Effects of nonresponse on survey estimates in the analysis of competing exponential risks

Ingrid Lyberg

SCB

R&D Report
Statistics Sweden
Research - Methods - Development
1991:4

Från trycket	Mars 1991
Producent	Statistiska centralbyrån, avdelningen för individstatistik
Ansvarig utgivare	Åke Lönnqvist
Förfrågningar	Ingrid Lyberg, 08-783 43 76

ISSN 0283-8680
Printed in Sweden

INLEDNING

TILL

R & D report : research, methods, development / Statistics Sweden. – Stockholm : Statistiska centralbyrån, 1988-2004. – Nr. 1988:1-2004:2.
Häri ingår Abstracts : sammanfattningar av metodrapporter från SCB med egen numrering.

Föregångare:

Metodinformation : preliminär rapport från Statistiska centralbyrån. – Stockholm : Statistiska centralbyrån. – 1984-1986. – Nr 1984:1-1986:8.

U/ADB / Statistics Sweden. – Stockholm : Statistiska centralbyrån, 1986-1987. – Nr E24-E26

R & D report : research, methods, development, U/STM / Statistics Sweden. – Stockholm : Statistiska centralbyrån, 1987. – Nr 29-41.

Efterföljare:

Research and development : methodology reports from Statistics Sweden. – Stockholm : Statistiska centralbyrån. – 2006-. – Nr 2006:1-.

R & D Report 1991:4. Effects of nonresponse on survey estimates in the analysis of competing exponential risks / Ingrid Lyberg.
Digitaliserad av Statistiska centralbyrån (SCB) 2016.

urn:nbn:se:scb-1991-X101OP9104

EFFECTS OF NONRESPONSE ON SURVEY ESTIMATES IN THE ANALYSIS OF COMPETING EXPONENTIAL RISKS*

by

Ingrid Lyberg

Statistics Sweden

March 1991

---

* An earlier version of this report was published in Stockholm Research Reports in Demography, No. 12, Department of Statistics, University of Stockholm.

CONTENTS

Summary

A model for studying the effects of nonresponse in competing risks analysis is proposed. The response probabilities are assumed to depend on whether, and from what cause, decrement has occurred during an observation period with right censoring. The model has been used to study nonresponse effects on estimates of transition intensities in the 1981 Swedish Fertility Survey. Some empirical results from that survey are presented to give realistic estimates of the parameters in the model.

The effects of nonresponse on technical bias, variance, and variance estimators of occurrence exposure rates (estimated intensities) are investigated by means of the model. It is shown that the technical bias (i.e., the bias due to ratio estimation) is often insignificant compared with the standard error, which in turn can often be estimated in an approximately unbiased manner by the usual variance estimator even in the nonresponse situation.

The nonresponse bias of estimates of transition intensities and transition probabilities is also investigated. It is shown that the nonresponse bias may be *very large* if the response probabilities for decrements and survivors differ greatly. Two methods to adjust for the nonresponse bias are investigated. Both require accurate estimates of the ratios between the response probabilities for decrements and survivors. If this requirement is not met, the adjustment methods may in fact increase the nonresponse bias.

1. Introduction

Most surveys suffer from nonresponse. This affects estimates not only by reducing the number of observations and thereby increasing the random error, but also by introducing bias in the estimates. The bias arises from the fact that the response behavior often is associated with the variables under study. The only certain way to avoid nonresponse bias is to make sure that all sampled units (or subsampled units from an original nonresponse group) respond. This is seldom possible in practice due to budget and time constraints, persistent refusals, etc. The effects of nonresponse and the costs of reducing it must also be balanced against other sources of error in a survey. Therefore, some nonresponse must usually be accepted.

Nonresponse effects (and adjustment methods) can sometimes be studied empirically by comparing estimates based on the responses obtained with corresponding estimates based on the whole target sample, provided data are available from an external source. This has been possible for the 1981 Swedish Fertility Survey, where information from the Swedish Fertility Register was used to study nonresponse effects on estimates of transition intensities, transition probabilities, test statistics, etc (Lyberg, 1983). The register, which has been described by Johansson and Finnäs (1983) and by Quist (1990), contains information about fertility and nuptiality for all Swedish women born in 1926-60.

Such empirical studies have limitations, however. Their results concern specific variables for which there is accurate information in the register. These variables may not be the most important ones in the survey, and the results may not be valid for the main survey variables. Thus, empirical studies should be carried out within the framework of a theoretical model. Such a model may make possible inference beyond the empirical findings. A theoretical model is also a necessary means during the design phase for the effective allocation of resources.

This paper presents the theoretical model used in the nonresponse study of the 1981 Swedish Fertility Survey. The model concerns an event history analysis where the transition behavior can be described by a competing risks model and the response behavior is assumed to depend on the outcome of the individual life history.

A competing risks model is a Markov chain with a continuous time parameter, one transient state (State 0) and some (finite) number K of absorbing states. (More complex hierarchical Markov chain models for event history analyses can often be decomposed into a number of sequential competing risks models. See, for instance, the analyses performed on the Swedish Fertility Register and the 1981 Swedish Fertility Survey by Quist and Rennermalm (1985).) The transition intensities of a competing risks model are defined as

$$\mu_j(t) = \lim_{h \downarrow 0} P_j(t, t+h) / h , \qquad j = 1, 2, \ldots, K,$$

where $P_j(t,t+h)$ is the transition probability from State 0 to state j during the time interval (t,t+h) among individuals who still belong to State 0 at time t. In this paper we assume that the transition intensities are constant, i.e., $\mu_j(t) = \mu_j$ for all relevant t, and $j=1,2,\ldots, K$.

Central rates (occurrence/exposure rates) are maximum likelihood estimators of the constant transition intensities μ_j. Their asymptotic properties are well.known, see, e.g., Hoem and Funck Jensen (1982, Chapter 4.1). Some attention has also been paid to their small-sample properties (Beyer et al., 1976; Vaeth, 1977; and Shou and Vaeth, 1980). So far almost nothing is known about the effects of nonresponse on central rates used as estimates of transition intensities.

In Section 2 I start with a short discussion of different approaches to model nonresponse and present the model used in this paper. This model describes how the life-history segments of the respondents (i.e., the histories actually observed) are generated by a probabilistic response mechanism. In Section 3 I present some results from the empirical nonresponse study of the 1981 Swedish Fertility Survey in order to give an impression of what estimates are realistic for the response probabilities defined in the theoretical model.

In Section 4 I return to the theoretical model and investigate the nonresponse effects on the technical bias, on the variance and on the usual variance estimator of central rates used as estimators of transition intensities. In the nonresponse situation, those central rates may be correlated and the usual variance estimator may be biased (even asymptotically). Most of the time, however, the correlation and bias are *very* small. Nevertheless, I present a consistent variance-covariance estimator to be used in cases where there is any doubt. The technical bias behaves in the same manner in the nonresponse situation as in the complete response situation, i.e., it can usually be ignored.

The nonresponse bias is investigated in Section 5. The bias can be expressed as a function of the ratios between the response probabilities for the decrements (from a specific cause) and for the survivors, as defined in the nonresponse model. If these ratios differ greatly from 1, the nonresponse bias of central rates becomes very large compared with the estimated transition intensities and compared with the standard errors of the estimates. When transition and survival probabilities are concerned, the nonresponse bias seems to be smaller if the probabilities are estimated via the estimates of transition intensities rather than by the proportions of decrements and survivors.

In Section 6 I investigate two methods of adjusting for the nonresponse bias. Both methods require estimates of the ratios between the response probabilities for the decrements and survivors. If these estimates are not accurate, the adjustment methods may increase the nonresponse bias. Other adjustment methods are discussed briefly.

2 The Nonresponse Model

2.1 Nonresponse modeling

Before presenting the nonresponse model used in this essay I give a brief review of nonresponse modeling as presented in the literature. The review is far from comprehensive. The purpose is only to illustrate the variety of approaches to model nonresponse that have been applied so far.

