Journal of Official Statistics, Vol. 16, No. 1, 2000, pp. 61–76

The Effect of Different Rotation Patterns on the Revisions of Trend Estimates

David G. Steel and Craig H. McLaren[1]

The X11 and X11ARIMA procedures are widely used to produce seasonally adjusted and trend estimates from time series obtained from sample surveys. The surveys are often based on designs in which there is sample overlap between different periods. The degree of overlap is determined by the pattern of inclusion of selected units over time, i.e., the rotation pattern. An important issue in analysing the series is that trend estimates at the end of the series are revised as estimates for recent periods are added. This article considers the effects of different rotation patterns on the mean squared error of the revisions of trend estimates.

Key words: X11ARIMA; mean squared error; trend; revision; rotation pattern; sampling error.

1. Introduction

Most government statistical agencies publish seasonally adjusted series. We believe that the main aim of producing a seasonally adjusted series is to assess the current underlying trend in the series. Kenny and Durbin (1982) make the observation that ''... those using the series for policy analysis frequently say that they are more interested in the underlying trends ...'' T.M.F. Smith in a discussion of Steel (1997) asks ''... why are we estimating the trend plus irregular series? Why are we not estimating the underlying trend?'' The underlying trend can be assessed by smoothing the seasonally adjusted series to filter out much of the irregularity in the series.

The Australian Bureau of Statistics (ABS) publishes trend estimates produced by applying moving averages developed by Henderson (1916) to the seasonally adjusted series (ABS, 1987). The ABS (1987) and Dagum (1996) note that publishing trend estimates for the current period leads to revisions when data for later periods become available. These revisions may concern analysts as important decisions are based on the published trend figures. Estimates of trend can be used to detect turning points and other important changes in a series and revisions introduce an element of uncertainty which may delay identification of these changes.

Since seasonally adjusted estimates and trend estimates are obtained by processes applied to the original series of survey estimates, they are influenced by the error structure of the original series. A sample survey is often used to generate the original series. Typically, the survey does not use an independent sample at each time period but involves a

[1] School of Mathematics and Applied Statistics, University of Wollongong, NSW 2522, Australia;
email: david_steel@uow.edu.au and craigmc@uow.edu.au
Acknowledgments: This research was supported by the Australian Research Council and the Australian Bureau of Statistics.

rotation pattern in which selected units are retained in the sample for several periods. The rotation pattern used will affect the sampling error structure of the original series. The seasonally adjusted series will exhibit fluctuations due to both irregular components and the sampling error. The estimated trend may also contain a component due to sampling error. In some cases the autocorrelation structure of the sampling error may be such that its impact is effectively eliminated by the trend filter.

Tallis (1995) suggested that filtered time series should be included among the outputs considered in developing the design of a sample survey. McLaren and Steel (1997, 1998) considered the effect of different rotation patterns on the sampling variance of seasonally adjusted and trend estimates under different realisations of X11 and X11ARIMA. They found that using rotation patterns with no monthly overlap considerably reduces the sampling variance of trend estimates and the difference between two consecutive trend estimates obtained using either X11 or X11ARIMA. This article considers how changing the rotation pattern affects the mean squared error (MSE) of the revision of trend estimates. We concentrate on monthly labour force surveys (MLFSs).

2. Preliminaries

2.1. Estimating trend

Consider a univariate time series of estimates obtained from a sample survey with values $\{y_t : t = 1, \ldots, T\}$. The observed value y_t at time t can be written as

$$\begin{aligned} y_t &= Y_t + e_t \\ &= T_t + S_t + I_t + e_t \end{aligned} \tag{1}$$

where Y_t is the true unknown value of the series at time t and e_t is the sampling error. The true value comprises trend-cycle, seasonal and irregular components T_t, S_t and I_t, respectively. In some cases a multiplicative decomposition may be more appropriate. Many statistical agencies produce seasonally adjusted series by attempting to estimate S_t and remove it from the series using the X11 method developed by Shiskin et al. (1967) or X11ARIMA developed by Dagum (1980 and 1988). Trend estimates can then be produced by smoothing the seasonally adjusted estimates using an appropriate filter.

Kenny and Durbin (1982) defined the concept of an historical trend estimate as one that is calculated at a time point t sufficiently far from the end of the series that the addition of new data points does not alter the trend estimate calculated for time t. We define a trend estimate for time t based on the series up to and including time period $t + k$, as

$$\begin{aligned} \hat{T}_{t|t+k} &= \boldsymbol{c}'_{t|t+k}\boldsymbol{y}^{sa}_{t+k} \\ &= \boldsymbol{a}'_{t|t+k}\boldsymbol{y}_{t+k} \end{aligned} \tag{2}$$

where $\boldsymbol{c}_{t|t+k}$ is a trend filter, $\boldsymbol{y}^{sa}_{t+k}$ is the vector of seasonally adjusted estimates ending k periods on from t, $\boldsymbol{y}_{t+k}$ is a vector of observed values of the original series ending k periods on from t and $\boldsymbol{a}_{t|t+k}$ is a filter which includes both seasonal adjustment and trend estimation applied to the original series. The initial trend estimate at the end of the series, $\hat{T}_{t|t}$, is found by setting $k = 0$. The historical trend is written at $\hat{T}_{t|\infty}$.

It is assumed that y_t is an unbiased estimate of Y_t so $E_p(y_t) = Y_t$, where the subscript p

denotes that the expectation is with respect to the sampling design. This means that $E_p(\hat{T}_{t|t+k}) = \boldsymbol{a}'_{t|t+k}\boldsymbol{Y}_{t+k}$ where $\boldsymbol{Y}_{t+k}$ is the vector of true unknown values ending at $t+k$. We treat $T_{t|t+k} = \boldsymbol{a}'_{t|t+k}\boldsymbol{Y}_{t+k}$ as the true unknown trend for the series of true values for time t given values up to time $t+k$ for the filter $\boldsymbol{a}_{t|t+k}$. Kenny and Durbin (1982) note that there is no unique definition of trend and that different filters may be used according to the degree of smoothness and sensitivity desired.

