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Use of Deflators in Business Surveys: An Analysis Based on Italian Micro Data

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This article aims at evaluating the deflation method used in the Bank of Italy's business surveys. Here the deflators of investments and revenues for each business are individually collected. Currently, deflation for computing changes of monetary values at constant prices is based on sector averages of individual deflators. The option to deflate individual changes with the corresponding individual deflator is also explored. Efficiency gains appear to be linked to the existing correlation between nominal change and the deflator chosen and to the extent of measurement error. Some theoretical results are exposed, linking the deflation method used to the classical Laspeyres and Paasche formulae. Several estimators of real rates of change of revenues and investments, based on individual and average deflation, are then selected and tested in a 6-year simulation study. In the absence of error, the individual deflation of revenues significantly decreases the MSE of the estimators. When unbiased and symmetrical measurement errors in deflation are accounted for, regression-based average deflators and individual deflators of revenue rates of change perform better than the other deflation techniques in terms of MSE of the revenue real growth rate.

Key words: Deflation; rates of change; price indices; mean square error; simulation study; measurement error; reliability theory.

1. Introduction

Measuring economic aggregates at constant prices often poses formidable practical and methodological problems. In practice, it seldom happens that reliable information on quantities and prices is at hand simultaneously. Business surveys are often used to study the evolution of aggregates such as revenues from sales and investments. Research interest often lies in changes expressed in real terms, i.e., net of price growth rate (from now on we will use the expression growth rate for price to indicate both negative and positive price changes). Delicate issues in deflation techniques arise naturally in studies on changes in productivity and quality, nourishing an ample literature. For the basic problem of measuring "real" changes in monetary aggregates over time, nominal values have thus to be deflated before comparison. The common practice of official statistical institutes is to deflate nominal values with an average deflator. On the one hand, this is the only viable

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strategy if deflators come from external sources; on the other hand, there are of course surveys where deflators are directly collected, typically those about sales, but also in these cases the common choice is to use average deflators (see for example ISTAT 2002 and Eurostat 2002).

In the absence of measurement errors, individual deflators are unbiased and therefore every nominal change should be deflated via its individual price growth rate. On the other hand, average deflators might be more stable than individual ones, particularly when deflating monetary values from small domains through average deflators calculated over larger domains. Moreover, averages would likewise compensate for measurement errors that do not follow a systematic pattern and are a simple and effective replacement for missing individual deflators.

If we shift our focus from deflators to the evolution of aggregate real changes, we can wonder how much this is affected by the deflator used for their construction. Our article attempts to tackle this issue.

In a bootstrap experiment, the bias of real change, based on some forms of average deflator is accordingly computed against the one based on the individual deflator. The main interest lies therefore in how much distortion is introduced by giving up individual deflation and whether a compensating decrease in variance adds up to a lower mean squared error (MSE). Previous direct attempts in the literature to analyse this trade-off are not easy to find.

Dealing with a monetary variable, if we define the individual nominal change as the ratio between individual monetary levels at times t and $t-1$, its correlation with the corresponding individual deflator varies according to the relationship existing between volumes and prices. The correlation is perfect if volumes are fixed, because the nominal change will rise proportionally with the increase of prices; the correlation is positive (albeit not perfect) if prices and quantities are positively correlated, but is reduced and may become negative in the opposite case.

If a significant positive correlation exists, we show that, in order to calculate the real change of an aggregate, the use of average deflators instead of individual ones may cause an increase in its variance through the attenuation of that positive correlation. Only regression-based average deflators are on a par with the individual deflator in terms of MSE of the resulting rate of change. These results are robust under classical measurement error scenarios.

The study is based on data from the Bank of Italy's Industrial Business Survey. The amount of missing deflators being fairly modest (around 11%), we will not deal with the topics of nonresponse and imputation for individual deflators here. Observations without individual deflators were simply not considered.

Section 2 is devoted to a theoretical overview of deflators and real changes of monetary variables, while Sections 3 and 4 present the Bank of Italy's business survey and the currently used deflation technique. Section 5 compares an estimator of the real rate of change based on individual deflators with an array of other real ratios of change based on average deflators in a simulation experiment aimed at assessing their MSE. Section 6 presents an alternative experiment simulating the presence of measurement error in the deflators, with the illustration of their effect on the MSE of the real changes. Section 7 summarises and concludes.

2. Deflators and Real Changes

Assessing the changes in monetary values net of price growth rate (the real rate of change) is a typical issue in economics (Guarini and Tassinari 1990). Real changes are rarely measured directly. Nominal values are usually deflated via price indices.

The dilemma existing between measuring quantities directly and dividing their monetary value by a price index is normally resolved in favour of the latter, essentially for practical reasons. The theoretical difficulties involved in individualising an adequate "representative good" in terms of which to measure the quantities of other goods now concern instead the need for an adequate choice of numeraire (the price index). This raises some delicate methodological points which we will not handle here (see, for example Hill 1971). These points, however, are less relevant when estimating percentage changes.

If separate data for quantities and prices are not collected, a business survey may in any case gather individual data about the total nominal levels at time 1, $A_1 = q_1 p_1$ and at time 0, $A_0 = q_0 p_0$ and the price growth rate $d = p_1/p_0$ between times 1 and 0, as declared by the individual firm. Hereafter A stands alternatively for revenues or investments.

Variables A_0, A_1, and $d = p_1/p_0$ are observed; q_0, q_1 are not. The relationship between nominal values (A), prices (p) and quantities (q) measured at $t = 0, 1$ follows the obvious equivalence

$$\frac{q_1}{q_0} = \frac{A_1/p_1}{A_0/p_0} = \frac{A_1/A_0}{p_1/p_0} = \frac{A_1/A_0}{d}$$

Real changes are thus calculated by dividing the nominal rate by the price index (the deflator $d = p_1/p_0$). For aggregate changes, the conventional Laspeyres and Paasche formulae for quantities are normally used. They can be written (indexing individual data by subscript i) as

$$R_L = \frac{\sum_i q_{i,1} p_{i,0}}{\sum_i q_{i,0} p_{i,0}} = \frac{\sum_i (A_{i,1}/d_i)}{\sum_i A_{i,0}} \tag{1}$$

$$R_P = \frac{\sum_i q_{i,1} p_{i,1}}{\sum_i q_{i,0} p_{i,1}} = \frac{\sum_i A_{i,1}}{\sum_i A_{i,0} d_i} \tag{2}$$

In business surveys, when it comes to evaluating real changes of monetary variables, they are often measured through ratios like (1) or (2). We will call such ratios real changes (the transformation of a ratio into a per cent rate of change by subtracting the unit from it and multiplying the result by 100 is a simple change of scale).

Using m for a given domain of study and d_i^* for the chosen deflator (either individual or average), we write the indices again, explicitly inserting the domain and showing them as weighted averages of the individual ratios $(A_{i,1}/A_{i,0})/d_i^*$:

$$ {}^{m}R_{\mathrm{L}} = \frac{\sum_{i\in m}(A_{i,1}/d_i^*)}{\sum_{i\in m}A_{i,0}} = \frac{\sum_{i\in m}\left(\frac{A_{i,1}/A_{i,0}}{d_i^*}\right)A_{i,0}}{\sum_{i\in m}A_{i,0}} \tag{3} $$

$$ {}^{m}R_{\mathrm{P}} = \frac{\sum_{i\in m}A_{i,1}}{\sum_{i\in m}A_{i,0}d_i^*} = \frac{\sum_{i\in m}\left(\frac{A_{i,1}/A_{i,0}}{d_i^*}\right)A_{i,0}d_i^*}{\sum_{i\in m}A_{i,0}d_i^*} \tag{4} $$

(survey weights are for simplicity left out of these formulae). In the Laspeyres formula, individual real ratios are weighted with base levels $A_{i,0}$. In the Paasche formula, inflated base levels $A_{i,0}d_i^*$ are used instead. We will also refer to these formulae as Laspeyres and Paasche quantity indices.

If ${}^{m}\bar{d}$ indicates an average deflator over the domain m, it follows that, if ${}^{*}d_i = {}^{m}\bar{d}$ for all i, then the two have the same value. That is:

If the chosen deflator is constant on the domain, the Laspeyres and Paasche quantity indices coincide with the ratio between the nominal change $\sum_{\mathrm{i\in m}}A_{i,1}/\sum_{\mathrm{i\in m}}A_{i,0}$ *and the deflator* ${}^{m}\bar{d}$.