The presentation emphasizes unit (or total) nonresponse, although there is no principal difference between unit and item (partial) nonresponse. In almost every survey there exists some information in the frame for all units in the sample, including the nonrespondents. Such information could be used to model and treat unit nonresponse in the same way as item nonresponse. In practice, however, unit nonresponse is often modeled and treated globally for all missing items while item nonresponse is treated by item-specific direct or indirect imputation (or classified as "no answer" in tabulations and analyses). In their review of methods for treating missing survey data, Kalton and Kasprzyk (1986) show that nearly all imputation methods presented can be described as relying on special cases of a general regression model of the relation between the variable with missing data and some auxiliary variables.

Survey statisticians have worked with nonresponse problems for decades. In 1977 the *Panel of Incomplete Data* was established by the US Committee on National Statistics. The panel's work was published in three volumes of which the third consists of proceedings of the 1979 Symposium on Incomplete Data (Panel on Incomplete Data, 1983). The panel did not end up with a unified theory for treating and modeling nonresponse. On the contrary, the volumes present a wide spectrum of viewpoints reflecting the practitioners' informal methods to provide reasonably accurate statistics to a reasonable cost and the theoreticians' efforts to make inference based on parametric models and maximum-likelihood estimation or on extension of the randomization theory. There is an agreement, however, that the treatment of nonresponse relies on implicit or explicit modeling of the *response mechanism* or on direct modeling of the values of the nonrespondents.

There are two main approaches to inference about finite population quantities from sample surveys, the *randomization approach* and the *model-based approach* (Rubin, 1983). The former, also referred to as *design-based inference*, treats values in the population as fixed and the inference is based on the distribution generated by the sample selection mechanism. The model-based approach treats values in the population as random variables and the inference is based on the model specified for these variables and likelihood inference. In the presence of nonresponse both approaches yield biased estimates if the response mechanism is related to the survey variables. In that case the response mechanism is *nonignorable* and has to be incorporated in the inference model to yield unbiased estimates.

Little (1983) provides a conceptual framework and a review of methods for handling nonresponse in parametric model-based inference. Most of these methods rely on the assumption of ignorable response mechanism and are not directly related to survey data. For handling nonignorable nonresponse it has been proposed to model the response mechanism by assuming that the survey variable y_i is observed when another interval scaled variable u_i lies below a threshold value c. The conditional probability that y_i is observed is then obtained from the regression of u_i on y_i and other observed variables w_i. By means of such a model it can be investigated how the nonresponse affects the likelihood based inference.

If auxiliary variables are observed for the total sample the likelihood inference can be made in two stages to adjust for nonresponse bias. Brehm (1990) has applied an approach suggested by Heckman (1976, 1979) that involves two-stage analysis. The response mechanism is analyzed in a first stage that provides values for an additional regressor in the outcome model. The coefficients of these additional regressors are estimates of the covariances between the errors in the outcome model and the response model. Brehm's analyses concern political research (categorical variables) and he used three sets of auxiliary variables to model the response mechanism: *administrative variables* of the survey process (amount of persuasion, number of calls), *behavioral variables* (four latent variables describing attitudes towards strangers, etc, found by LISREL analysis on refusals' and reluctant respondents' recorded reasons for not participating), and *demographic variables* (sex, income, respondent's and interviewer's race). The corrections induced changes in the estimated outcome model coefficients that were consistent, sensible and substantively important.

Other researchers have more directly combined an outcome model with a response propensity model. Fay (1986) proposes log-linear causal models for modeling ignorable and nonignorable response mechanisms when survey data are categorical. Stasny (1986 and 1987) models the outcomes of a categorical survey variable and the response mechanism in a panel survey. She combines a Markov-chain outcome model with a Markov-chain response model where the transition probabilities between response categories depend on the outcome of the survey variable. Her approach allows a person to be nonrespondent at both of two interview periods, but she has to put constraints on the models to get an estimable number of parameters.

For a comprehensive review of likelihood-based approaches to model and treat nonresponse I refer to the textbook by Little and Rubin (1987).

There seems to have developed a consensus that inference based on randomization theory, like likelihood analysis, must rely on models that cannot be tested by means of observed data when there is nonresponse (Särndal and Hui 1981; Little, 1982).

Within the framework of randomization theory the response mechanism can be regarded as either deterministic or stochastic. With the deterministic approach the population is thought to be divided in two strata, the response stratum and the nonresponse stratum. The well-known Hansen-Hurwitz plan for subsampling among nonrespondents rely on the assumption of deterministic response behavior (Hansen and Hurwitz, 1946; Cochran, 1977, pp. 370-374). An early application of modeling a stochastic response mechanism is the well-known procedure of Politz and Simmons (1949, 1950).

Lindström and Lundström (1974) proposed a method to investigate the magnitude of the nonresponse error by introducing a parametric response propensity function, $p(x)=ax^2+bx+c$, where $0 \leq p(x) \leq 1$ and x is the survey variable that is assumed to be continuous with a frequency function $f(x)$. By means of that model they investigate the variance and bias of the unadjusted mean for the respondents for selected values of a and b and various frequency functions. Lindström and Lundström define their response propensity function as the "probability of selecting a responding unit among units with the same variable value x". Their hesitation to directly define "response probabilities" probably reflects that this concept was not commonly accepted among survey statisticians at that time.

The articles in the three volumes edited by the Panel on Incomplete Data, for instance: Oh and Scheuren (1983), Platek and Gray (1983) and Cassel et al (1983), and later literature, show a development towards modeling the response mechanism as stochastic rather than deterministic. The concept of *response probabilities* seems now to have been commonly accepted within the framework of design-based inference.

Oh and Sheuren assume a "uniform response mechanism", i.e., constant response probabilities within disjoint subpopulations, and they regard the response mechanism as "quasi-randomization". Dalenius (1983) argues that "the response mechanism should be formulated *given the sample s*, with consideration given to the survey operations to which the units in the *particular sample s* are exposed ." Särndal and Swensson (1987) make this possible by regarding the outcome from the response mechanism as generated by the second phase in two-phase-sampling. They assume that the individual response probabilities are constant within *response homogeneity groups* of the primary sample. (The number of such groups and their definition are not necessarily the same for all possible samples.) Given the response model is true, (approximately) unbiased estimators follow from results obtained under the assumption of "true" probabilistic two-phase sampling. Swensson and Särndal also conducted a simulation study. This study showed that a regression estimator performed better than the "simple expanded" estimator concerning variance, sensitivity to wrong assumptions regarding the response mechanism, and difference between coverage rates of confidence intervals and nominal rate.

Bethlehem (1988) proposes a model with individual response probabilities (not dependent on the particular sample) to investigate the properties of the Horvitz-Thompson estimator (HT-estimator) and the generalized regression estimator. Both estimators are modified for the nonresponse situation: The modified HT-estimator of the population mean is defined as $\hat{y} = (\Sigma y_k/\pi_k) / (\Sigma 1_k/\pi_k)$, where π_k is the sample inclusion probability. (Bethlehem also formulates poststratification as a special case of the modified general regression estimator). Ekholm and Laaksonen (1990) model individual response probabilities and use estimates of such probabilities in a "model based Horvitz-Thompson estimator". (The inverse of the product of the sample inclusion probability and the response probability is used to weight individual values.) This approach was applied in the 1985 Finnish Household Budget Survey, where estimates of the individual response probabilities were "predicted" by means of a logit model.

Recent development of nonresponse modeling emphasizes the behavioral aspect of nonresponse (Fay, 1986, Groves, 1989, Cialdini, 1990). "The causes of noncontact nonresponse are likely to be very different from the causes of refusal nonresponse" (Groves, 1989, p. 183). This has implications both for the administration of a survey and for the inference from the survey. Therefore, response models should be extended to *survey participation models* that account for all relevant factors affecting "participation probability", not just covariates related to the sampled person. For instance, the well-known variability between different interviewers' response rates is a factor that could be used in an adjustment model. For a fuller description of this matter I refer to chapters 4 and 5 in the textbook by Groves (1989), and to Cialdini (1990) and Brehm (1990).

2.2 The nonresponse model for a competing risks model

Most nonresponse models reviewed in the previous section concern estimation of fixed population quantities or linear regression analysis. Exceptions are Fay's log-linear causal models, Stasny's Markov-chain models, and Brehm's two-stage correction used in connection with outcome models for dichotomous dependent variables. The nonresponse model proposed in this paper concerns traditional competing risks analysis. The relevant parameters are the transition intensities or other *model parameters* derived from them.