The estimate of trend for time t will change as data for periods after time t become available. This occurs because at the end of a seasonally adjusted series there are insufficient data points to enable the use of symmetric trend filters and asymmetric filters have to be used. When new data points become available, this additional information leads to trend estimates being revised as the filters used evolve to a symmetric form. As more information becomes available the revised trend estimates will approach the historical trend for that time point.

The extent of revision to the initial trend estimate has deterred some organisations from publishing trend estimates, at least for the last few months. The revision between the trend estimate for time t made at time t and that made for time t at time $t+k$ is

$$\begin{aligned}\hat{T}_{t|t} - \hat{T}_{t|t+k} &= \boldsymbol{a}'_{t|t}\boldsymbol{y}_t - \boldsymbol{a}'_{t|t+k}\boldsymbol{y}_{t+k} \\ &= (\boldsymbol{a}^*_{t|t} - \boldsymbol{a}_{t|t+k})'\boldsymbol{y}_{t+k} \\ &= \boldsymbol{b}'_{t|t+k}\boldsymbol{y}_{t+k} \qquad (3)\end{aligned}$$

where the filter $\boldsymbol{a}^*_{t|t} = (\boldsymbol{a}'_{t|t}, 0, \ldots, 0)'$ contains k zeros and $\boldsymbol{b}_{t|t+k} = \boldsymbol{a}^*_{t|t} - \boldsymbol{a}_{t|t+k}$ is the trend revision filter.

Linacre and Zarb (1991) noted that the last three trend estimates are those most susceptible to being revised. In practice, there will be a point where the degree of revision will be negligible. A minimal difference will usually be observed between the trend estimate $\hat{T}_{t|t+3}$ and trend estimates up to the historical trend, $\hat{T}_{t|\infty}$. Dagum (1996) commented that the size of revisions is negligible after three months' values are added to the series. We will focus on the cases of one month revisions ($k=1$) and three-month revisions ($k=3$). For example the one-month revision is the difference between the initial estimate and the estimate made for the same time point when one observation is added to the series.

The revision $\hat{T}_{t|t} - \hat{T}_{t|t+k}$ is different from the sampling error. The sampling error of $\hat{T}_{t|t}$ is $\hat{T}_{t|t-}T_{t|t}$, the difference between the estimated trend for time t and the true unknown initial trend for time t that would be obtained if the trend filter was applied to the series of true values ending at t. Similarly the sampling error of $\hat{T}_{t|t+k}$ is $\hat{T}_{t|t+k-}T_{t|t+k}$. Users do not observe the sampling error but observe the revision of the estimates, which can be decomposed as

$$\begin{aligned}\hat{T}_{t|t} - \hat{T}_{t|t+k} &= \hat{T}_{t|t} - T_{t|t} + T_{t|t} - \hat{T}_{t|t+k} \\ &= \hat{T}_{t|t} - T_{t|t} + T_{t|t} - T_{t|t+k} + T_{t|t+k} - \hat{T}_{t|t+k} \\ &= (\hat{T}_{t|t} - T_{t|t}) + (T_{t|t} - T_{t|t+k}) - (\hat{T}_{t|t+k} - T_{t|t+k}) \qquad (4)\end{aligned}$$

Hence, the observed revision can be expressed in terms of the sampling error of $\hat{T}_{t|t}$ and $\hat{T}_{t|t+k}$ and the revision of the trend in the true series, $T_{t|t} - T_{t|t+k}$, due to the use of an initial asymmetric filter which is different from the symmetric filter. The revision based on the true series is unobservable and will not be affected by sampling error and therefore the choice of rotation pattern.

One approach to reducing the revisions of trend estimates is to design trend filters which minimise them. Kenny and Durbin (1982) conducted an empirical study which showed that prediction of the future values past the end of the series reduced the difference between an initial and final trend estimate for a particular time point. Prediction enables asymmetric filters to be used at the end of a series which have properties similar to the symmetric filters used within the middle of the series. A widely used example of using prediction at the end of a series is X11ARIMA (Dagum 1980, 1988) which extends the original series at either end by an ARIMA model. A number of authors, for example Pierce (1980) and Dagum (1982), have conducted empirical studies which illustrate that the use of X11ARIMA reduces the size of the revisions in the seasonally adjusted series compared with X11. Another method suggested by Dagum (1996) consists of extending a modified trend series with ARIMA forecasts and then applying a smoothing filter.

The current short-term direction of the trend will be important to many users and can be analysed by looking at the estimate of the one-month change in the trend estimates for the two most recent periods

$$\Delta\hat{T}_{t|t} = \hat{T}_{t|t} - \hat{T}_{t-1|t}$$

Thus the revision of the estimate of the one-month change in trend at the end of a series should also be considered. The revision of this estimate can be written in terms of the revision of the trend estimates

$$\begin{aligned}\Delta\hat{T}_{t|t} - \Delta\hat{T}_{t|t+k} &= (\hat{T}_{t|t} - \hat{T}_{t-1|t}) - (\hat{T}_{t|t+k} - \hat{T}_{t-1|t+k}) \\ &= (\hat{T}_{t|t} - \hat{T}_{t|t+k}) - (\hat{T}_{t-1|t} - \hat{T}_{t-1|t+k}) \qquad (5)\end{aligned}$$

This suggests that a reduction in the revision of the trend level estimate results in a reduction in the revision of the estimate of the change in trend.

2.2. *Rotation patterns*

The degree of sample overlap between any two periods is determined by the rotation pattern, which is the pattern of selected units' inclusion in the survey over time. Rotation patterns vary in the number of times a unit is included in the survey and the time interval between inclusions. Most of the rotation patterns used for monthly surveys are special cases of the general class of *a*-*b*-*a*(*m*) rotation patterns in which selected units are included for *a* consecutive months, removed from the survey for *b* months and then included again for a further *a* months. The pattern is repeated so that selected units are included for a total of *m* occasions. Setting $b = 0$ gives an *in-for-m* rotation pattern. The Canadian and Australian MLFSs use an *in-for*-6 and *in-for*-8 rotation patterns respectively. The U.S. Current Populations Survey uses a 4-8-4(8) rotation pattern.

2.3. *Trend estimates within seasonal adjustment packages*

Henderson filters are used within X11 and X11ARIMA in the production of seasonally adjusted estimates. The ABS (1993) also use Henderson filters to smooth the seasonally adjusted estimates to produce the published trend estimates.

