of Business and Economic Statistics, 2, 291–320.

Bordignon, S. (1988). Destagionalizzazione delle serie storiche delle forze di lavoro. Research paper, Department of Statistics, University of Padua, Italy.

Box, G.E.P. and Jenkins, G.M. (1970). Time Series Analysis: Forecasting and Control. San Francisco: Holden Day.

Burman, J.P. (1980). Seasonal Adjustment by Signal Extraction. Journal of the Royal Statistical Society, Ser. A, 143, 321–327.

Cleveland, W.S., Dunn, D.M., and Terpenning, I. (1978). SABL: A Resistant Seasonal Adjustment Procedure with Graphical Methods to Interpretation and Diagnosis. In Seasonal Analysis of Economic Time Series, ed. A. Zellner, Washington, D.C.: U.S. Government Printing Office, 201–231.

Daddi, P. and D'Esposito, M. (1985). Analysis of X11ARIMA and SABL Seasonal Adjustment Procedures. METRON, vol. XLIII, 93–115.

Dagum, E.B. (1980). The X11ARIMA Seasonal Adjustment Method. Statistics Canada, Catalogue No. 12-564E, Ottawa, Canada.

Dagum, E.B. (1988). The X11ARIMA/88 Seasonal Adjustment Method—Foundations and User's Manual. Research paper. Time Series Research and Analysis Division, Statistics Canada, Ottawa.

Granger, C.W.J. (1978). Seasonality: Causation, Interpretation and Implications. In Seasonal Analysis of Economic Time Series, ed. A. Zellner, Washington, D.C.: U.S. Government Printing Office, 33–46.

Grether, D.M. and Nerlove, M. (1970). Some Properties of 'Optimal' Seasonal Adjustment. Econometrica, 38, 682–703.

Lothian, J. (1984). The Identification and Treatment of Moving Seasonality in X11ARIMA. In Proceedings of the Business and Economic Statistics Section, American Statistical Association, 166–171.

Marris, S.N. (1960). The Treatment of Moving Seasonality in Census Method II. In Seasonal Adjustment in Electronic Comporters, Organization for Economic Cooperation and Development, Paris, 257–309.

Nerlove, M. (1964). Spectral Analysis of Seasonal Adjustment Procedures. Econometrica, 32, 241–286.

Pierce, D. (1978). Seasonal Adjustment when both Deterministic and Stochastic Seasonality are Present. In Seasonal Analysis of Economic Time Series, ed. A. Zellner, Washington, D.C.: U.S. Government Printing Office, 242–269.

Sims, C. (1978). Comments. In Seasonal Analysis of Economic Time Series, ed. A. Zellner, Washington, D.C.: U.S. Government Printing Office, 47–49.

Tukey, J. (1978). Comments. In Seasonal Analysis of Economic Time Series, ed. A. Zellner, Washington, D.C.: U.S. Government Printing Office, 50–53.

Wecker, W.E. (1978). Comments. In Seasonal Analysis of Economic Time Series, ed. A. Zellner, Washington, D.C.: U.S. Government Printing Office, 274–279.

Received October 1989
Revised February 1991