Following Cassel et al. (1983), I assume probabilistic response behavior that is connected with the variables under study. The response probability approach has also been applied by Platek (1978), Andersson (1979), Little (1982), Hoem (1983), and others.

Consider a population of individuals (units) whose current life-history segments may be described by a competing risks model. According to some sampling plan, n units are selected at random from the population. The sampling plan is assumed to be noninformative, i.e., there is independence between life histories and selection. This means that the *life histories sampled* may be regarded as independent outcomes of the same stochastic process, provided that the population is homogeneous and that there is between-unit independence. (For a more detailed discussion of this matter, see Hoem, 1983, Chapter 2; and Hoem, 1989.)

For the units sampled, we try to obtain information about the transition behavior during some risk period. For various reasons, however, we fail to observe some units, the nonrespondents. For the respondents we here assume that complete information is obtained without measurement errors.

Let the response behavior of an individual be independent of the sample design and of the response behavior of other individuals (although this is not always true in an interview survey). We assume that the response behavior is probabilistic and depends on the actual life history in the same manner for all units. This means that the *life histories observed* can be regarded as independent outcomes of the same stochastic process, defined by the competing risks model and a nonresponse model. Such a model is defined below.

We consider n units observed from some time 0 until time z (right censoring) under the competing risks model with *constant* intensities $\mu_1, \mu_2, \ldots, \mu_K$. For each unit we define $T = \min(U,z)$, where U is the time of transition out of State 0. Let $Q_j = 1$ and $Q_k = 0$ for $k \neq j$ and $j,k=1,2,\ldots, K$ if transition due to cause j occurs before time z. If no transition occurs before time z, then $T = z$ and $Q_j = 0$ for all j. Let $Q = \Sigma Q_j$, which means that $Q = 1$ if $T < z$ and $Q = 0$ if $T = z$, and let $R = 1$ if the individual responds, $R = 0$ otherwise. The response behavior (variable R) is assumed to be independent between units and to satisfy

$$P(R=1 \mid Q_j=1) = r_j \qquad \text{for } j=1,2,\ldots, K$$

and

$$P(R=1 \mid Q=0) = r_0.$$

Thus, the response behavior depends on whether decrement occurs during the time period (0,z) and from what cause, but *not on when it happens.* (I do not distinguish between causes of nonresponse, since most of the nonresponse in the 1981 Fertility Survey was refusals, 11% compared with 2% noncontacts.)

The *variables observed* are R, $Q_j^*=R \cdot Q_j$, $Q^*=R \cdot Q$, and $T^*=R \cdot T$. By this definition, the values of the observed variables are equal to 0 for the nonrespondents.

We use an extra subscript, v (v=1,2,..., n) to designate unit (individual) number. The observed central rates then become

$$\hat{\mu}^*_j := \Sigma\, Q^*_{jv} \,/\, \Sigma\, T^*_v \,, \tag{2.1}$$

where the numerator is the total number of decrements due to cause j (j=1,2,..., K) and the denominator is the total exposure time among all respondents.

3 Some Empirical Results

In 1981, Statistics Sweden conducted a fertility survey among Swedish women born in 1936–60. A sample of 4 966 women was drawn by simple random sampling from each of five strata, a stratum being one of the five-year birth cohorts (1936–40, 1941–45,..., 1956–60) which constitute the target population. Interviews were made with 4 300 respondents (87 percent), a comparatively high rate for a fertility survey. The response rates were higher among women who had children (still) living with them than among other women. For women born in 1941–45 the response rate was 89 percent among those with children compared with 72 percent among the other women. (Since children here refer to children 17 years or younger, the difference would probably be larger if we could account for all children born.) This suggests an association between response behavior and fertility.

The data from the survey have been used in several substantial analyses. A technical documentation of the survey is given in Lyberg (1984) and substansive descriptive results are given in Information i prognosfrågor (1982:4, 1983:4, and 1984:4). Results based on life history analyses are presented in Quist and

Rennermalm (1985) and in several reports from the Section of Demography at University of Stockholm (a late reference, including a list of other reports, is Hoem (1990)).

The expected broad use of data from the survey called for a thorough investigation of possible effects of the selective nonresponse on estimates. In particular, there was a need to investigate the effects of nonresponse in life history analyses. The investigation conducted consists of two parts; an empirical study based on comparisons with register data and a theoretical part presented in this essay. Some results from the empirical investigation of the connection between family history and response behavior, and of the nonresponse effects on central rates (Lyberg, 1983) are shown in Figures 1 and 2 and in Tables 1-3. The results are based on data from the Swedish Fertility Register mentioned earlier. Since only information about response behavior is collected from the survey, the results are not confounded by any measurement errors in the survey.

With few exceptions, the response rate is higher at a given age among women who left State 0 because of birth ($\hat{r}_1$) or marriage ($\hat{r}_2$) than among those who remained in the state ($\hat{r}_0$). This means that the age-specific central rates based on respondents only ($\hat{\mu}_j^*$) are higher than those based on the whole target sample ($\hat{\mu}_j$). In other words, the birth and marriage intensities are overestimated because of the nonresponse. (It is only if $z\mu$ is very large and both r_j and μ_j are much larger for one cause of decrement than for the other, that the intensity for the more rare cause might be underestimated although the response probability for the decrements is larger than for the survivors.)

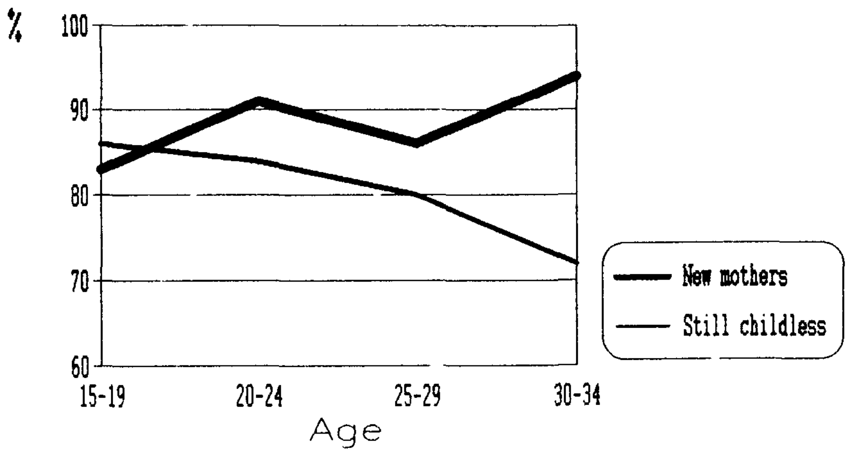


Figure 1. Response rates in the 1981 Swedish Fertility Survey among those still childless at the end of a given five-year age interval and among those who gave birth to their first child in the age interval. Women born in 1941-45. Percent.

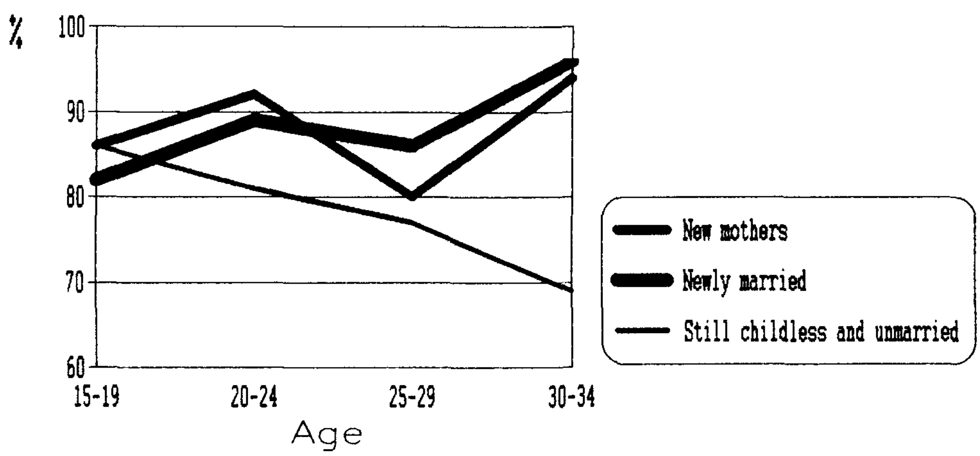


Figure 2. Response rates in the 1981 Swedish Fertility Survey among those still unmarried and childless at the end of a given five-year age interval and among those who gave birth to their first child or married in the age interval. Women born in 1941-45. Percent.