X11 consists of an iterative application of linear filters where the user can specify, to a degree, a level of smoothness and a choice of seasonal filters. The use of moving averages results in a symmetric filter for the central values, and asymmetric filters for the values at the beginning and end of the series. The X11ARIMA method is an extension of X11 and extrapolates the original series at both ends by an ARIMA model.

We consider trend estimates obtained by applying a Henderson moving average to seasonally adjusted estimates obtained from both X11 and X11ARIMA. The seasonally adjusted and trend estimates produced by these two packages can be approximated by linear filters. Dagum et al. (1996) developed a Cascade method approach, where the Cascade filters are a result of the convolution of the various linear filters used within X11 and X11ARIMA. We consider two combinations of Cascade filters which result in filters used to seasonally adjust a series:

1. Standard X11 Cascade filter: This corresponds to a choice of a 13 term Henderson moving average for estimation of trend (H13), 3×3 moving average (ma) for the first estimation of the seasonal factors ($S1_{3\times3}$), 3×5 ma for estimation of seasonal factors ($S2_{3\times5}$), and no modification for outliers.
2. Standard X11 Cascade filter with ARIMA forecasts: This corresponds to a choice of a H13, $S1_{3\times3}$, $S2_{3\times5}$, and extended forecasts from a (0, 1, 1)(0, 1, 1)$_{12}$ model with parameters $\theta = 0.4, \Theta = 0.6$, and no modification for outliers.

For each combination, a set of filters for trend estimation can be found by multiplying the seasonal adjustment filters by an appropriate trend filter. We considered the application of a 13 term Henderson moving average to the seasonal adjustment filters which is used by the ABS for most monthly series. Different filters can be found corresponding to $\boldsymbol{a}_{t|t}$ and also $\boldsymbol{a}_{t|t+k}$. A single filter for the difference between two trend estimates, given by (3), can also be calculated.

3. Revision of Trend Estimates

Kenny and Durbin (1982) evaluated different methods of trend estimation at the end of a series using the criterion of the difference between the historical and current trend estimates. We use a similar criterion and consider the expectation of

$$(\hat{T}_{t|t} - \hat{T}_{t|t+k})^2 \tag{6}$$

which we call the MSE of the revision.

Wolter and Monsour (1981) discussed two different concepts for variability for survey-based time series, namely, sampling variance and total variance. The expectation of (6) can be considered using these two concepts. Conditioning on the true unknown values means that the only variability recognised is that due to sampling. As the sample design, in particular the rotation pattern, only affects this component of variability we will focus on it. Alternatively, an unconditional expectation can be taken, i.e., assuming the true values Y_t are the realisation of some stochastic process.

3.1. Conditional on the population

Conditioning on the vector $\boldsymbol{Y}_{t+k} = (Y_1, \ldots, Y_{t+k})'$ and finding the MSE of the revision between two trend estimates for time t, assuming $E_p(\boldsymbol{y}_{t+k}) = \boldsymbol{Y}_t$, gives

$$\begin{aligned} R^c_{t|t+k} &= E_p\{(\hat{T}_{t|t} - \hat{T}_{t|t+k})^2\} \\ &= V_p(\hat{T}_{t|t} - \hat{T}_{t|t+k}) + \{E_p(\hat{T}_{t|t} - \hat{T}_{t|t+k})\}^2 \\ &= \boldsymbol{b}'_{t|t+k} V_p(\boldsymbol{y}_{t+k}) \boldsymbol{b}_{t|t+k} + \{E_p(\boldsymbol{b}'_{t|t+k} \boldsymbol{y}_{t+k})\}^2 \\ &= \boldsymbol{b}'_{t|t+k} V_p(\boldsymbol{y}_{t+k}) \boldsymbol{b}_{t|t+k} + (\boldsymbol{b}'_{t|t+k} \boldsymbol{Y}_{t+k})^2 \end{aligned} \tag{7}$$

where $\boldsymbol{b}_{t|t+k}$ is the trend revision filter.

The sampling variance, $V_p(\boldsymbol{y}_{t+k}) = V(\boldsymbol{y}_{t+k}|\boldsymbol{Y}_{t+k})$ will depend on the correlation structure of the series of sample estimates $\boldsymbol{y}_t$ and hence can be affected by the rotation pattern used. An appropriate model is derived in Section 4. It is important to note that the second term in (7) depends on the filter combination chosen and the true underlying series, but does not depend on the sampling variance and hence is unaffected by the rotation pattern and other features of the sampling design, such as sample size.

3.2. *Unconditional on the population*

Consider the true unknown values $\boldsymbol{Y}_{t+k}$ to be a realisation of a stochastic process with covariance matrix $V_\zeta(\boldsymbol{Y}_{t+k}) = \boldsymbol{\Sigma}_{t+k}$ and the mean vector of $E_\zeta(\boldsymbol{Y}_{t+k}) = \boldsymbol{\mu}_{t+k}$. The subscript ζ denotes an expectation with respect to the distribution of the true values.

Taking expectations of the conditional expectation found in (7) gives the unconditional expectation

$$\begin{aligned} R^u_{t|t+k} &= E\{(\hat{T}_{t|t} - \hat{T}_{t|t+k})^2\} \\ &= E_\zeta\{E_p(\hat{T}_{t|t} - \hat{T}_{t|t+k})^2\} \\ &= E_\zeta\{\boldsymbol{b}'_{t|t+k} V_p(\boldsymbol{y}_{t+k}))\boldsymbol{b}_{t|t+k} + (\boldsymbol{b}'_{t|t+k}\boldsymbol{Y}_{t+k})^2\} \\ &= \boldsymbol{b}'_{t|t+k} E_\zeta\{V_p(\boldsymbol{y}_{t+k})\}\boldsymbol{b}_{t|t+k} + V_\zeta(\boldsymbol{b}'_{t|t+k}\boldsymbol{Y}_{t+k}) + \{E_\zeta(\boldsymbol{b}'_{t|t+k}\boldsymbol{Y}_{t+k})\}^2 \qquad (8) \\ &= \boldsymbol{b}'_{t|t+k} V(\boldsymbol{y}_{t+k})\boldsymbol{b}_{t|t+k} + (\boldsymbol{b}'_{t|t+k}\boldsymbol{\mu}_{t+k})^2 \qquad (9) \end{aligned}$$

where $V(\boldsymbol{y}_{t+k}) = E_\zeta(V_p(\boldsymbol{y}_{t+k})) + V_\zeta(\boldsymbol{Y}_{t+k})$ since $E_p(\boldsymbol{y}_{t+k}) = \boldsymbol{Y}_{t+k}$

The first variance term in (8) depends on the rotation pattern. The other terms will be unaffected by the rotation pattern but depend on the filters chosen and also on the properties of the true series $\boldsymbol{Y}_{t+k}$. In practice the term affected by the rotation pattern in (8) will behave similarly to the variance term in (7). We therefore concentrate on the revision of the trend values conditional on the true series, which avoids modeling $\boldsymbol{Y}_{t+k}$.