If we introduce the Laspeyres and the Paasche price indices

$$ {}^{m}P_{\mathrm{L}} = \frac{\sum_{i\in m}A_{i,0}d_i}{\sum_{i\in m}A_{i,0}} \tag{5} $$

$$ {}^{m}P_{\mathrm{P}} = \frac{\sum_{i\in m}A_{i,1}}{\sum_{i\in m}(A_{i,1}/d_i)} \tag{6} $$

the previous result can also be seen as a consequence of Uggè's result (1946; see also Predetti 1994, pp. 37–39):

$$ {}^{m}R_{\mathrm{P}} = {}^{m}R_{\mathrm{L}} + \frac{\mathrm{Cov}(p_1/p_0, q_1/q_0)}{{}^{m}P_{\mathrm{L}}} $$

since the covariance between individual price indices and individual quantity indices vanishes if individual price indices are all equal (as implied by the choice of an average). Such issues appear to have been empirically addressed in a paper by Horner and Coleman (1971) that analyses the effect of grouping of products for the purpose of constructing a quantity index.

If the same domain m is used both for average deflation and real ratios of changes, the nominal change is decomposed into the product of a price index and a real ratio of change deflated with individual deflators, in the following way (Predetti 1994, pp. 47–48):

$$ \frac{\sum_{i\in m}A_{i,1}}{\sum_{i\in m}A_{i,0}} = {}^{m}P_{\mathrm{P}} \cdot {}^{m}R_{\mathrm{L}} \tag{7} $$

$$\frac{\sum_{i \in m} A_{i,1}}{\sum_{i \in m} A_{i,0}} = {}^{m}P_{\mathrm{L}} \cdot {}^{m}R_{\mathrm{P}} \tag{8}$$

For each unit i, there are therefore three choices for the deflator d_i^* used in (3) and (4):

1) as the individual deflator d_i, collected at the respondent's level;
2) as an average ${}^{m}\bar{d}$ of the individual deflators, calculated over all the units of a domain m;
3) as an average $\bar{d}$ of individual deflators, calculated within some other domain.

For cases 2) and 3), the average can either come from within the survey or from an external source.

The use of average deflators calculated within cells independent of the domain of study for which the real aggregate change is calculated is a common approach of survey data dissemination. The underlying assumption is that price dynamics of firms are presumably driven by similar factors within the same cell. In this case, the decompositions (7) and (8) no longer hold.

The choices of the mean and of the weighting factors for the average deflators are essentially driven by empirical considerations. The most used aggregate price index is a Laspeyres price index, with periodically updated weights. More complex price indices are used in special applications, such as the NBER manufacturing productivity database (Bartelsman and Gray 1996).

3. An Outline of the Bank of Italy's Industrial Business Survey

The Bank of Italy has been conducting business surveys since 1972 (Banca d'Italia 2005). The original target population, composed of manufacturing firms with 50 employees or more until 1998, was progressively enlarged to include all the industrial firms (including the energy, mining and quarrying sectors) with 50 employees or more as of 1999 and, from 2001 on, those with more than 19 employees. The survey is run every year. Interviews are conducted in the first months of the year $t+1$ and collect data for the years $t-1$ and t, together with forecasts for the current year $t+1$.

The sampling design is stratified with a single stage. The sample size is determined by Neyman's optimum allocation to strata criterion, in order to minimise the variance of the means of the main variables of interest (investments, revenues and number of employees) within the size classes. The sample units are originally chosen at random and always recontacted, provided that they still belong to the target population. Refusals and firms no longer in the target population are routinely replaced with similar units.

Much care is devoted to data quality checks. The panel survey structure enables us to monitor data consistency across time within the same firm (see Duncan et al. 1989). Outliers are spotted through selective editing techniques (Cox et al. 1995), in order to limit the respondent burden.

Firms that were subject to mergers or acquisitions during the years $t-1$ or t are used for the estimates only if data are collected for the same set of local units and employees for the three years $t-1$, t and $t+1$. Such collection is performed either by fictitiously putting back the merger/acquisition event to the beginning of year $t-1$ or by postponing it to the end of year $t+1$. The method may lead to biased totals, but provides stable estimates of per capita values and rates of change, which are the main interests of the survey.

The utilisation of one survey to estimate rates of change, instead of two contiguous surveys, provides estimates less influenced by structural changes of the enterprises (which would be difficult to consider) and by different measurement error patterns (best treated within a single survey). The population used for the construction of the grossing up weights refers to year $t-3$ (the most recent available when estimates are produced) and is therefore kept fixed for the three years considered, without taking into account possible births and deaths of enterprises.

The estimation process takes place under the fixed population approach (Särndal et al. 1992). The target population can be represented as a set of $\{1, \ldots, i, \ldots, N\}$ labels, each associated with an array $\mathbf{X}_i$ containing, for the unit i, the selection probability and the values of the variables. Every element of the array is assumed a constant.

The weighting process that produces the final weights involves two steps. First, every firm in the sample receives a weight given by the ratio between the total number of firms and the actual sample size in the stratum (strata are formed by the combination of size classes and economic sectors). Second, a post-stratification adjustment to the geographical location of firms' headquarters is performed. In order to limit the amount of the post-stratification adjustments, this is based on four aggregations (North-West; North-East; Centre; South and Islands) of the 20 Italian regions.

4. The Deflation Technique Used in the Bank of Italy's Business Survey

In the Bank of Italy's survey, the sample units with 50 employees or more are asked to provide, for revenues and investments: *a*) the total monetary values for the three years considered; *b*) the deflator, as evaluated by the firm on the basis of their available information. For revenues, firms are asked the overall price growth rate against the previous year for the goods they sell. For both revenues and investments, they provide the overall price growth rate the following year, which for revenues is the average planned price hike/reduction of the goods sold, while for investment it is the firm's forecast of the price dynamics of the investment goods it is going to buy.

Up to and including 2002 the investments deflator for the current year was the previous year's forecast of price growth rate for investment goods. Starting from 2003, deflators for the current year are asked for investment goods too and accordingly used to construct average deflators.

The survey collects individual deflators on the assumption that price growth rates gathered directly within the sample will be much more suitable than deflators coming from external surveys. These would carry a double source of distortion:

1) the basket of goods used for its calculation might not be representative of the goods dealt with in the survey;
2) firms from which data are collected to build the index might not be representative of the survey target population.

Direct collection of price growth rates entails of course the presence of measurement error, but even official price indices are not immune to them. Survey planners thought that the advantage of direct collection in terms of unbiasedness would outweigh the drawback of the introduction of a new error source in the survey process.

Average deflators were initially chosen as this seemed a robust and viable way of resolving typical measurement issues. They are calculated within cells formed by the economic sectors, where price growth rates for firms are presumably influenced by the same factors. In order to rank firms' contributions according to their size, for both revenues and investments the weight is the product between the sample weight and the nominal value of revenues for year $t-1$. The average deflator is a Laspeyres price index of the type shown in Formula (5), which we indicate with ${}^{s}d$, where s stands for the generic economic sector in the set $S = \{1, 2, \ldots, s, \ldots\}$. To calculate the real ratio of change for the domain m, Formula (3) is used which, after inserting the survey weights w_i, becomes

$$ {}^{m}R_{\mathrm{L}}^{s} = \frac{\sum_{s \in S} I(m \cap s) \frac{1}{{}^{s}d} \sum_{i \in m \cap s} A_{i,1} w_i}{\sum_{i \in m} A_{i,0} w_i} \quad \text{where } I(m \cap s) = \begin{cases} 1 & \text{if } m \cap s \neq \varnothing \\ 0 & \text{otherwise} \end{cases} \tag{9} $$

As a simple example of how (9) works, let us suppose that our domain of interest m is the class size, labelled "50–99," composed of firms with number of employees between 50 and 99. If we use average deflators by economic sectors, let S' be the subset of economic sectors of S which are found within firms belonging to the 50–99 class size. In such a case (9) becomes

$$ {}^{50-99}R_{\mathrm{L}}^{s} = \frac{\sum_{s \in S'} (1/{}^{s}d) \sum_{i \in 50-99} A_{i,1} w_i}{\sum_{i \in 50-99} A_{i,0} w_i} $$

As (8) shows, when the domain m coincides with one of the cells used for the calculation of average deflators, (9) is equal to the Paasche estimator (4) based on individual deflators (see Appendix A).