Table 1. Age-specific first birth rates based on the whole sample and the respondents only in the 1981 Swedish Fertility Survey. Response rates among those still childless, by age, and among those who gave birth to their first child in each age interval. Women born in 1941-45.

Age	First birth rates per 1000		Response rates %		Number of women in the sample	
	target sample $\hat{\mu}_1$	respond-ents $\hat{\mu}_1^*$	still childless women $\hat{r}_0$	new mothers $\hat{r}_1$	still child-less	new mothers
					1180	-
15	1	1	86		1179	
16	12	10	86		1165	
17	24	27	86	83	1137	188
18	56	57	86		1075	
19	80	72	86		992	
20	111	118	86		888	
21	104	105	86		800	
22	132	140	86	91	701	479
23	120	128	85		622	
24	193	220	84		513	
25	158	162	82		438	
26	142	154	82		380	
27	144	151	81	86	329	265
28	130	133	81		289	
29	154	177	80		248	
30	108	133	78		223	
31	84	106	76		205	
32	50	54	75	94	195	67
33	31	42	74		189	
34	44	61	72		181	
35	40	55	71	(100)	174	7

Table 2. Age-specific first birth and first marriage rates among childless, unmarried women based on the whole sample and the respondents only in the 1981 Swedish Fertility Survey. Response rates among those still childless and unmarried, by age, and among those who left that state due to birth or marriage in each age interval. Women born in 1941-45.

Age	Birth rates per 1000 sample $\hat{\mu}_1$	Birth rates per 1000 respondents $\hat{\mu}_1^*$	Marriage rates per 1000 sample $\hat{\mu}_2$	Marriage rates per 1000 respondents $\hat{\mu}_2^*$	Response rates % among still childless and unmarried $\hat{r}_0$	Response rates % among new mothers $\hat{r}_1$	Response rates % among newly married $\hat{r}_2$	Number of women still childless & unmarried in the sample
								1180
15	1	1	0	0	86			1179
16	10	8	3	4	86			1163
17	19	22	10	8	86	86	82	1130
18	34	34	47	45	86	(103)*	(144)*	1042
19	31	30	78	74	86			933
20	34	39	129	134	86			791
21	36	36	173	184	85			640
22	26	29	178	181	84	92	89	521
23	30	34	202	219	83	(102)	(499)	413
24	46	53	174	185	81			332
25	16	12	141	137	82			284
26	41	42	86	93	81			250
27	39	32	112	105	82	80	86	215
28	46	50	122	151	80	(51)	(131)	181
29	104	118	92	118	77			149
30	35	47	57	66	75			136
31	30	31	15	20	75			130
32	16	22	65	89	72	94	96	119
33	18	25	53	75	70	(16)	(26)	110
34	28	41	19	27	69			105
35	30	44	40	59	66	(100)	(100)	98
						(3)	(4)	

* Number of new mothers (newly married) in the whole sample.

Table 3. Duration-specific first birth and divorce rates among childless, first married women based on the whole sample and the respondents in the 1981 Swedish Fertility Survey. Response rates among those still childless and married, by duration in marriage ("age"), and among those who left that state due to birth or divorce in each duration interval. Women born in 1941-45 and first married as childless at age 20-24.

Age **	Birth rates per 1000		Divorce rates per 1000		Response rates % among			Number of women still childless and married in the sample
	sample	respondents	sample	respondents	still childless and married	new mothers	newly divorced	
	$\hat{\mu}_1$	$\hat{\mu}_1^*$	$\hat{\mu}_3$	$\hat{\mu}_3^*$	$\hat{r}_0$	$\hat{r}_1$	$\hat{r}_3$	
					89			502
0	499	519	0	0	87			303
1	500	493	0	0	88			183
2	369	395	7	8	86	91	(100)	127
3	410	482	50	60	79	(422)*	(8)*	81
4	105	136	26	34	76			71
5-10	104	122	56	41	79	86	(53)	28
						(28)	(15)	

* Number of new mothers (newly divorced) in the whole sample.
** Age refers to number of years in marriage (duration).

Among those 502 sampled women born in 1941-45 who married as childless in age 20-24 only 23 (4.6%) divorced as childless during the first eleven years of marriage. Sixteen 16 (70% or 68-71% with 95% confidence) of those divorced women participated in the survey. Among those 450 women who did not divorce but gave birth to a child within the first eleven years of marriage no less than 408 (91%) participated in the survey. Among those 28 women who were still married and childless after eleven years of marriage 22, (79% or 77-80% with 95% confidence) participated in the survey. (The number of women in the three groups does not add to 502 - one woman is "lost". She might have been temporarily emigrated and thereby not covered by the register for some period.) These differences in response rates mean that the central rates based on respondents overestimate the marital fertility and underestimate the divorce intensities (except for the first five years).

4 Technical Bias, Variance and Covariance of Central Rates

In this section the theoretical model presented in Section 2 is used to investigate the effects of nonresponse on the technical bias due to ratio estimation, on the variance and on the covariance of central rates used as estimates of transition intensities. I present some well-known results for the complete response situation and investigate whether they are valid when there is nonresponse as well. It is found that they usually are. Most of the time, the technical bias can be ignored. If those who decrement from various causes have response behaviors which differ from each other and from those of the survivors, then the central rates for different causes are in fact correlated, and the usual variance estimator is biased. However, the correlation and the bias of the variance estimator are probably insignificant for realistic values of intensities and response probabilities. Finally we present an approximately unbiased covariance-variance estimator.

4.1 The complete response situation

When all units respond, the central rates in (2.1) are maximum likelihood estimators of the intensities μ_j ($j=1,2,\ldots, K$). In that case, the rates are asymptotically independent and normally distributed with expected values and variances (see, for instance, Hoem and Funck Jensen (1982), and the Appendix):

$$E(\hat{\mu}_j) \approx \mu_j \, [1 + (q-pz\mu)/nq^2] \tag{4.1}$$

and

$$V(\hat{\mu}_j) \approx \mu_j\mu/nq \qquad \text{for } j=1,2,\ldots, K \tag{4.2}$$

where $q=1-p = 1-\exp(-z\mu)$ and $\mu=\Sigma\mu_j$.

The second term in the brackets of (4.1) is the (approximate) *technical bias.* This bias decreases faster than the standard error as the number of observations increases. The relation between the technical bias and the standard error is shown in Table 4, based on results provided by Beyer et al (1976) and Vath (1977). Their results are calculated from exact formulas for expected value and variance, but calculations based on the approximate expressions in (4.1) and (4.2) with z=1 give the same results down to the second decimal in most cases.

Table 4. Relative technical bias and ratio between technical bias and standard error of occurrence/exposure rates without nonresponse, by sample size n, for z=1.

	Transition intensities			Exact values			Approximate values		
	μ	μ_j	μ_j/μ	n=10	n=30	n=50	n=10	n=30	n=50
relative	.1	all		.056	.018	.010	.052	.017	.010
bias	1.0	all		.074	.023	.014	.066	.022	.013
technical	.1	.01	.1	.02	.01	.01	.016	.009	.007
bias/		.05	.5	.04	.02	.02	.036	.021	.016
s.e.		.10	1.0	.05	.03	.02	.050	.029	.023
technical	1.0	.10	.1	.05	.03	.02	.053	.030	.024
bias/		.50	.5	.12	.07	.05	.118	.068	.053
s.e.		1.	1.0	.16	.10	.07	.166	.096	.074

It appears that the technical bias is very small compared to the standard error for small values of μ and μ_j/μ. The technical bias can give a significant contribution to the mean square error only if μ is very large and the number of observations is very small.