We also consider the revision of the change in trend estimates given in (5) by choosing the appropriate combination of linear filters. For example the MSE of the revision of the estimate of one-month change is

$$\begin{aligned} \Delta R^c_{t|t+k} &= E_p\{(\Delta\hat{T}_{t|t} - \Delta\hat{T}_{t|t+k})^2\} \\ &= V_p(\Delta\hat{T}_{t|t} - \Delta\hat{T}_{t|t+k}) + \{E_p(\Delta\hat{T}_{t|t} - \Delta\hat{T}_{t|t+k})\}^2 \\ &= \boldsymbol{b}^{*'}_{t|t+k} V_p(\boldsymbol{y}_{t+k})\boldsymbol{b}^*_{t|t+k} + (\boldsymbol{b}^{*'}_{t|t+k}\boldsymbol{Y}_{t+k})^2 \end{aligned} \tag{10}$$

where $\boldsymbol{b}^*_{t|t+k} = (0, \boldsymbol{a}'_{t|t}, 0, \ldots, 0)' - (0, \boldsymbol{a}'_{t|t+k})' - ((\boldsymbol{a}'_{t-1|t}, 0, \ldots, 0) - \boldsymbol{a}'_{t-1|t+k})'$.

4. A Model for the Sampling Variance

A model for the variance term in (7) is required that reflects the design complexities of an MLFS. Some simplifying assumptions can be made. Consider a simple random sample taken without replacement and the estimates of proportions y_t, for which

$$V_p(y_t) = \frac{(1-f_t)Y_t}{f_t N_t}(1 - Y_t) \tag{11}$$

where, for time t, Y_t is the true proportion, N_t is the size of the population, n_t is the sample size and $f_t = n_t/N_t$ is the sampling fraction. For example, the sampling fraction for the Australian MLFS is approximately 1/170 and is effectively constant over the length of the series considered. In practice, more complex designs will be used, which means the variance is multiplied by a design effect d_t. If there are no major changes to the sample design or the population structure, at least over the effective length of the filters being considered, then d_t does not change and changes in variance are mainly determined by changes in Y_t. For example, the proportion of employed people from the Australian MLFS varied by approximately 10 percentage points over the length of the series (1978–1998) and this variation will lead to some change in the variance.

The rotation schemes used for MLFSs imply that the sample at any particular time will consist of a number of panels. A panel is a set of units that are included and removed from the survey at the same time. Most MLFSs use multistage sampling and when a panel is rotated out of the survey it is replaced by another panel drawn from the same primary sampling unit. We assume that the estimate at time t is, at least approximately, the average of estimates from each rotation group and that estimates from different rotation groups are independent. A model is needed for the sampling correlation between y_t and y_{t+s} that reflects this design. McLaren and Steel (1998) developed a model in which it is assumed that the sampling correlation between estimates obtained from the same rotation group s periods apart is $r(s)$ if no rotation has occurred and $d(s)$ if a rotation has occurred. These assumptions imply that the sampling correlation between y_t and y_{t+s} is

$$\rho(y_{t+s}, y_t) = d(s) + k(s)(r(s) - d(s)) \tag{12}$$

where $k(s)$ is the sample overlap factor associated with the rotation pattern at lag s. The sampling covariance is then

$$C_p(y_{t+s}, y_t) = \frac{1-f_t}{f_t N_t}\sqrt{Y_{t+s}(1 - Y_{t+s})}\sqrt{Y_t(1 - Y_t)}\rho(y_{t+s}, y_t) \tag{13}$$

This model assumes that the correlations at lag s are constant across time when they are likely to change. For example, the correlation will be affected by the amount of change in labour force status occurring in the population. An approximate indication of the amount by which the autocorrelations vary can be obtained by the use of gross flows estimates from the matching sample in the survey. Gross flows use matched samples across two different time points to determine the flow into and out of a particular variable of interest. Using data from the Australian MLFS for the period 1978–1998 we found that the lag one correlations varied by 0.05 for the proportion of employed persons and 0.18 for the proportion of unemployed persons. Analysis, not included here, has shown that the effect of different rotation patterns is not particularly sensitive to changes of this magnitude in the correlation structure.

We consider the proportions given by the number of unemployed and employed persons divided by the civilian population aged fifteen or over for the Australian MLFS. Bell (1998) calculated estimates for $r(s)$ and $d(s)$ in (12) for these variables by treating the rotation groups as replicates and measuring the autocorrelation at the rotation group level. These values are given in Table 1. Lee (1990) used the same approach to estimate sampling correlations for the Canadian MLFS. A model given by Bell (1998) was used to extrapolate values beyond the given lags.

5. Results

The MSE of the revision given by (7) comprises a variance and a bias component

$$\begin{aligned} R^c_{t|t+k} &= \boldsymbol{b}'_{t|t+k} V_p(\boldsymbol{y}_{t+k}) \boldsymbol{b}_{t|t+k} + (\boldsymbol{b}'_{t|t+k} \boldsymbol{Y}_{t+k})^2 \\ &= V + A^2 \end{aligned}$$

where $V = \boldsymbol{b}'_{t|t+k} V_p(\boldsymbol{y}_{t+k}) \boldsymbol{b}_{t|t+k}$ is the variance of $\boldsymbol{b}'_{t|t+k}\boldsymbol{y}_{t+k}$ and $A = \boldsymbol{b}'_{t|t+k}\boldsymbol{Y}_{t+k}$ is the revision in the series that would be observed if there was no sampling error. The term A will depend on how the true series is evolving and will not depend on the sample design, including the rotation pattern. The magnitude of the bias component relative to the variance will play an important role in the MSE of the revision. As sample size decreases the variance increases, so the contribution of the variance component becomes more important. To assist interpretation of the results we introduce the ratio

$$\phi = \frac{A^2}{V}$$

which indicates the relative importance of the variance component. The smaller ϕ the greater influence the rotation pattern will have. For comparison we choose V in ϕ to be the variance for complete rotation.