5. The First Experiment

Our purpose is to compare aggregate real changes based on individual deflators against those based on average deflators, using the Bank of Italy's survey data. In order to have homogeneous data at our disposal for several consecutive survey years, we restrict our attention to the manufacturing firms with 50 employees or more.

We will focus our attention on the Laspeyres estimator (3), to which we apply the individual deflation and various forms of average deflators (including the one currently used). The Paasche expression (4) will be considered only with individual deflators, in order to assess the different behaviour of individual deflation at the "other extreme."

We assume the Laspeyres quantity index based on individual deflation to be unbiased. It is the benchmark against which all the other estimators are compared.

The reliability of individual deflators should be judged on the ground of the firm's ability to provide them. Firms seem prepared for such an assessment. Investment and sales are recurrent business activities for which enterprises set up accurate accounting and planning procedures, including an assessment of price growth rate. A further factor increasing the overall quality of collected data is the panel nature of the sample (albeit with a limited degree of attrition). Repeated survey participations help firms understand the questions they are asked, thereby minimising nonsampling error. This picture contrasts with other instances of price movement evaluations, such as those asked in household surveys, in which interviewees might think about the problem of prices for the first time and therefore be very widely off the mark.

All the estimators under review are weighted means of the individual terms $(A_{i,1}/A_{i,0})/d_i^*$ and their variance is not easily tractable, as it nonlinearly involves three random variables $A_{i,0}$, $A_{i,1}$ and d_i. Appendix B suggests an explanation of the inverse link between the variance of the aggregate real change and the correlation between individual ratios $A_{i,1}/A_{i,0}$ and the deflators d_i^*.

Measurement errors are not treated in this first experiment. We evaluate the performance of the estimators through their mean squared error (MSE), which requires the calculation of the bias and variance for each of them.

5.1. *Description*

The bootstrap (Efron 1982; Davison and Hinkley 1997) appears to be a flexible choice to appraise the components of the MSE of the estimators of real changes. We selected 14 different estimators – indexed from 1) to 14). The first thirteen, of which 1) uses individual deflators, are calculated with the Laspeyres Formula (3), while the estimator 14) follows the Paasche Formula (4) with individual deflators. The estimators 2)–5) use average Laspeyres deflators, 6)–9) use Paasche deflators, while 10)–13) rely on deflators derived from regressions with dummy variables. Such models can take into account the effect of more than one discrete variable at a time. Dummies formed by variable interactions were not considered, since preliminary data analyses showed they were not relevant in the models.

The formulae of the estimators are presented below. The symbol m again denotes the domain of interest, while o (*overall*) indicates the whole target population.

1) $$^{m}R_{\mathrm{L}} = \frac{\sum_{i \in m} (A_{i,1}/d_i) w_i}{\sum_{i \in m} A_{i,0} w_i}$$

(d_i are the individual deflators provided by the firms in the sample);

2) $$^{m}R^{o}_{\mathrm{L}} = \frac{\sum_{i \in m} (A_{i,1}/{}^{o}d_{\mathrm{L}}) w_i}{\sum_{i \in m} A_{i,0} w_i} \text{ with } {}^{o}d_{\mathrm{L}} = \frac{\sum_{i \in o} w_i A_{i,0} d_i}{\sum_{i \in o} w_i A_{i,0}}$$

(*overall* Laspeyres deflator);

3) $$^{m}R^{s}_{\mathrm{L}} = \frac{\sum_{s \in S} I(m \cap s) \frac{1}{{}^{s}d_{\mathrm{L}}} \sum_{i \in m \cap s} A_{i,1} w_i}{\sum_{i \in m} A_{i,0} w_i} \text{ with } {}^{s}d_{\mathrm{L}} = \frac{\sum_{i \in s} w_i A_{i,0} d_i}{\sum_{i \in s} w_i A_{i,0}}$$

(Laspeyres deflator by *economic sectors* $S = \{1, 2, \ldots, s, \ldots\}$, defined as: food products, beverages and tobacco; textiles, clothing, leather, shoes; chemicals, rubber, plastic products; nonmetal minerals; engineering; other manufacturing). The indicator function $I(m \cap s)$ for this formula and all the ones that follow indicates that we consider only the price indices ${}^{s}d$ calculated for the sectors where some sample units belonging to m can be found. This is the estimator currently employed for the production of the survey results;

4) $${}^{m}R_{\mathrm{L}}^{C} = \frac{\sum_{c \in C} I(m \cap c) \frac{1}{{}^{c}d_{\mathrm{L}}} \sum_{i \in m \cap c} A_{i,1} w_i}{\sum_{i \in m} A_{i,0} w_i} \text{ with } {}^{c}d_{\mathrm{L}} = \frac{\sum_{i \in c} w_i A_{i,0} d_i}{\sum_{i \in c} w_i A_{i,0}}$$

(Laspeyres deflator by *size classes* $C = \{1, 2, \ldots, c, \ldots\}$, defined in terms of number of employees as 50–99; 100–199; 200–499; 500–999; 1,000 and more);

5) $${}^{m}R_{\mathrm{L}}^{G} = \frac{\sum_{g \in G} I(m \cap g) \frac{1}{{}^{g}d_{\mathrm{L}}} \sum_{i \in m \cap g} A_{i,1} w_i}{\sum_{i \in m} A_{i,0} w_i} \text{ with } {}^{g}d_{\mathrm{L}} = \frac{\sum_{i \in g} w_i A_{i,0} d_i}{\sum_{i \in g} w_i A_{i,0}}$$

(Laspeyres deflator by *geographical areas* $G = \{1, 2, \ldots, g, \ldots\}$, defined as North-West; North-East; Centre; South and Islands);

6) $${}^{m}R_{\mathrm{L}}^{O} \text{ same as 2) with } {}^{o}d_{\mathrm{P}} = \frac{\sum_{i \in o} w_i A_{i,1}}{\sum_{i \in o} (w_i A_{i,1} / d_i)}$$

(*overall* Paasche deflator) instead of ${}^{o}d_{\mathrm{L}}$;

7) $${}^{m}R_{\mathrm{P}}^{S} \text{ similar to 3) with } {}^{s}d_{\mathrm{P}} = \frac{\sum_{i \in s} w_i A_{i,1}}{\sum_{i \in s} (w_i A_{i,1} / d_i)}$$

(Paasche deflator by *economic sectors*) instead of ${}^{s}d_{\mathrm{L}}$;

8) $${}^{m}R_{\mathrm{P}}^{C} \text{ similar to 4) with } {}^{c}d_{\mathrm{P}} = \frac{\sum_{i \in c} w_i A_{i,1}}{\sum_{i \in c} (w_i A_{i,1} / d_i)}$$

(Paasche deflator by *size classes*) instead of ${}^{c}d_{\mathrm{L}}$;

9) $${}^{m}R_{\mathrm{P}}^{G} \text{ similar to 5) with } {}^{g}d_{\mathrm{P}} = \frac{\sum_{i \in g} w_i A_{i,1}}{\sum_{i \in g} (w_i A_{i,1} / d_i)}$$

(Paasche deflator by *geographical areas*) instead of ${}^{g}d_{\mathrm{L}}$;

10) $${}^{m}R_{\mathrm{L}}^{S+C} = \frac{\sum_{s\in S}\sum_{c\in C} I(m\cap s\cap c)\frac{1}{{}^{s,c}d}\sum_{i\in m\cap s\cap c} A_{i,1}w_i}{\sum_{i\in m} A_{i,0}w_i}$$

where ${}^{s,c}d$ is the predicted value from a linear regression of the individual deflators on dummy variables representing economic sectors and size classes;

11) $${}^{m}R_{\mathrm{L}}^{S+G} = \frac{\sum_{s\in S}\sum_{g\in G} I(m\cap s\cap g)\frac{1}{{}^{s,g}d}\sum_{i\in m\cap s\cap g} A_{i,1}w_i}{\sum_{i\in m} A_{i,0}w_i}$$

where ${}^{s,g}d$ is the predicted value from a linear regression of the individual deflators on dummy variables representing economic sectors and geographical areas;