4.2 The nonresponse situation

In the nonresponse situation, the approximate moments of the central rates in (2.1) become (see Appendix, Theorem 1):

$$E(\hat{\mu}_j^*) \approx \mu_j^*\{1 + (q-pz\mu)r_Q[1-(r_Q-r_0)\ \frac{pz\mu(z\mu-q)}{q^*(q-pz\mu)}]/n(\mu t^*)^2\}\ , \quad (4.3)$$

$$V(\hat{\mu}_j^*) \approx \mu^*\mu_j^*[1-q_j^*a^*]/nq^* \quad (4.4)$$

and

$$\mathrm{Cov}(\hat{\mu}_i^*,\hat{\mu}_j^*) \approx -\ \mu_i^*\mu_j^*a^*/n \qquad \text{for } i,j=1,2,\ldots,\ K \quad (4.5)$$

where

$$a^* = \frac{pz\mu(z\mu-2q+pz\mu)}{q(\mu t^*)^2}\ (r_Q - r_0)$$

and where

$$r_Q = \Sigma_j\ \bar{\mu}_j r_j \qquad (\text{with } \bar{\mu}_j=\mu_j/\mu)$$

is the overall expected response rate among all departures,

$$q_j^* = E(Q_j^*) = r_j\ \bar{\mu}_j q\ , \qquad q^* = E(Q^*) = r_Q q$$

and

$$t^* = E(T^*) = [qr_Q-pz\mu(r_Q-r_0)]/\mu\ ,$$

$$\mu_j^* = q_j^*/t^*, \qquad \mu^* = q^*/t^*.$$

If $r_j = r_0 = 1$ for all j, i.e., if all units respond with probability 1, then (4.3) and (4.4) become equal to (4.1) and (4.2), respectively, and (4.5) becomes equal to 0. If the response probabilities are equal (a rather unrealistic assumption), i.e., if $r_j = r_0 = r$ for all j, then the covariance expression in (4.5) again becomes equal to 0 and the expressions in (4.3) and (4.4) become

$$E(\hat{\mu}_j^*) \approx \mu_j^*\{1 + (q-pz\mu)/nrq^2\}\ ,$$

and

$$V(\hat{\mu}_j^*) \approx \mu\mu_j/nrq, \qquad \text{for } j=1,2,\ldots,\ K$$

where

$$r = qr_Q + pr_0$$

is the overall expected response rate. Thus, if the response probabilities are equal for all units, the only effect of nonresponse on the technical bias and on the variance is that they both increase as the expected number $n\cdot r$ of observations decreases.

We have made a series of trial calculations of the standard error and technical bias; some of these calculations are listed in Table 5, which shows the effect of unequal response probabilities when the overall expected response rate, $r = qr_1 + pr_0$, is 85 percent.

Table 5. Increase in relative standard error (coefficient of variation) and relative technical bias of $\hat{\mu}^*$ due to selective nonresponse with an overall expected response rate of 85 percent, for z=1. One cause of decrement.

Transition intensity $\mu_j = \mu$	Response probability %		Increase in	
	r_1	r_0	relative standard error	relative technical bias
0.01	95	84.9	1.03	1.18
	75	85.1	1.15	1.18
0.10	95	84	1.03	1.18
	75	86	1.15	1.18
1.00	95	68	1.01	1.16
	76.3	100	1.16	1.17
All	85	85	1.08	1.18

With equal response probabilities of r = 0.85 for all units, irrespective of the outcome of the life history, the relative standard error and technical bias will increase by 8 percent and 18 percent, respectively, due to nonresponse. The increase is larger for the technical bias than for the standard error. Further calculations suggest that the standard error seems to be more sensitive to the difference between r_1 and r_0. For the technical bias, the increase seems to be approximately proportional to the inverse of the overall expected response rate r, no matter how much r_1 and r_0 differ. This is generally valid for small intensities, $\mu < 1$. Thus, most of the time, the technical bias behaves in the same manner in the nonresponse situation as in the complete response situation: the bias is proportional to the inverse of the number of observations and is insignificant if this number is large enough.

The central rates for different risks become correlated if the response probabilities differ, as is shown in (4.5). The term a^* usually becomes very small, however, and the correlation between the central rates can be ignored. This is illustrated in Table 6.

Table 6. Approximate correlation between estimators of transition intensities for two causes of decrement, for z=1. Percent

Response probabilities %			Transition intensities						
			$\mu =$	0.1	1.0	3.0		5.0	
			$\mu_1 =$	0.05	0.5	0.3	1.5	0.5	2.5
r_1	r_2	r_0	$\mu_2 =$	0.05	0.5	2.7	1.5	4.5	2.5
95	95	75		-0.0	-1.3	-1.4	-2.4	-0.7	-1.1
95	75	55		-0.1	-2.7	-2.2	-4.2	-1.0	-1.9
75	95	55		-0.1	-2.7	-2.8	-4.2	-1.2	-1.9
75	55	95		0.0	1.3	3.6	3.9	2.2	2.3
55	75	95		0.0	1.3	1.5	3.9	0.8	2.3
55	55	95		0.0	1.7	3.4	5.7	2.1	3.5

In the nonresponse situation, the usual variance estimator becomes

$$v(\hat{\mu}_j^*) = \hat{\mu}_j^*/\Sigma T_v^*, \qquad \text{for } j=1,2,\dots, K, \tag{4.6}$$

where ΣT_v^* is the total exposure time among the respondents. When there is no nonresponse, this estimator is approximately unbiased. In the presence of nonresponse, its expected value is approximately

$$E[v(\hat{\mu}_j^*)] \approx \mu_j^*/nt^* = \mu_j^*\mu^*/nq^*$$

$$= V(\hat{\mu}_j^*)[1 + q_j^*a^*/(1-q_j^*a^*)]. \tag{4.7}$$

The second term in the brackets of (4.7) is the approximate relative bias of the variance estimator in (4.7). Like the covariance in (4.5), this bias depends on a^*, which is equal to 0 if the response probabilities are equal, i.e., if $r_j = r_0$ for all j. If the response probabilities differ, the variance estimator is biased, but the bias is usually *very* small (see Table 7 and Figure 3).

Table 7. Relative bias of the variance estimator in (4.6) for one cause of decrement, for z=1. Percent.

Response Probabilities % r_1	r_0	Transition intensity 0.01	0.1	0.5	1
95	55	.00	.19	3.04	7.58
95	75	.00	.05	.97	2.68
55	75	−.00	−.03	−.71	−2.31
55	95	−.00	−.04	−.95	−3.31

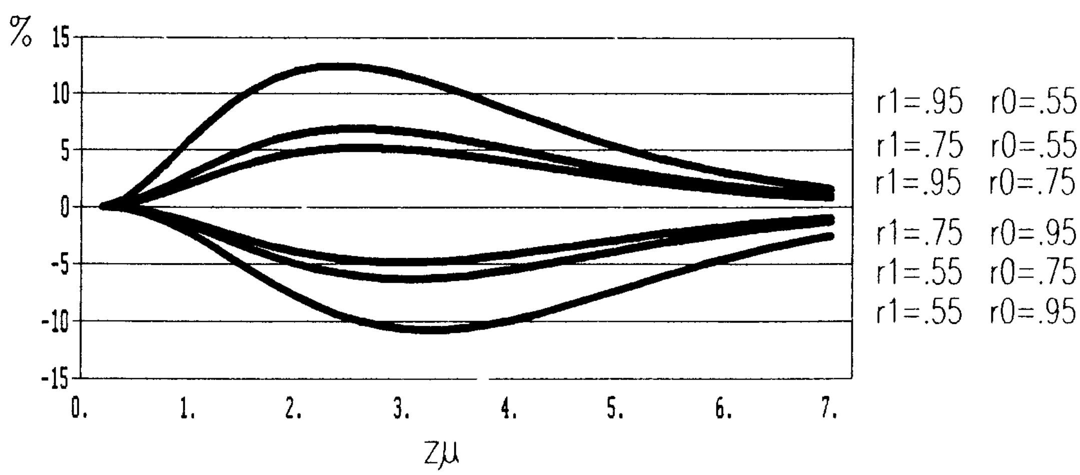


Figure 3. Relative bias of the variance estimator in (4.6) for one cause of decrement with intensity μ and observation until time z. Selected values of response probabilities for decrements (r_1) and for survivors (r_0), respectively. Percent.