We investigate three different situations, namely a constant series, a linear series and a series with a turning point, and assess how they affect the bias component. For a constant series, $Y_t = c_0$, the bias component is zero and $\phi = 0$, because the weights comprising $\boldsymbol{b}_{t|t+k}$ sum to one. The MSE of the revision is then entirely due to the variance which depends on the chosen rotation pattern. The other extreme will occur when the

Table 1. Rotation group autocorrelations

Proportion of employed persons (Australian MLFS)

lag	1	2	3	4	5	6	7	8
$r(s)$	0.80	0.71	0.64	0.57	0.50	0.45	0.40	0.36
$d(s)$	0.15	0.15	0.14	0.13	0.12	0.11	0.11	0.10

Proportion of unemployed persons (Australian MLFS)

lag	1	2	3	4	5	6	7	8
$r(s)$	0.62	0.52	0.44	0.37	0.31	0.26	0.22	0.19
$d(s)$	0.11	0.11	0.10	0.09	0.09	0.08	0.08	0.07

bias component dominates the variance. This will occur when the period of interest is very volatile or contains a sharp turning point. The effect of the different rotation patterns, which affect only on the variance, will be small for this case. In other cases, the bias component may be approximately the same magnitude as the variance and the choice of different rotation patterns will have an effect on the MSE of the revisions. We will consider the case of an approximately linear series. The relative effect of different rotation patterns on the variance term is approximately the same for the constant and linear series and turning point situation, but the bias will differ. We consider these three different situations using the proportion of employed and unemployed persons from the Australian MLFS.

Tables 2 to 4 summarise the effect of different rotation patterns for each of these situations and the two Cascade filter combinations for the revisions of the initial estimates of the trend. Tables 5 to 7 summarise the effects for estimates of the one-month change in trend. These tables give, for each of the two variables and a selection of rotation patterns, the ratio of the MSE of the revision for the chosen rotation patterns divided by the MSE of the revision that would be obtained when there is complete sample rotation each month.

5.1. Constant series

A series of constant values was chosen within the observed range for each variable, with $Y_t = 0.57$ for the proportion of employed persons and $Y_t = 0.054$ for the proportion of unemployed persons. Periods of approximately constant proportions have been observed in the series.

Consider the revisions of the initial estimates of the trend. The MSE ratios for Cascade filter combination 1 (X11) for the one-month revisions in the proportion of unemployed and employed are given in columns 1 and 2 of Table 2. Columns 5 and 6 of Table 2 give corresponding results for Cascade filter combination 2 (X11ARIMA). These results indicate that the MSE of the one-month revision can be reduced by choosing a rotation

Table 2. Ratio of the MSE of revision of trend estimates - constant series

	Combination 1				Combination 2			
	$R^c_{t\|t+1}$		$R^c_{t\|t+3}$		$R^c_{t\|t+1}$		$R^c_{t\|t+3}$	
Rotation scheme	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	0.86	0.91	0.65	0.77	0.95	0.97	0.73	0.83
1-2-1(8)	0.84	0.90	0.61	0.75	0.92	0.96	0.69	0.81
1-1-1(6)	0.76	0.84	0.63	0.76	0.86	0.90	0.72	0.81
2-2-2(8)	0.72	0.82	0.81	0.92	0.71	0.80	0.88	0.94
2-10-2(4)	0.93	0.95	1.26	1.21	0.74	0.82	1.08	1.08
3-3-3(6)	0.84	0.89	1.21	1.18	0.64	0.73	1.04	1.04
4-8-4(8)	0.77	0.84	1.23	1.20	0.54	0.67	0.94	0.99
6-6-6(12)	0.62	0.74	1.00	1.06	0.46	0.61	0.80	0.89
in-for-6	0.61	0.74	0.99	1.05	0.47	0.61	0.82	0.90
in-for-8	0.53	0.69	0.84	0.96	0.42	0.58	0.74	0.85
no rotation	0.38	0.58	0.58	0.79	0.31	0.50	0.53	0.72

Table 3. Ratio of the MSE of revisions of trend estimates - linear series

Rotation scheme	Combination 1				Combination 2			
	$R^c_{t\|t+1}$		$R^c_{t\|t+3}$		$R^c_{t\|t+1}$		$R^c_{t\|t+3}$	
	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	0.95	0.91	0.80	0.86	0.98	0.99	0.81	0.89
1-2-1(8)	0.94	0.90	0.78	0.85	0.97	0.98	0.78	0.88
1-1-1(6)	0.91	0.85	0.78	0.85	0.94	0.96	0.80	0.88
2-2-2(8)	0.90	0.84	0.89	0.95	0.88	0.92	0.91	0.97
2-10-2(4)	0.97	0.97	1.15	1.13	0.89	0.93	1.05	1.05
3-3-3(6)	0.94	0.90	1.12	1.12	0.85	0.90	1.03	1.04
4-8-4(8)	0.91	0.87	1.13	1.13	0.81	0.88	0.96	1.00
6-6-6(12)	0.86	0.77	1.00	1.04	0.78	0.85	0.85	0.95
in-for-6	0.86	0.77	1.00	1.04	0.78	0.85	0.87	0.95
in-for-8	0.83	0.72	0.91	0.98	0.77	0.84	0.81	0.92
no rotation	0.77	0.62	0.76	0.88	0.72	0.81	0.66	0.85

pattern with high monthly overlap. The worst case occurs when there is complete rotation each month. Rotation patterns with the same degree of monthly sample overlap have very similar MSE ratios. The best option is no rotation, but this is not a practical option due to the load imposed on respondents and likely deterioration in response rate. These rotation patterns used in Canada (*in-for*-6) and Australia (*in-for*-8) perform well. Under combination 1 changing from a 4-8-4(8) to an *in-for*-8 pattern reduces the MSE of the one-month revision of a trend estimate for employed by 31 percent and for unemployed by 18 percent. Under combination 2, changing from a 4-8-4(8) to an *in-for*-8 pattern reduces the one-month MSE of the revision for employment by 22 percent and for unemployment by 13 percent.