12) $${}^{m}R_{\mathrm{L}}^{C+G} = \frac{\sum_{c\in C}\sum_{g\in G} I(m\cap c\cap g)\frac{1}{{}^{c,g}d}\sum_{i\in m\cap c\cap g} A_{i,1}w_i}{\sum_{i\in m} A_{i,0}w_i}$$

where ${}^{c,g}d$ is the predicted value from a linear regression of the individual deflators on dummy variables representing class sizes and geographical areas;

13) $${}^{m}R_{\mathrm{L}}^{S+C+G} = \frac{\sum_{s\in S}\sum_{c\in C}\sum_{g\in G} I(m\cap s\cap c\cap g)\frac{1}{{}^{s,c,g}d}\sum_{i\in m\cap s\cap c\cap g} A_{i,1}w_i}{\sum_{i\in m} A_{i,0}w_i}$$

where ${}^{s,c,g}d$ is the predicted value from a linear regression of the individual deflators on dummy variables representing economic sectors, class sizes and geographical areas;

14) $${}^{m}R_{\mathrm{P}} = \frac{\sum_{i\in m} A_{i,1}}{\sum_{i\in m} A_{i,0}d_i}$$

with individual deflators d_i, as in 1).

The domain m is, at the most aggregate level, the whole Italian manufacturing sector (restricted to enterprises with 50 employees or more) and, more analytically, all the possible values of *sector of economic activity*, *geographical area* and *size class* (employees). Survey results are usually broken down separately at this level of detail.

500 sample replicates are drawn with replacement from the original sample for the six surveys covering the years 1997–2002. The draws are made within the survey strata, with sample size the same as in the case of the original sample: this means that there is no need to compute new weights for each replication (the loss of precision due to not computing the post-stratification adjustment is negligible).

If X denotes any of the estimators 1)–14), the variance $\sigma^2(X)$ and the mean $\bar{X}$ over all the replicates are calculated for each domain. We suppose ${}^mR_L$ to be unbiased and consequently its value μ on the original sample is regarded as the "true" one. The mean squared error (MSE) of X takes the form

$$\mathrm{MSE}(X) = \sigma^2(X) + (\bar{X} - \mu)^2$$

If $X = {}^m R_L$, the second term (the bias component) is negligible and vanishes as the number of bootstrap replications increases. The bias of $X = {}^m R_P$ is intrinsically due to the different form of the Paasche quantity index (4).

5.2. *Results*

Table 1 shows for all the estimators:

(1) the MSE expressed in percentage of MSE(${}^mR_L$);
(2) the variance expressed in percentage of $\sigma^2({}^mR_L)$;
(3) the variance in terms of relative contribution to its own MSE;
(4) the bias in terms of relative contribution to its own MSE.

As shown in Section 2, real changes 2)–5) are equivalent to the Paasche estimator 14) whenever the domains of interest coincide with the cells used to calculate the Laspeyres deflators. Estimators 6)–9) are equivalent to 1) if the domains of interest coincide with the cells used to calculate the Paasche deflator and the same equivalence holds for the overall domain, however the cells used to calculate the Paasche deflators are selected. All these properties are presented in Appendix A and are a consequence of (7) and (8).

The simulation results appear multi-faceted. For *revenues*, the MSE and the variances tend to increase if average deflators replace individual ones (Table 1), which can be explained by the correlation between individual deflators d_i and nominal ratios $A_{i,1}/A_{i,0}$, being almost always statistically significant (Table 2).

A significant positive correlation should produce a shrinkage effect on the variance (see Appendix C): this effect is weakened with the average deflators, causing the increase of the variance and hence of the MSE of the real changes. Another factor augmenting the MSE is the bias component (Table 1). Apart from the Paasche estimator (and its equivalents) for domains different from the total, only the estimators deflated with regression-based price indices feature performances comparable to the benchmark. This fact can be explained by the composite structure of these deflators, which manages to preserve the correlation between individual nominal ratios and deflators.

Quite surprisingly, on average the bias in the calculation of real changes of revenues generated by replacing individual deflators with averages is not offset by a variance reduction and therefore always entails a cost in terms of MSE.

In contrast, for *investment*, MSE and variances tend to be virtually equal to those of the benchmark (Table 1). This result too can be justified (as seen in Table 2) by the correlation between individual deflators and nominal ratios, quite low and not statistically significant except for one year. In such a case, the replacement of individual deflators with averages does not produce reductions in variance and MSE.

Two possible factors could explain the small correlation between nominal rates of change and individual deflators for investment. The first one is economic and pertains to

Table 1. Average MSE, variance and bias[a] of real changes (percentages)

Real change	Total				Sector of economic activity[b]				Class size[c]				Geographical area[d]			
		Variance				Variance				Variance				Variance		
	MSE as % of the MSE of ^mR_L	As % of the variance of ^mR_L	As % of its own MSE	Bias as % of its own MSE	MSE as % of the MSE of ^mR_L	As % of the variance of ^mR_L	As % of its own MSE	Bias as % of its own MSE	MSE as % of the MSE of ^mR_L	As % of the variance of ^mR_L	As % of its own MSE	Bias as % of its own MSE	MSE as % of the MSE of ^mR_L	As % of the variance of ^mR_L	As % of its own MSE	Bias as % of its own MSE
Revenues																
1) ^mR_L	100.0	100.0	100.0	0.0	100.0	100.0	100.0	0.0	100.0	100.0	100.0	0.0	100.0	100.0	100.0	0.0
2) $^mR_L^O$	207.6	184.9	85.8	14.2	465.6	271.7	31.0	69.0	250.2	178.2	43.7	56.3	211.8	156.7	39.8	60.2
3) $^mR_L^S$	137.4	124.8	92.0	8.0	120.1	116.5	97.1	2.9	153.1	114.1	55.9	44.1	145.5	113.8	39.7	60.3
4) $^mR_L^C$	146.6	132.7	90.8	9.2	308.2	188.2	34.8	65.2	129.3	124.9	96.1	3.9	178.5	137.4	34.7	65.3
5) $^mR_L^G$	187.2	167.5	87.4	12.6	329.8	198.9	34.5	65.5	225.1	164.9	42.8	57.2	114.4	107.3	57.6	42.4
6) $^mR_P^O$	100.0	100.0	100.0	0.0	405.9	233.6	26.5	73.5	216.2	158.4	38.9	61.1	165.6	116.6	38.1	61.9
7) $^mR_P^S$	100.0	100.0	100.0	0.0	100.0	100.0	100.0	0.0	150.0	114.5	51.0	49.0	132.2	100.8	39.8	60.2
8) $^mR_P^C$	100.0	100.0	100.0	0.0	301.3	181.5	30.9	69.1	100.0	100.0	100.0	0.0	155.6	114.5	35.1	64.9
9) $^mR_P^G$	100.0	100.0	100.0	0.0	320.9	187.7	29.4	70.6	204.6	151.5	38.4	61.6	100.0	99.2	100.0	0.0
10) $^mR_L^{S+C}$	114.3	104.8	94.1	5.9	119.9	115.5	96.8	3.2	127.8	121.8	96.1	3.9	146.1	117.1	36.6	63.4
11) $^mR_L^{S+G}$	134.7	122.6	92.5	7.5	119.9	116.5	97.2	2.8	151.1	113.9	55.3	44.7	108.1	101.8	57.6	42.4
12) $^mR_L^{C+G}$	132.9	120.9	92.3	7.7	239.4	151.8	41.6	58.4	129.6	124.7	96.2	3.8	112.0	103.8	57.9	42.1
13) $^mR_L^{S+C+G}$	112.3	103.2	94.5	5.5	119.3	114.7	96.9	3.1	126.9	120.8	96.1	3.9	106.3	99.1	57.2	42.8
14) ^mR_P	207.6	184.9	85.8	14.2	120.1	116.5	97.1	2.9	129.3	124.9	96.1	3.9	114.4	107.3	57.6	42.4
Investment																
1) ^mR_L	100.0	100.0	100.0	0.0	100.0	100.0	100.0	0.0	100.0	100.0	100.0	0.0	100.0	100.0	100.0	0.0
2) $^mR_L^O$	99.7	99.3	99.6	0.4	98.9	97.7	98.5	1.5	99.8	98.8	99.2	0.8	100.2	99.7	99.7	0.3
3) $^mR_L^S$	99.9	99.5	99.6	0.4	99.6	98.8	98.9	1.1	99.8	98.8	99.1	0.9	100.2	99.7	99.7	0.3
4) $^mR_L^C$	99.8	99.4	99.5	0.5	99.1	97.8	98.5	1.5	99.7	98.9	99.2	0.8	100.5	100.0	99.7	0.3