Thus, we can usually rely on the common variance estimator even in the nonresponse situation and also ignore the small correlation between different central rates. If we have any doubts, however, we can use the following estimator (derived in our Appendix, Theorem 1), which estimates the variances and covariances approximately unbiasedly:

$$cov(\hat{\mu}_i^*, \hat{\mu}_j^*) = \frac{n}{n-1} \Sigma \,(Q_{iv}^* - \hat{\mu}_i^* T_v^*) \,(Q_{jv}^* - \hat{\mu}_j^* T_v^*) \,/\, [\, \Sigma\, T_v*]^2 , \qquad (4.8)$$

for i,j = 1,2,..., K. The term n in (4.8) is the number of *sampled* units which belong to State 0 during the relevant observation period. (This number corresponds to the figures in the last columns in Tables 2 and 3.) Usually this number is unknown. Replacing $n(n-1)^{-1}$ by 1 results in a slight underestimation. Replacing $n(n-1)^{-1}$ by $n_r(n_r-1)^{-1}$, where n_r is the number of observed units during the relevant period, gives a slight overestimation instead.

5. Nonresponse Bias

In the preceding section I showed that the technical bias of the central rate $\hat{\mu}_j^*$ usually is insignificant. This means that $E(\hat{\mu}_j^*) \approx q_j^*/t^* = \mu_j^*$, which, however, is usually *not* approximately equal to the transition intensity μ_j. The estimator suffers from nonresponse bias. This bias is serious, since it cannot be estimated from observed data. Also, the nonresponse bias does not decrease as the number of observations increases, unlike the technical bias. On the contrary, the ratio between the nonresponse bias and the standard error increases as the number of observations increases.

In this section it is shown how the nonresponse bias of central rates can be expressed as a function of the differences between the response probabilities of the decrements from various causes and for the survivors. I discuss how large the nonresponse bias may be; it turns out that it may be very large if the response probabilities differ greatly. I have also investigated the effects of nonresponse on estimates of transition and survival probabilities. It was found that the nonresponse bias seems to be less important if such probabilities are estimated via the central rates rather than directly by the proportions of decrements and survivors. respectively.

5.1 Nonresponse bias of central rates

The nonresponse bias of the central rate $\hat{\mu}_j^*$ (j=1,2,..., K) is approximately

$$b(\hat{\mu}_j^*) \approx q_j^*/t^* - \mu_j = \frac{r_j \mu_j q}{t^* \mu} - \mu_j$$

$$= \mu_j \frac{q(r_j - r_Q) + pz\mu(r_Q - r_0)}{t^* \mu} \tag{5.1}$$

$$\approx \sigma(\hat{\mu}_j^*)\,[q(r_j - r_Q) + pz\mu(r_Q - r_0)]\sqrt{\frac{\mu_j}{\mu}\frac{n}{r_j q}}, \tag{5.2}$$

where $\sigma(\hat{\mu}_j^*) = \sqrt{V(\hat{\mu}_j^*)} \approx \mu_j^* \mu^*/nq^*$. The approximation in (5.2) is based on the assumption that the term $q_j^* a^*$ in (4.7) is insignificant, which is usually true (see the preceding section).

The nonresponse bias of $\hat{\mu}_j^*$ (j=1,2,..., K) is positive if $r_j > r_Q > r_0$ and negative if $r_j < r_Q < r_0$. If the response probabilities are equal, i.e., $r_j = r_0$ for all j, there is no nonresponse bias. At the end of this section I present values of the relative nonresponse bias (Table 13) and of the ratio between the nonresponse bias and the standard error (Table 14). Tables 8 and 9 are excerpts from those tables. The results show that the nonresponse bias may be very large if the response probabilities of decrements and survivors differ greatly.

Table 8. Relative nonresponse bias, $b(\hat{\mu}_j^*)/\mu_j$. Percent; z=1

Expected response rates[1], %			Transition intensities $\mu_j = 0.1$		
r_j	r_0	r_{-j}	$\mu = 0.1$	$\mu = 0.5$	$\mu = 1$
95	75	95	25	19	14
95	75	75	25	25	25
75	95	95	- 20	- 20	- 20
75	95	75	- 20	- 17	- 13

1) r_{-j} is the overall expected response rate among decrements from any other cause than j.

Table 9. Ratio between nonresponse bias and the standard error, $b(\hat{\mu}_j^*)/\sigma(\hat{\mu}_j^*)$, for sample size n=100. Percent; z=1

Expected response rates[1], %			Transition intensities $\mu_j = 0.1$		
r_j	r_0	r_{-j}	$\mu = 0.1$	$\mu = 0.5$	$\mu = 1$
95	75	95	60	44	30
95	75	75	60	55	49
75	95	95	- 68	- 62	- 56
75	95	75	- 68	- 50	- 34

In our fertility survey, the differences between the response probabilities for decrements and survivors increased with age (see Tables 1 and 2). This is natural, since the group of survivors becomes more and more homogeneous with respect to fertility and nuptiality (and response behavior) with increasing age. This means that the nonresponse bias becomes more serious when we analyze the life histories for high ages. This is illustrated in Table 10, where the transition intensities and respond probabilities are based on the corresponding estimates in Table 2.

Table 10. Relative nonresponse bias and ratio between the nonresponse bias and the standard error for realistic values of transition intensities and response probabilities. Sample size n=100; Percent; z=1.

Transition intensities			Response probabilities, %			Relative nonresponse bias of		Ratio between the bias and the s.e. of	
μ_1	μ_2	μ	r_1	r_2	r_0	$\hat{\mu}_1^*$	$\hat{\mu}_2^*$	$\hat{\mu}_1^*$	$\hat{\mu}_2^*$
.03	.05	.08	86	82	86	0	−4	0	−9
.03	.15	.18	92	89	81	13	9	18	29
.03	.15	.18	80	86	77	3	11	4	33
.02	.05	.07	94	96	69	34	37	35	59

If there is only one cause of decrement, the expected value and relative nonresponse bias of $\hat{\mu}^*$ become

$$E(\hat{\mu}*) \approx \mu* = \mu/[1 - \frac{p \cdot z \cdot \mu}{q}(1 - \frac{1}{w})] \qquad (5.3)$$

and

$$rb(\hat{\mu}*) \approx (\mu* - \mu)/\mu = (w-1)/[1 + wg(\mu)], \qquad (5.4)$$

respectively, where

$$g(\mu) = \frac{q}{pz\mu} - 1 = \sum_{v=2}^{\infty} \frac{(z\mu)^{v-1}}{v!} \qquad (5.5)$$

and $w = r_1/r_0$.

The term $g(\mu)$ increases with $z\mu$. Thus, the relative nonresponse bias is insignificant for very large values of $z\mu$, and is approximately equal to $(w-1) = (r_1 - r_0)/r_0$ if $z\mu$ is much smaller than 1. Figure 4 shows the relative nonresponse bias for various values of $z\mu$ and of the ratio w. Realistic values of that ratio are given in Table 11.