Table 4. Ratio of the MSE of revisions of trend estimates - turning point

Rotation scheme	Combination 1				Combination 2			
	$R^c_{t\|t+1}$		$R^c_{t\|t+3}$		$R^c_{t\|t+1}$		$R^c_{t\|t+3}$	
	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	0.97	0.98	0.96	0.97	0.99	1.00	0.98	1.00
1-2-1(8)	0.96	0.98	0.96	0.96	0.98	1.00	0.98	1.00
1-1-1(6)	0.94	0.97	0.96	0.96	0.96	0.99	0.98	1.00
2-2-2(8)	0.93	0.97	0.98	0.99	0.93	0.99	0.99	1.00
2-10-2(4)	0.98	0.99	1.03	1.03	0.94	0.99	1.00	1.00
3-3-3(6)	0.96	0.98	1.02	1.03	0.91	0.98	1.00	1.00
4-8-4(8)	0.95	0.97	1.02	1.03	0.89	0.98	1.00	1.00
6-6-6(12)	0.91	0.95	1.00	1.01	0.87	0.98	0.99	1.00
in-for-6	0.91	0.95	1.00	1.01	0.87	0.98	0.99	1.00
in-for-8	0.89	0.95	0.98	1.00	0.86	0.97	0.98	1.00
no rotation	0.85	0.93	0.96	0.97	0.83	0.97	0.97	0.99

Table 5. Ratio of the MSE of revision of one-month change in trend estimates - constant series

Rotation scheme	Combination 1				Combination 2			
	$\Delta R^c_{t\|t+1}$		$\Delta R^c_{t\|t+3}$		$\Delta R^c_{t\|t+1}$		$\Delta R^c_{t\|t+3}$	
	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	0.87	0.91	0.55	0.71	0.95	0.97	0.62	0.75
1-2-1(8)	0.85	0.90	0.51	0.68	0.92	0.96	0.58	0.73
1-1-1(6)	0.73	0.82	0.62	0.76	0.86	0.90	0.72	0.81
2-2-2(8)	0.70	0.81	0.78	0.90	0.71	0.80	0.90	0.97
2-10-2(4)	0.91	0.94	1.33	1.26	0.74	0.82	1.17	1.15
3-3-3(6)	0.84	0.88	1.33	1.27	0.64	0.73	1.20	1.16
4-8-4(8)	0.72	0.81	1.33	1.28	0.54	0.67	1.07	1.09
6-6-6(12)	0.56	0.70	1.06	1.11	0.46	0.61	0.90	0.98
in-for-6	0.56	0.70	1.05	1.10	0.47	0.61	0.93	0.99
in-for-8	0.50	0.66	0.87	1.00	0.42	0.58	0.83	0.93
no rotation	0.35	0.55	0.60	0.82	0.31	0.50	0.61	0.79

Results for the three-month revision are given in columns 3, 4, 7 and 8 of Table 2 and the conclusions change considerably. These results are of particular interest as the three-month revision gives an indication of the difference between the initial and final trend estimate. For combination 1, the best performing rotation patterns are now 1-1-1(6), 1-2-1(5) and 1-2-1(8), which not only perform considerably better than the independent sample, but also considerably better than the rotation patterns with high monthly overlap. The differences between rotation patterns are more pronounced for the three-month revision. The rotation patters 2-10-2(4), 4-8-4(8) and 3-3-3(6) now have a higher MSE ratio when compared with the other rotation patters, including complete rotation. In general,

Table 6. Ratio of the MSE of revision of one-month change in trend estimates - linear series

Rotation scheme	Combination 1				Combination 2			
	$\Delta R^c_{t\|t+1}$		$\Delta R^c_{t\|t+3}$		$\Delta R^c_{t\|t+1}$		$\Delta R^c_{t\|t+3}$	
	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	0.96	0.91	0.66	0.91	0.98	0.99	0.64	0.81
1-2-1(8)	0.95	0.90	0.63	0.91	0.97	0.98	0.60	0.80
1-1-1(6)	0.91	0.81	0.72	0.93	0.94	0.96	0.73	0.86
2-2-2(8)	0.90	0.81	0.83	0.97	0.88	0.92	0.90	0.98
2-10-2(4)	0.97	0.95	1.25	1.08	0.89	0.93	1.17	1.12
3-3-3(6)	0.95	0.89	1.25	1.08	0.85	0.90	1.19	1.13
4-8-4(8)	0.91	0.82	1.25	1.08	0.81	0.87	1.07	1.08
6-6-6(12)	0.85	0.71	1.05	1.03	0.77	0.85	0.91	1.00
in-for-6	0.85	0.71	1.03	1.03	0.78	0.85	0.93	1.00
in-for-8	0.83	0.66	0.90	1.00	0.76	0.84	0.84	0.96
no rotation	0.78	0.56	0.70	0.95	0.71	0.81	0.62	0.86

Table 7. Ratio of the MSE of revision of one-month change in trend estimates - turning point