Table 1. Continued

Real change	Total				Sector of economic activity[b]				Class size[c]				Geographical area[d]			
		Variance				Variance				Variance				Variance		
	MSE as % of the MSE of ^mR_L	As % of the variance of ^mR_L	As % of its own MSE	Bias as % of its own MSE	MSE as % of the MSE of ^mR_L	As % of the variance of ^mR_L	As % of its own MSE	Bias as % of its own MSE	MSE as % of the MSE of ^mR_L	As % of the variance of ^mR_L	As % of its own MSE	Bias as % of its own MSE	MSE as % of the MSE of ^mR_L	As % of the variance of ^mR_L	As % of its own MSE	Bias as % of its own MSE
5) $^mR_L^G$	99.7	99.3	99.6	0.4	99.1	97.8	98.5	1.5	99.9	98.9	99.1	0.9	100.4	99.8	99.7	0.3
6) $^mR_P^O$	100.0	100.0	100.0	0.0	99.1	97.8	98.5	1.5	100.0	98.9	99.1	0.9	100.4	99.8	99.7	0.3
7) $^mR_P^S$	100.0	100.0	100.0	0.0	100.0	100.0	100.0	0.0	100.0	98.9	99.1	0.9	100.5	99.9	99.7	0.3
8) $^mR_P^C$	100.0	100.0	100.0	0.0	99.2	97.8	98.5	1.5	100.0	100.0	100.0	0.0	100.3	99.7	99.7	0.3
9) $^mR_P^G$	100.0	100.0	100.0	0.0	99.3	97.9	98.5	1.5	100.0	98.9	99.1	0.9	100.0	100.0	100.0	0.0
10) $^mR_L^{S+C}$	99.9	99.6	99.6	0.4	99.7	98.8	98.9	1.1	99.7	99.0	99.2	0.8	100.2	99.7	99.7	0.3
11) $^mR_L^{S+G}$	99.8	99.5	99.6	0.4	99.7	98.8	98.9	1.1	99.9	98.9	99.1	0.9	100.2	99.5	99.7	0.3
12) $^mR_L^{C+G}$	99.8	99.5	99.5	0.5	99.2	97.9	98.6	1.4	99.8	99.0	99.2	0.8	100.7	100.1	99.6	0.4
13) $^mR_L^{S+C+G}$	99.9	99.6	99.6	0.4	99.7	98.9	98.9	1.1	99.8	99.1	99.2	0.8	100.4	99.8	99.7	0.3
14) ^mR_P	99.7	99.3	99.6	0.4	99.6	98.8	98.9	1.1	99.7	98.9	99.2	0.8	100.4	99.8	99.7	0.3

Source: Bank of Italy's Industrial Business Surveys 1997–2002 (Manufacturing firms with 50 employees or more).

[a] Estimators averaged over all the possible categories and all the years.

[b] Food products, beverages and tobacco; textiles, clothing, leather, shoes; chemicals, rubber, plastic products; nonmetal minerals; engineering; other manufacturing.

[c] 50–99; 100–199; 200–499; 500–999; 1,000 and more.

[d] North-West; North-East; Centre; South and Islands, as the location of the enterprise's administrative headquarters.

Table 2. Correlation coefficient between individual nominal rates of change and individual deflators[a]

Year	Revenues	Investments
1997	0.16**	0.04
1998	0.06	0.01
1999	0.16**	0.05
2000	0.21**	0.09**
2001	0.09**	0.00
2002	0.12**	−0.01

Source: Bank of Italy's Industrial Business Surveys 1997–2002 (Manufacturing firms with 50 employees or more).
[a] Referring to test for the null hypothesis H_0: $\rho = 0$, ** denotes a p-value lower than 0.01, * between 0.01 and 0.05; no asterisk denotes a p-value larger than 0.05.

the fact that long-term investment decisions are unlikely to be much influenced by short-term price growth rate (see, for example European Commission 2001). The second one is the forecasting error attached to the use of a prospective price growth, which is increased by the respondent's preference for round values when asked for a forecast percentage, whereas a more widespread range of values tends to be used for percentages referring to the past. We found an empirical proof of this behaviour in the 2003 survey, for which actual investment deflators were collected for the first time (see Section 5.3 for further details) and exhibited a larger variance than the previous forecasts.

5.3. The Experiment with Actual Investment Deflators in the 2003 Survey

The survey has traditionally collected data about the forecast price growth rate of investment goods, because this factor influences the decision to invest, together with other long-term factors. Discussion within the survey staff, stimulated by the results of the present article, suggested the insertion in the survey questionnaire, starting from 2003, of the actual price growth rate in investment. The correlation coefficient between nominal rates of change and actual individual deflators is still not significantly different from zero for the year 2003 and, apart from a few outliers, forecast and actual deflators are indeed very close. A simulation experiment using actual deflators along the same lines as the previous one produced results (Table 3) very similar to those shown in Table 1. Results with forecast deflators are very similar and omitted for brevity.

6. The Experiment with Measurement Errors

6.1. Presence of Measurement Error

Lichtenberg and Griliches (1986) pioneered the study of measurement error in industrial price deflators. They individualised two main sources of measurement error in the prices, i.e., the one coming from the multiplicity of concepts of price (list price versus actual transaction price; shipment versus order price; and so on) and the one coming from quality change. Deflators provided by the firms in the Bank of Italy's business survey may also be contaminated by such sources of measurement error.

The available deflators have been treated so far as if they were free of measurement error, yet we must allow for its presence. Another source of this error is the fact that the

Table 3. Average MSE and variance[a] of real changes for investment using actual deflators (percentages)

Real change	Total		Sector of economic activity[b]		Class size[c]		Geographical area[d]	
	MSE as % of the MSE of ${}^{m}R_{L}$	As % of the variance of ${}^{m}R_{L}$	MSE as % of the MSE of ${}^{m}R_{L}$	As % of the variance of ${}^{m}R_{L}$	MSE as % of the MSE of ${}^{m}R_{L}$	As % of the variance of ${}^{m}R_{L}$	MSE as % of the MSE of ${}^{m}R_{L}$	As % of the variance of ${}^{m}R_{L}$
1) ${}^{m}R_{L}$	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2) ${}^{m}R_{L}^{O}$	101.5	101.4	101.7	100.1	102.2	101.5	102.6	102.2
3) ${}^{m}R_{L}^{S}$	101.7	101.6	99.8	98.6	102.3	101.7	102.6	102.3
4) ${}^{m}R_{L}^{C}$	101.4	101.4	100.8	99.4	101.2	101.0	101.8	101.5
5) ${}^{m}R_{L}^{G}$	101.5	101.4	101.6	100.0	102.1	101.4	101.2	100.9
6) ${}^{m}R_{P}^{O}$	100.0	100.0	101.4	99.8	101.8	101.0	102.1	101.8
7) ${}^{m}R_{P}^{S}$	100.0	100.0	100.0	100.0	101.7	101.2	102.2	101.9
8) ${}^{m}R_{P}^{C}$	100.0	100.0	100.6	99.3	100.0	100.0	101.2	101.0
9) ${}^{m}R_{P}^{G}$	100.0	100.0	101.4	99.8	101.6	101.0	100.0	100.0
10) ${}^{m}R_{L}^{S+C}$	101.5	101.5	100.0	98.8	101.3	101.2	101.8	101.5
11) ${}^{m}R_{L}^{S+G}$	101.6	101.6	99.9	98.6	102.1	101.6	101.2	101.0
12) ${}^{m}R_{L}^{C+G}$	101.5	101.5	100.7	99.3	101.3	101.2	101.1	100.9
13) ${}^{m}R_{L}^{S+C+G}$	101.6	101.6	100.0	98.8	101.5	101.3	101.1	100.9
14) ${}^{m}R_{P}$	101.5	101.4	99.8	98.6	101.2	101.0	101.2	100.9

Source: Bank of Italy's Industrial Business Surveys 2003 (Manufacturing firms with 50 employees or more).