Table 11. Values of $w = r_1/r_0$ for selected realistic values of r_1 and r_0.

r_1	r_0						
	.3	.4	.5	.6	.7	.8	.9
.3	1	.8	.6	.5	.4	.4	.3
.4	1.3	1	.8	.7	.6	.5	.4
.5	1.7	1.3	1	.8	.7	.6	.6
.6	2	1.5	1.2	1	.9	.8	.7
.7	2.3	1.8	1.4	1.2	1	.9	.8
.8	2.7	2.0	1.6	1.3	1.1	1	.9
.9	3.0	2.3	1.8	1.5	1.3	1.1	1

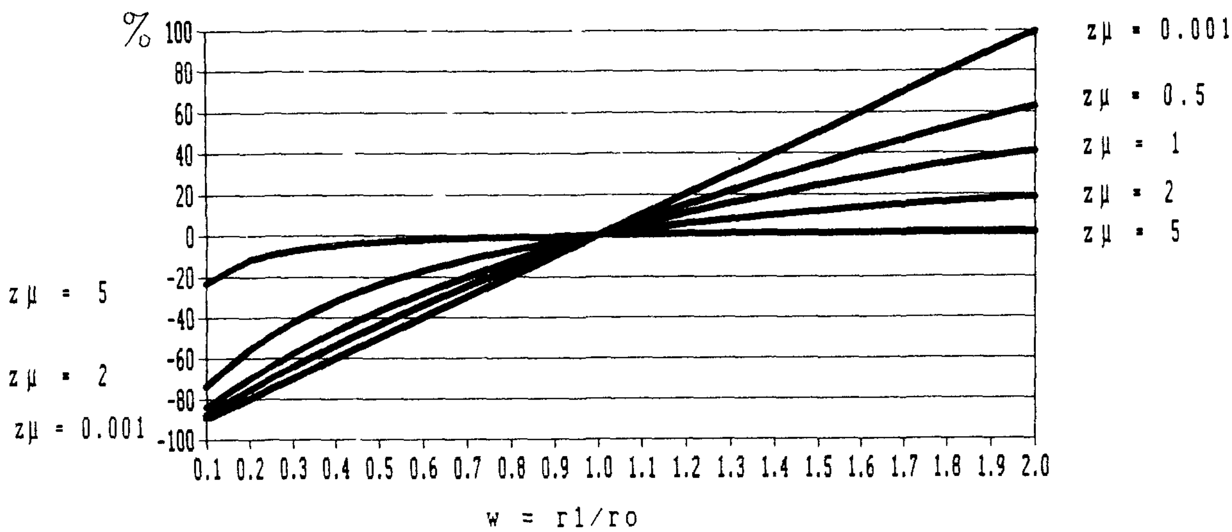


Figure 4. Relative nonresponse bias of $\hat{\mu}^*$. Percent

5.2 Nonresponse bias of estimated transition and survival probabilities

One often wants to estimate the *transition probabilities* $q_j = P_j(0,z) = q\mu_j/\mu = [1-\exp(-z\mu)]\mu_j/\mu$, for $j=1,2,\ldots, K$, and the *survival probability* $p=1-q= \exp(-z\mu)$. Various estimation methods are available. We discuss two of them. One is based on the estimated transition intensities and the other on the observed proportions of survivors and decrements due to the various causes. In the nonresponse situation those estimators become (for z=1):

$$q'_j \quad = q'\hat{\mu}^*_j/\hat{\mu}^* \qquad\qquad \text{for } j=1,2,\ \ldots,\ K,$$

where $q' = 1-p'$,

$$p' \quad = 1-q' \quad = \exp(-z\hat{\mu}^*) \tag{5.6}$$

and

$$q''_j \quad = \quad \sum_{v=1}^{n_r} Q^*_{jv}\ /n_r \qquad\qquad \text{for } j=1,2,\ \ldots,\ K,$$

$$p'' \quad = \quad 1-q'' \quad = 1 - \sum_{j=1}^{K} q''_j \ , \tag{5.7}$$

respectively, where n_r is the number of respondents. Both methods give consistent estimators when there is no nonresponse. The latter estimators are also unbiased in that case. The former estimators , however, have slightly smaller asymptotic variances. In the nonresponse situation both methods give rise to nonresponse bias:

$$\begin{aligned} rb(p') &= -\frac{p}{q}\ rb(q') \ = \frac{E(p')}{p} - 1 \\ &\approx \quad \exp[-(w-1)z\mu h(z\mu)] - 1 \end{aligned}$$

and

$$\begin{aligned} rb(p'') &= -\frac{p}{q}\ rb(q'') \ = \frac{E(p'')}{p} - 1 \\ &\approx -\ (w-1)q/[p+wq]\ , \end{aligned}$$

where $h(z\mu) = 1/[1 + wg(z\mu)]$ with $g(z\mu)$ given in (5.5). Both estimators underestimate p and overestimate q if $w=r_Q/r_0>1$ and vice versa. (As before, r_Q and r_0 are the response probabilities among decrements of any cause and among survivors, respectively). It can be shown that the magnitudes of the relative biases increase with $|w-1|$.

Calculations with realistic values, ($0.02 \leq p,q \leq .98$ and $0.1 \leq w \leq 5$), show the following relations for the relative biases of the estimators:

$$0 < rb(q') < rb(q'') \quad \text{and}$$
$$rb(p'') < rb(p') < 0 \quad \text{if } r_Q > r_0$$

and

$$rb(q'') < rb(q') < 0 \quad \text{and}$$
$$0 < rb(p') < rb(p'') \quad \text{if } r_Q < r_0 .$$

Calculations for $.1 \leq w \leq 5$ and $.02 \leq p,q \leq .98$ also show that:

$0 \leq |rb(p'')| - |rb(p')|$ decreases with p for all p, and increases with $|w-1|$ except for small w; $w < w(p)_{lim} < 1$,

and that

$0 \leq |rb(q'')| - |rb(p')|$ increases with q except for large q; $q > q_{lim}$, and increases with $|w-1|$ except for small w; $w < w(q)_{lim} < 1$,

where, for instance, :

p	q	$w(p)_{lim}$	$w(q)_{lim}$	w	q_{lim}
.8	.2	.6	.6	.2	.96
.5	.5	.5	.5	.8	.86
.3	.7	.4	.4	1.2	.80
.3	.7	.4	.4	2.6	.54

For realistic values of w and p=1-q, however, the estimators' relative nonresponse biases differ by a few percentage points only. Examples of the results presented above are shown in Table 12 and Figures 5 and 6 below, and in Table 15 and at the end of this section.

Table 12. Relative nonresponse bias of estimated survival probabilities obtained via estimated intensities (p') and observed proportions of survivors (p"), respectively. Percent.

Parameter values		Response probabilities, %			
		$r_1=95$ $r_0=75$ $w=1.27$		$r_1=75$ $r_0=95$ $w=0.79$	
z·μ	p %	rb(p')	rb(p")	rb(p')	rb(p")
0.01	99	- 0.3	- 0.3	0.2	0.2
0.10	91	- 2.5	- 2.3	2.0	2.0
1	37	-13.0	-14.4	14.4	15.4
2	14	-13.2	-18.7	16.7	22.3
5	1	- 3.5	-20.9	4.6	26.4