Rotation scheme	Combination 1				Combination 2			
	$\Delta R^c_{t\|t+1}$		$\Delta R^c_{t\|t+3}$		$\Delta R^c_{t\|t+1}$		$\Delta R^c_{t\|t+3}$	
	emp	unemp	emp	unemp	emp	unemp	emp	unemp
complete	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1-2-1(5)	0.97	0.99	0.96	0.96	0.99	1.00	0.98	0.99
1-2-1(8)	0.96	0.99	0.95	0.95	0.98	1.00	0.98	0.99
1-1-1(6)	0.93	0.97	0.96	0.96	0.97	0.99	0.99	1.00
2-2-2(8)	0.92	0.97	0.98	0.99	0.93	0.99	0.99	1.00
2-10-2(4)	0.98	0.99	1.03	1.04	0.94	0.99	1.01	1.00
3-3-3(6)	0.96	0.98	1.03	1.04	0.91	0.98	1.01	1.00
4-8-4(8)	0.93	0.97	1.03	1.04	0.89	0.98	1.00	1.00
6-6-6(12)	0.89	0.95	1.01	1.02	0.87	0.98	1.00	1.00
in-for-6	0.89	0.95	1.00	1.02	0.87	0.98	1.00	1.00
in-for-8	0.87	0.95	0.99	1.00	0.86	0.97	0.99	1.00
no rotation	0.84	0.93	0.96	0.97	0.83	0.97	0.98	1.00

under combination 2, the gains for the 1-b-1(m) designs relative to a completely independent design are less than for combination 1, while the gains for rotation patterns with a high monthly overlap increase. Columns 7 and 8 of Table 2 illustrate that rotation patterns with high monthly overlap, such as *in-for*-8, perform nearly as well as the 1-b-1(m) designs under combination 2. The 2-10-2(4) and 3-3-3(6) patterns both still have slightly higher MSE ratios than for a complete rotation, although they perform better than under combination 1.

Changing from a 4-8-4(8) to a 1-2-1(8) pattern under combination 1 reduces the MSE of the three-month revision in a trend estimate by 50 percent for employed and 38 percent for unemployed. For combination 2 the gains from changing from 4-8-4(8) to a 1-2-1(8) pattern are 26 percent for employed and 18 percent for unemployed.

Results for the revisions of the estimate of one month change of the trend are given in Table 5. For the one-month revisions the results are similar to those already discussed. For the three-month revisions there are substantial gains in using 1-2-1(m) rotation patterns for both X11 and X11ARIMA. For example for X11ARIMA, using 1-2-1(8) instead of 4-8-4(8) and *in-for*-8 gives gains of 46 percent and 30 percent for employed, respectively.

The differences in the results for one- and three-month revisions can be explained by the properties of the filters applied. The one-month revision filter gives high weight to short-term cycles. This means that rotation patterns with a high degree of sample overlap, which include short and longer term cycles, will reduce the MSE of the one-month revision in trend to a greater degree than rotation patterns containing only short term cycles, i.e., the 1-b-1(m) patterns. The three-month revision filter applies less weight to the shorter term cycles. The 1-b-1(m) rotation patterns are therefore more suited to these filters.

5.2. Approximately linear series

For many periods, the Australian MLFS series are locally approximately linear. To give an indication of what would occur in practice, a range of values was chosen in the proportion of employed and unemployed series which gave an approximately linear series over the time period of interest. For the employed series this period included June 1987, while for the unemployment series the period included October 1988. Successive data values were then added to these months to calculate the revision of the trend estimates, which was taken as the bias component. The resulting MSEs of revisions are presented in Table 3 for the trend estimates and in Table 6 for the estimates of one-month change in the trend.

Consider the one-month revisions of the estimates of the trend. The MSE ratios are similar, but larger in magnitude across the different rotation patterns, when compared with a constant series for both filter combinations. Gains achieved by changing rotation patterns are smaller when compared with the constant series as there is now a contribution for A which is not affected by rotation. Rotation patterns with high monthly overlap reduce the MSE of the one-month revision for a linear series. Rotation patterns with the same design of monthly sample overlap have very similar MSE ratios.

The parameter ϕ for the particular month under consideration for combination 1 for a one-month revision is 1.69 for employment and 0.086 for unemployment. Under combination 2 these values become 1.45 and 1.66, respectively. The rotation pattern will still have an appreciable effect on the MSE of the one-month revision, but not as much for a constant series. For example, changing from a 4-8-4(8) to an *in-for*-8 pattern reduces the MSE of the one month revision of a trend estimate for employment by 9 percent and for unemployment 17 percent for combination 1. However, under combination 2, changing from a 4-8-4(8) to an *in-for*-8 pattern reduces the MSE of the one-month revision for employment by 6 percent and for unemployment by 4 percent.

Again, when looking at the three month revision the conclusions change considerably. The best performing rotation patterns are the 1-1-1(6), 1-2-1(5) and 1-2-1(8). For example, changing from a 4-8-4(8) to a 1-2-1(8) pattern under combination 1 reduces the MSE of the three-month revision by 32 percent for employed and 25 percent for unemployed. For combination 2 the gains from changing from 4-8-4(8) to 1-2-1(8) are 18 percent for employed and 12 percent for unemployed. The gains are again smaller when compared with the constant series. The *in-for*-8 design performs almost as well as the 1-2-1(m) and 1-1-1(6) designs under combination 2 for the three-month revision. The magnitude of ϕ for the three-month revision ranged from 0.39 for employment under combination 2 up to 0.74 for employment under combination 1. This range indicates that the rotation pattern will still have an impact on the MSE of the three-month revision. Note that we have used survey estimates y_t to calculate values of ϕ. As the survey estimate will be more variable than the true series, these values are likely to overestimate the actual values of ϕ.

For the MSE of the revision of the estimates of change in the trend estimates similar conclusions apply as for the constant series. There are considerable gains in using a 1-2-1(m) design for the three month revision. The fundamental conclusion still applies for an approximately linear series for the three-month revisions for both the trend estimates and their one-month change. That is, the 1-b-1(m) designs can generally achieve gains over the other widely used rotation patterns.

5.3. Turning point

Months were chosen in the employed and unemployed series corresponding to a turning point. For the employed series September 1990 was used, while for the unemployed series October 1990 was used. The MSE of the revision of trend estimates and one-month change in them were calculated and the results are presented in Tables 4 and 7.

The bias component completely dominates the variance in these cases. There is no substantial difference between the MSE of the revisions for the different rotation patterns for either the revision of the initial estimates or the estimates of change. For the one-month revision for employment ϕ was 3.27 under combination 1 and 3.04 under combination 2. For unemployment ϕ was 4.71 under combination 1 and 15.20 under combination 2. Examining columns 1 and 2 and columns 5 and 6 in Table 4 shows that there is still a slight advantage in changing rotation patterns, although not as great as in the linear or constant series case. For a three-month revision values of ϕ range from 5.95 for unemployment under combination 1 and 40.46 for unemployment under combination 2. The rotation pattern now has practically no influence on the MSE of the revision.