[a] Estimators averaged over all the possible categories and all the years.

[b] Food products, beverages and tobacco; textiles, clothing, leather, shoes; chemicals, rubber, plastic products; nonmetal minerals; engineering; other manufacturing.

[c] 50–99; 100–199; 200–499; 500–999; 1,000 and more.

[d] North-West; North-East; Centre; South and Islands, as the location of the enterprise's administrative headquarters.

individual deflator is not derived directly from the firm's balance sheet. It might rather be considered an average of individual growth rates taken on an unknown set of goods and computed through an unknown formula.

The most straightforward way to assess the amount of measurement error in individual deflators would be to use a subsample of control for the estimation of an error distribution, which could estimate the bias and adjust for response error (see Lessler and Kalsbeek 1992). Such a subsample is not available. An alternative could be a benchmark deflator to compare with. Unfortunately the Italian National Statistical Institute (ISTAT) releases deflators (both for goods sold by the industrial firms and purchased by them as investment goods) only by sectors of economic activity, regardless of firm size and geographical location (see, for example ISTAT 2002), making any comparison structurally biased by the use of different cells for the calculation of average deflators.

Anyhow, we can study the effect of measurement error in deflators using only the available survey data. For this purpose we employ the reliability ratio, a basic tool in nonsampling error modelling.

6.2. *The Reliability Ratio*

The *reliability ratio* λ_X of the measurement of a variable X (Heise 1969; Fuller 1987; for a recent application see Biancotti et al. 2004) is the ratio between the standard deviation of X without error and that of the same X measured with an error not correlated with X and identically and independently distributed over all the measures of X.

If we are dealing with two random variables X and Y, let respectively ρ_{XY} be their correlation coefficient measured without error and $\hat{\rho}_{XY}$ the one measured with error. A straightforward derivation from the definition of reliability ratio (Biemer and Trewin 1997) is

$$\hat{\rho}_{XY} = \lambda_X \lambda_Y \rho_{XY} \tag{10}$$

Under the classical assumptions, λ_X and λ_Y are positive real numbers smaller than one, so that measurement error attenuates the "true" correlation coefficient ρ_{XY} to a smaller value $\hat{\rho}_{XY}$ with the same sign.

If the interest lies in determining a lower bound for λ_Y or λ_X, it is easy to show that both lie in the interval $[\hat{\rho}_{XY}/\rho_{XY}, 1]$. If only $\hat{\rho}_{XY}$ is available, all one can say is that

$$\min(\lambda_X, \lambda_Y) \geq \hat{\rho}_{XY} \tag{11}$$

The result still holds if X and Y represent two independent measurements of the same random variable. In such a case $\rho_{XY} = 1$ and we can assert that:

> *If X and Y are two independent measurements of the same random quantity, $\hat{\rho}_{XY}$ is a lower bound for their reliability ratio.*

$\hat{\rho}_{XY}$ therefore represents a worst-case estimate of the amount of measurement error.

6.3. *Simulation of Measurement Error in the Deflators*

Two independent measures of the individual deflators in the same survey are unfortunately not available. The previous year's survey asks, however, for forecast deflators of revenues.

We therefore choose them as a *special* second measure. A slight modification in the definition of the reliability ratio is required. It is easy to show that, if X and Y are respectively the forecast and the actual value for the individual deflator of revenues, both measured with error, with X also affected by an additive forecasting error nonnegatively correlated with X, inequality (11) still holds (see Appendix C). Table 4 shows survey values of $\hat{\rho}_{XY}$ varying between 0.33 and 0.54.

These numbers help us answer the following question. Is the amount of measurement error large enough to justify the use of average deflators instead of individual ones? A high level of measurement error could indeed attenuate the desired correlation between individual deflators and individual nominal ratios.

A way to get further insights is to simulate the presence of measurement error by artificially adding an error structure to our sample data. We focus on what happens by simulating the presence of a symmetrical, zero-mean measurement error and adopt accordingly the classical measurement error model $d_i = d_i^* + \varepsilon_i$, where the d_i^* are the "true" measures and the error terms are *iid*. As an exploratory analysis proves that neither factor interactions nor further variables in the model were significant, we choose to estimate d_i^* through a simple linear model with stratification and post-stratification variables as the only covariates.

Let $\hat{d}_i$ be the predicted individual deflators from the model. The residuals are defined as $e_i = d_i - \hat{d}_i$. The measurement errors are simulated by randomly reassigning a variable quota of the residuals e_i to the sample units. For this purpose e_i is split into two parts $(1 - a)e_i$ and ae_i $(0 \leq a \leq 1)$, the latter being *randomly reassigned* within each year's sample. Four values of a are used (0.25, 0.50, 0.75, 1.00), corresponding to a growing quota of the residuals being reassigned up to complete random reassignment. For each bootstrap sample of the experiment described in Section 5, the four error quotas are applied and the MSE of the estimators is recalculated accordingly. Table 5 shows the relative MSE of the real changes for revenues with measurement error generated for the deflators.

With an increase in the error quota, the relative MSE of the real changes based on average deflators tends to decrease, particularly in the smaller domains. The reason is that the correlation between the individual deflators and nominal revenue changes (which is the

Table 4. Individual deflators of revenues. Correlation coefficient $\hat{\rho}_{XY}$ between forecast and actual data[a]

Year	Correlation coefficient
1997	0.43**
1998	0.33**
1999	0.37**
2000	0.51**
2001	0.54**
2002	0.51**
1997–2002[b]	0.45

Source: Bank of Italy's Industrial Business Surveys 1997–2002 (manufacturing firms with 50 employees or more).

[a] Referring to test for the null hypothesis H_0: $\rho = 0$, ** denotes a p-value lower than 0.01, * between 0.01 and 0.05; no asterisk denotes a p-value larger than 0.05.

[b] Pooled data.

Table 5. Revenues: average MSE of the real changes for the error simulations[a] (percentages)

Domain	Real changes													
	${}^{m}R_{\mathrm{L}}$	${}^{m}R_{\mathrm{L}}^{O}$	${}^{m}R_{\mathrm{L}}^{S}$	${}^{m}R_{\mathrm{L}}^{C}$	${}^{m}R_{\mathrm{L}}^{G}$	${}^{m}R_{\mathrm{P}}^{O}$	${}^{m}R_{\mathrm{P}}^{S}$	${}^{m}R_{\mathrm{P}}^{C}$	${}^{m}R_{\mathrm{P}}^{G}$	${}^{m}R_{\mathrm{L}}^{S+C}$	${}^{m}R_{\mathrm{L}}^{S+G}$	${}^{m}R_{\mathrm{L}}^{C+G}$	${}^{m}R_{\mathrm{L}}^{S+C+G}$	${}^{m}R_{\mathrm{P}}$
25% of residual reassigned at random														
Total	100.0	161.1	121.8	127.1	150.6	100.0	100.0	100.0	100.0	108.8	120.5	119.8	107.7	161.1
Sector of economic activity[b]	100.0	421.3	110.9	295.2	309.3	386.6	100.0	292.0	303.3	110.9	110.8	235.1	110.5	110.9
Class size[c]	100.0	235.1	152.4	116.5	214.7	216.1	151.6	100.0	202.8	116.2	150.0	116.9	115.7	116.5
Geographical area[d]	100.0	231.4	144.2	191.3	108.6	194.3	136.9	175.0	100.0	143.5	103.8	107.0	102.4	108.6
50% of residual reassigned at random														
Total	100.0	122.4	108.1	110.3	119.2	100.0	100.0	100.0	100.0	103.4	107.9	107.7	103.0	122.4
Sector of economic activity[b]	100.0	343.9	103.5	259.5	264.9	331.4	100.0	292.0	303.3	103.5	103.4	213.8	103.3	103.5
Class size[c]	100.0	207.9	146.2	105.5	192.3	201.2	146.7	100.0	188.3	105.8	143.2	105.8	105.6	105.5
Geographical area[d]	100.0	206.8	141.4	179.6	102.3	192.4	139.6	173.6	100.0	139.8	100.8	102.0	100.4	102.3
75% of residual reassigned at random														
Total	100.0	102.4	101.1	101.6	102.5	100.0	100.0	100.0	100.0	100.7	101.1	101.3	100.6	102.4
Sector of economic activity[b]	100.0	239.9	100.7	200.4	197.6	240.3	100.0	202.3	199.6	100.6	100.6	173.3	100.6	100.7
Class size[c]	100.0	165.0	130.8	100.6	154.3	164.9	131.7	100.0	154.4	100.9	127.2	100.6	100.7	100.6
Geographical area[d]	100.0	171.7	136.0	157.4	100.5	171.0	137.1	157.3	100.0	133.5	100.4	100.6	100.4	100.5
100% of residual reassigned at random														
Total	100.0	109.3	108.8	108.9	109.4	100.0	100.0	100.0	100.0	108.4	108.7	108.8	108.2	109.3
Sector of economic activity[b]	100.0	145.7	103.4	137.9	130.9	149.3	100.0	140.7	133.3	103.3	103.3	127.5	103.2	103.4
Class size[c]	100.0	118.7	109.4	103.0	112.8	118.5	109.3	100.0	111.9	103.0	106.3	102.9	102.8	103.0
Geographical area[d]	100.0	138.7	125.2	131.9	103.6	141.7	127.7	133.6	100.0	122.9	103.5	103.5	103.4	103.6

Source: Bank of Italy's Industrial Business Surveys 1997–2002 (manufacturing firms with 50 employees or more).

[a] Percentages of the MSE of ${}^{m}R_{\mathrm{L}}$.

[b] Food products, beverages and tobacco; textiles, clothing, leather, shoes; chemicals, rubber, plastic products; nonmetal minerals; engineering; other manufacturing.

[c] 50–99; 100–199; 200–499; 500–999; 1,000 and more.

[d] North-West; North-East; Centre; South and Islands, as the location of the enterprise's administrative headquarters.

major factor making ${}^{m}R_{\mathrm{L}}$ superior to the alternatives, under the hypothesis of the absence of measurement errors) is diluted as error quotas grow (Table 7).

For investment we have seen (Table 1) that, for data without errors, the low correlation between individual deflator and nominal investment changes accounts for the hardly noticeable differences shown for the relative MSE. Quite obviously indeed, in all the error scenarios the MSE of the real changes is very similar to that of absence of errors, because the correlation remains stably negligible (Tables 6 and 7). Given these results, from now on we focus exclusively on revenue deflators.

6.4. Application of the Reliability Theory to the Deflator of Revenues

For revenues, Table 8 presents the values of the *reliability ratio* λ corresponding to the four simulated levels of measurement error. They must be compared with those of its lower bound $\hat{\rho}_{XY}$ (Table 4, reproduced however in Table 8 for simplicity).

For the years 1997–2002, the average value of $\hat{\rho}_{XY}$ is very close to the average value of λ corresponding to the error simulations randomly reassigning 75% of the total residual. If we look at Table 5 for that level, we notice that for this scenario of measurement error, the MSE of ${}^{m}R_{\mathrm{L}}$ is lower than that of the alternatives, with the exception of ${}^{m}R_{\mathrm{L}}^{S+C+G}$ for all the domains, owing to the composite structure of the average deflator it uses.

7. Conclusions

In the Bank of Italy's yearly Industrial Business Survey each enterprise is asked to expressly state the price growth rate for revenues and investments, in addition to the nominal levels of these two aggregates. These data are used to deflate the nominal change in revenues and investments, in order to estimate the real changes that are published.

Having individual deflators at hand, we have the choice of deflating each individual nominal rate of change either by its own deflator or by some average deflator. By means of a simulation experiment, we compared the current practice (average deflators by sector of economic activity) against averages obtained by other stratification variables and against individual deflators. We also took into account the presence of measurement error in the deflators.

We found that individual deflators produce the lowest MSE for real changes of revenues in the absence of measurement error. When we take into account unbiased and symmetrical measurement error, simulations show interesting results, with only real changes using deflators based on regression models with dummy variables producing MSE levels close to those obtained through individual deflators. These results are observed under a significant positive correlation between individual nominal ratios and individual deflators.

For real changes in investment, where the previously mentioned correlation is absent, we showed the equivalence, in terms of their MSE levels, of using averages or individual deflators, with no evident advantage for the latter, with or without measurement error. The presence of forecasting error in the deflator is a factor that contributes to this result, but the fact that it still holds with retrospective deflators points to the underlying pattern of independence of major investment decisions from short-term price fluctuations as the leading cause.

Table 6. Investment: average MSE of the real changes for the error simulations[a] (percentages)

Domain	Real changes													
	${}^{m}R_{\mathrm{L}}$	${}^{m}R_{\mathrm{L}}^{O}$	${}^{m}R_{\mathrm{L}}^{S}$	${}^{m}R_{\mathrm{L}}^{C}$	${}^{m}R_{\mathrm{L}}^{G}$	${}^{m}R_{\mathrm{P}}^{O}$	${}^{m}R_{\mathrm{P}}^{S}$	${}^{m}R_{\mathrm{P}}^{C}$	${}^{m}R_{\mathrm{P}}^{G}$	${}^{m}R_{\mathrm{L}}^{S+C}$	${}^{m}R_{\mathrm{L}}^{S+G}$	${}^{m}R_{\mathrm{L}}^{C+G}$	${}^{m}R_{\mathrm{L}}^{S+C+G}$	${}^{m}R_{\mathrm{P}}$
25% of residual reassigned at random														
Total	100.0	100.0	100.1	100.1	100.0	100.0	100.0	100.0	100.0	100.1	100.1	100.1	100.1	100.0
Sector of economic activity[b]	100.0	99.4	99.9	99.6	99.5	99.5	100.0	99.5	99.6	100.0	99.9	99.6	100.0	99.9
Class size[c]	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.1	100.1	100.0
Geographical area[d]	100.0	101.5	101.4	101.9	101.6	101.5	101.4	101.3	100.0	101.5	101.3	102.0	101.5	101.6
50% of residual reassigned at random														
Total	100.0	100.2	100.2	100.2	100.2	100.0	100.0	100.0	100.0	100.2	100.2	100.3	100.2	100.2
Sector of economic activity[b]	100.0	99.9	100.1	100.0	99.9	99.9	100.0	99.9	99.9	100.2	100.1	100.0	100.2	100.1
Class size[c]	100.0	100.0	100.1	100.3	100.1	100.1	100.0	100.0	100.0	100.2	100.1	100.3	100.3	100.3
Geographical area[d]	100.0	102.8	102.5	103.2	102.8	102.7	102.3	102.3	100.0	102.7	102.4	103.2	102.5	102.8
75% of residual reassigned at random														
Total	100.0	100.3	100.2	100.3	100.4	100.0	100.0	100.0	100.0	100.2	100.3	100.4	100.3	100.3
Sector of economic activity[b]	100.0	100.3	100.3	100.4	100.2	100.4	100.0	100.3	100.2	100.3	100.3	100.4	100.3	100.3
Class size[c]	100.0	100.1	100.1	100.5	100.1	100.2	100.1	100.0	100.0	100.4	100.1	100.5	100.5	100.5
Geographical area[d]	100.0	104.0	103.6	104.5	104.0	103.8	103.1	103.3	100.0	103.8	103.4	104.3	103.5	104.0
100% of residual reassigned at random														
Total	100.0	100.4	100.2	100.4	100.5	100.0	100.0	100.0	100.0	100.2	100.3	100.5	100.3	100.4
Sector of economic activity[b]	100.0	100.7	100.3	100.8	100.6	100.9	100.0	100.8	100.6	100.4	100.3	100.7	100.4	100.3
Class size[c]	100.0	100.0	100.0	100.6	100.1	100.2	100.1	100.0	100.0	100.5	100.1	100.7	100.6	100.6
Geographical area[d]	100.0	105.1	104.6	105.7	105.0	104.9	104.0	104.2	100.0	104.8	104.3	105.3	104.4	105.0

Source: Bank of Italy's Industrial Business Surveys 1997–2002 (manufacturing firms with 50 employees or more).

[a] Percentages of the MSE of ${}^{m}R_{\mathrm{L}}$.