6. Conclusion

By choice of different rotation patterns we can significantly change the sampling error component in the MSE of the revision of the initial trend estimates and also the one-month change in trend estimates obtained under both X11 and X11ARIMA. Our results demonstrate that if the primary concern is the reduction of the MSE of the one-month revisions, then rotation patterns with high monthly overlap such as *in-for*-8 should be used. The *in-for*-8 rotation pattern performs slightly better than the *in-for*-6 rotation pattern, which performs better the 4-8-4(8), which performs considerably better than the 1-2-1(8) pattern.

Different conclusions are obtained for three-month revisions. Considerable reduction in the MSE of the three-month revision can be obtained by using rotation patterns with no monthly overlap such as 1-2-1(8) instead of those currently in use. For X11 the 1-2-1(8) pattern performed better than the *in-for*-8. Under X11ARIMA an *in-for*-8 rotation pattern performed almost as well as 1-2-1(8) when looking at a three-month revision of the trend estimates, but the 1-2-1(8) pattern performed much better for revisions in the estimates of one-month change in the trend estimates. For both X11 and X11ARIMA the *in-for*-6 pattern performed worse than the *in-for*-8, and the 4-8-4(8) performed worse than the *in-for*-6 pattern.

We believe that the three-month revisions of the estimates of the direction of the trend series are the more important as users will expect the trend estimates to have settled down after three months. Users analysing the current trends will be interested in how well the initial trends reflect the final trends, which are effectively those shown after three months of data have been added to the series.

The relative contribution of the term affected by the rotation pattern depends on the properties of the true series around the time point considered and the magnitude of the sampling error. When the period of interest contained a sharp turning point and the sampling error was relatively small as large national estimates were considered, the rotation pattern used was found not to make any significant difference in the MSE of the one-or three-month revisions. However, many periods in the series will be approximately constant or linear, in which case the choice of rotation pattern will have an effect. For subnational estimates and estimates for other subpopulations the sampling errors will be larger than for the national estimates and so the choice of rotation pattern is likely to have an important effect on the MSE of the revisions.

McLaren and Steel (1997, 1998) considered the effect of different rotation patterns on the sampling variance of seasonally adjusted and trend estimates under different realisations of X11 and X11ARIMA. They found that the 1-2-1(m) rotation patterns considerably reduced the sampling variance of the trend estimates and the estimates of one month change in trend estimates. The results here show that the 1-2-1(m) rotation patterns also perform well for three-month revisions of trend estimates.

6. References

Australian Bureau of Statistics (1987). A Guide to Smoothing Time Series - Estimates of ''Trend.'' Australian Bureau of Statistics, cat. 1316.0, Canberra.

Australian Bureau of Statistics (1993). A Guide to Interpreting Time Series Monitoring ''Trend'' - An Overview. Australian Bureau of Statistics, cat. 1348.0, Canberra.

Bell, P.A. (1998). Using State Space Models and Composite Estimation to Measure the Effects of Telephone Interviewing on Labour Force Estimates. ABS Working papers in Methodology no. 98/2.

Dagum, E.B. (1980). The X11ARIMA Seasonal Adjustment Method. Ottawa: Statistics Canada, cat. 12-564E.

Dagum, E.B. (1982). The Effects of Asymmetric Filters on Seasonal Factor Revisions. Journal of the American Statistical Association, 77, 380, 732–738.

Dagum, E.B. (1988). The X11ARIMA/88 Seasonal Adjustment Methods - Foundations and User's Manual. Ottawa: Statistics Canada.

Dagum, E.B. (1996). A New Method to Reduce Unwanted Ripples and Revisions in Trend-Cycle Estimates From X11ARIMA. Survey Methodology, 20, 1, 77–83.

Dagum, E.B., Chhab, N., and Chiu, K. (1996). Derivation and Properties of the X11ARIMA and Census X11 Linear Filters. Journal of Official Statistics, 12, 4, 329–347.

Henderson, R. (1916). Note on Graduation by Adjusted Average. Transactions of the American Society of Actuaries, 17, 43–48.

Kenny, P.B. and Durbin, J. (1982). Local Trend Estimation and Seasonal Adjustment of Economic and Social Time Series. Journal of the Royal Statistical Society, Series A, 145, 1–41.

Lee, H. (1990). Estimation of Panel Correlations for the Canadian Labour Force Survey. Survey Methodology, 16, 283–292.

Linacre, S. and Zarb, J. (1991). Picking Turning Points in the Economy. Australian Economic Indicators, April 1991. Australian Bureau of Statistics, cat. 1350.0.

McLaren, C.H. and Steel, D.G. (1997). The Effect of Different Rotation Patterns on the Sampling Variance of Seasonal and Trend Filters. Proceedings of the American Statistical Association, Section on Survey Research Methods, 790–795.

McLaren, C.H. and Steel, D.G. (1998). The Impact of Different Rotation Patterns on the Sampling Variance of Seasonally Adjusted and Trend Estimates. Preprint 20/98, School of Mathematics and Applied Statistics, University of Wollongong.

Pierce, D.A. (1980). Data Revisions with Moving Average Seasonal Adjustment Procedures. Journal of Econometrics, 14, 95–114.

Shiskin, J., Young, A.H., and Musgrave, J.C. (1967). The X-11 Variant of the Census Method II Seasonal Adjustment Program. Technical Paper 15, Bureau of the Census, U.S. Department of Commerce, Washington, D.C.

Steel, D.G. (1997). Producing Monthly Estimates of Unemployment and Employment According to the International Labour Office Definition, with Discussion. Journal of the Royal Statistical Society, Ser. A, 160, Part 1, 5–46.

Tallis, K. (1995). Survey Design and Signal Extraction: Meeting Changing User Expectations. Bulletin of the International Statistical Institute, Proceedings 50th Session, Book 3, 971–989.

Wolter, K.M. and Monsour, N.J. (1981). On the Problem of Variance Estimation for a Deseasonalized Series. In Current Topics in Survey Sampling, ed. D. Krewski, R. Platek, and J.N.K. Rao. New York: Academic Press.

Received December 1998
Revised April 1999