[b] Food products, beverages and tobacco; textiles, clothing, leather, shoes; chemicals, rubber, plastic products; nonmetal minerals; engineering; other manufacturing.

[c] 50–99; 100–199; 200–499; 500–999; 1,000 and more.

[d] North-West; North-East; Centre; South and Islands, as the location of the enterprise's administrative headquarters.

Table 7. Correlation coefficient between individual nominal rates of change and individual deflators[a] for the error simulations

Year	Percentage of residuals reassigned at random			
	25%	50%	75%	100%
Revenues				
1997	0.15**	0.12**	0.06*	0.01
1998	0.04	0.02	0.00	0.00
1999	0.15**	0.11**	0.05	0.00
2000	0.20**	0.14**	0.07**	0.00
2001	0.10**	0.10**	0.07**	0.05*
2002	0.12**	0.10**	0.05	0.00
Investment				
1997	0.03	0.01	−0.02	−0.04
1998	0.00	−0.02	−0.04	−0.05
1999	0.05	0.05	0.04	0.02
2000	0.09**	0.07	0.03	0.01
2001	0.01	−0.02	−0.03	−0.03
2002	−0.01	−0.02	−0.01	−0.01

Source: Bank of Italy's Industrial Business Surveys 1997–2002 (manufacturing firms with 50 employees or more).
[a] Referring to test for the null hypothesis H_0: $\rho = 0$, ** denotes a p-value lower than 0.01, * between 0.01 and 0.05; no asterisk denotes a p-value larger than 0.05.

In conclusion, the final choice between individual and average deflation depends on the overall reliability of the individual measurement. We find that, when a significant correlation between individual nominal ratios and individual deflators exists, price indices obtained from regressions with dummy variables (representing the strata and post-strata of the survey) work better than those derived by simple means. Individual deflation still emerges as a viable option, or at least, as a significant benchmark against which the effectiveness of synthetic measures of price growth rates should be gauged.

Table 8. Individual deflators of revenues. Reliability ratio for the bootstrap experiments with error simulation

Year	$\hat{\rho}_{XY}$	Percentage of residual reassigned at random			
		25%	50%	75%	100%
Reliability ratio					
1997	0.43	0.97	0.83	0.50	0.30
1998	0.33	0.94	0.75	0.35	0.19
1999	0.37	0.96	0.66	0.40	0.20
2000	0.51	0.97	0.82	0.57	0.34
2001	0.54	0.92	0.70	0.38	0.22
2002	0.51	0.92	0.69	0.38	0.22
Average reliability ratio[a]					
1997–2002[b]	0.45	0.95	0.74	0.43	0.24

Source: Bank of Italy's Industrial Business Surveys 1997–2002 (manufacturing firms with 50 employees or more).
[a] Simple mean of year-based reliability ratios.
[b] Pooled data.

Appendix A

The Paasche (Laspeyres) Quantity Index with Individual Deflators as a Particular Case of the Laspeyres Quantity Index Deflated with a Laspeyres (Paasche) Price Index

When the domain m coincides with one of the cells s used for the calculation of average deflators, Formula (9) in the text, becomes

$$ {}^{m}R_{\mathrm{L}}^{s} = \frac{\sum_{s\in S} I(m\cap s)\frac{1}{{}^{s}d}\sum_{i\in m\cap s} A_{i,1}w_i}{\sum_{i\in m} A_{i,0}w_i} = \frac{\frac{1}{{}^{m}d}\sum_{i\in m} A_{i,1}w_i}{\sum_{i\in m} A_{i,0}w_i} \tag{12} $$

where $I(m\cap s) = \begin{cases} 1 & \text{if } m\cap s \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$

If ${}^{m}d$ is the average deflator calculated as a Laspeyres price index, i.e., ${}^{m}d = \sum_{i\in m} w_i A_{i,0} d_i / \sum_{i\in m} w_i A_{i,0}$
we get

$$ {}^{m}R_{\mathrm{L}}^{s} = \frac{\frac{1}{{}^{m}d}\sum_{i\in m} A_{i,1}w_i}{\sum_{i\in m} A_{i,0}w_i} = \frac{\sum_{i\in m} A_{i,1}w_i}{\sum_{i\in m} A_{i,0}w_i d_i} $$

which is exactly a Paasche quantity index (4) with individual deflators.

If we deflate (9) with a Paasche price index instead, it is easy to obtain a Laspeyres quantity index with individual deflators. The same property holds if $m = o$ (the whole target population), no matter how the cells used to compute the Paasche price index are chosen, i.e.,

$$ {}^{o}R_{\mathrm{L}}^{s} = \frac{\sum_{s\in S} I(o\cap s)\frac{1}{{}^{s}d}\sum_{i\in o\cap s} A_{i,1}w_i}{\sum_{i\in o} A_{i,0}w_i} = \frac{\sum_{s\in S}\frac{1}{{}^{s}d}\sum_{i\in s} A_{i,1}w_i}{\sum_{i\in o} A_{i,0}w_i} = \frac{\sum_{i\in o} A_{i,1}w_i\frac{1}{d_i}}{\sum_{i\in o} A_{i,0}w_i} $$

if one takes into account that $o\cap s = s$ and that ${}^{s}d = \sum_{i\in s} w_i A_{i,1} / \sum_{i\in s} w_i A_{i,1}(1/d_i)$

Appendix B

Variance of the Real Rate of Change

We now briefly explore the behaviour of the variance of the real change expressed through a Laspeyres quantity index.

The aggregate real change contains the terms $(A_{i,1}/A_{i,0})/d_i^*$. Intuitively, its variance is inversely related to the correlation of the individual ratio $A_{i,1}/A_{i,0}$ with the deflator d_i^*, because the variance of $(A_{i,1}/A_{i,0})/d_i^*$ is relatively smaller whenever the correlation between the numerator and the denominator is positive (as compared to the case when this correlation is not significant or negative). Analytically, if we indicate $A_{i,1}/A_{i,0}$ with Y and d_i^* with X, an approximate formula for the variance of Y/X (see, for example Bishop et al. 1975) is

$$\mathrm{V}\left(\frac{Y}{X}\right) \cong \frac{[\mathrm{E}(Y)]^2}{[\mathrm{E}(X)]^2}\left\{\frac{\mathrm{V}(X)}{[\mathrm{E}(X)]^2}+\frac{\mathrm{V}(Y)}{[\mathrm{E}(Y)]^2}-2\frac{\mathrm{Cov}(X,Y)}{\mathrm{E}(X)\mathrm{E}(Y)}\right\}$$

that can also be easily expressed, in terms of variation and correlation coefficients as follows:

$$\mathrm{V}\left(\frac{Y}{X}\right) \cong \left[\frac{\mathrm{E}(Y)}{\mathrm{E}(X)}\right]^2\{[\mathrm{CV}(X)]^2+[\mathrm{CV}(Y)]^2-2\,\mathrm{Corr}(X,Y)\,\mathrm{CV}(X)\,\mathrm{CV}(Y)\}$$

This reasoning does not explicitly introduce measurement error models. Obviously the variance increases whenever measurement errors weaken the correlation.

Appendix C

Reliability Ratio for Forecasts with Additive Forecasting Error

With additive forecasting error on X, we define λ_X in a slightly different way from the one currently employed in the literature. Let $X = X^* + \gamma + \varepsilon_1$, where γ is the forecasting error term. In this case we define $\lambda_X = \mathrm{Var}(X^* + \gamma)/\mathrm{Var}(X)$. The inequality $0 \le \lambda_X \le 1$ still holds, provided the forecasting error is nonnegatively correlated with the "true" measure X^* and uncorrelated with the measurement error. Under these hypotheses, equality $\hat{\rho}_{XY} = \lambda_X\lambda_Y\rho_{XY}$ still holds. Since $0 \le \lambda_Y \le 1$, it trivially follows that $\hat{\rho}_{XY} \le \rho_{XY}$. These inequalities, together with $\rho_{XY} \le 1$ and $\hat{\rho}_{XY}$ and ρ_{XY} having the same sign, prove that $\hat{\rho}_{XY} \le (\hat{\rho}_{XY}/\rho_{XY}) = \lambda_X\lambda_Y \le \lambda_Y$.

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