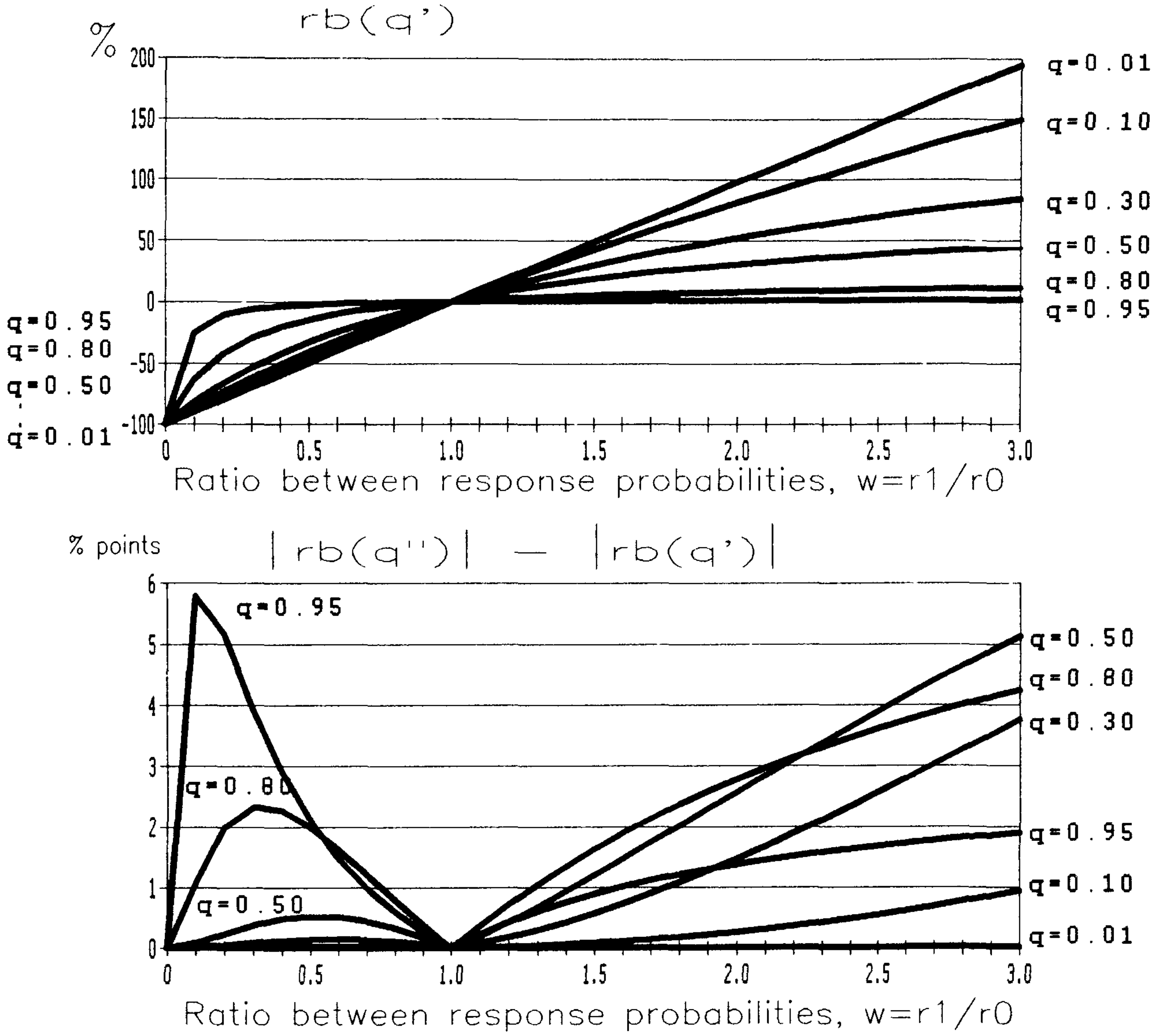


Figure 5. Relative nonresponse bias for $q'=\exp(-\hat{\mu}^*)$ and difference between $|rb(q'')|$ and $|rb(q')|$, by $w=r_1/r_0$ for selected values of q. Percent and percentage points.

Figure 5 shows how the size of the relative bias for the estimator q' increases when the difference between the response probabilities of decrements and survivors increases. For realistic values of q and w, the relative nonresponse of the estimator p" is only a few percentage points larger in magnitude than that of p'.

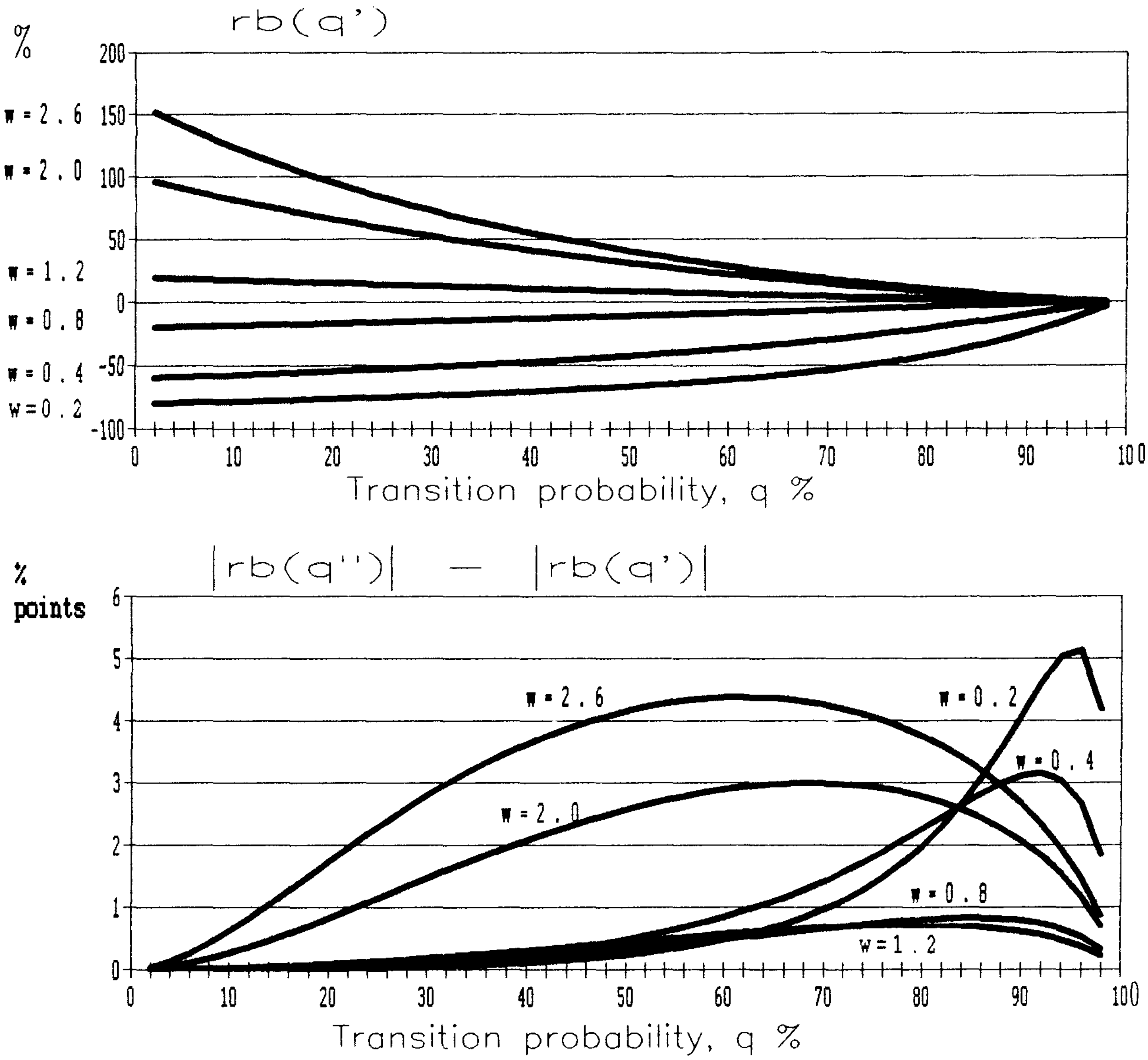


Figure 6. Relative nonresponse bias for $q'=\exp(-\hat{\mu}^*)$ and difference between $|rb(q'')|$ and $|rb(q')|$, by q for selected values of $w=r_1/r_0$. Percent and percentage points.

Figure 6 shows how the relative bias for the estimator q' decreases with q. The difference between $|rb(q'')|$ and $|rb(q')|$ also increases with q, if q is not very large. This difference is, however, only a few percentage points.

The estimators q' and p' that are based on estimated transition intensities perform better than the corresponding estimators q" and p" that are based on observed proportions of decrements and survivors. The former have slightly smaller asymptotic variances and slightly smaller nonresponse bias. However, the former require more information: One must know when the decrements occur to calculate the denominator of the central rate.

5.3 Summary of results

Tables 13 and 14 present the nonresponse bias for central rates used as estimates of transition intensities for selected values of μ_j and μ and various response probabilities. Table 13 presents the relative nonresponse bias and Table 14 the ratio between the nonresponse bias and the standard error for n=100.

Table 15 shows the relative nonresponse bias for estimates of transition intensities and of transition and survival probabilities when there is only one cause of decrement. The relative bias is large when the response probabilities differ greatly and the parameter estimated is small. Thus, the absolute nonresponse bias may be small but is nevertheless important when estimates of small-valued parameters are compared in the analysis.

Table 13. Relative nonresponse bias of $\hat{\mu}_j^*$ for selected values of μ_j and μ; z=1. Percent.

Expected[1] response rates %			Transition intensities							
			μ_j=0.01		μ_j=0.1			μ_j=0.01		
r_j	r_0	r_{-j}	μ=0.01	μ=0.1	μ=0.1	μ=0.5	μ=1	μ=0.5	μ=1	μ=5
95	95	75	0	1	0	4	9	0	5	22
		55	0	2	0	8	19	0	10	58
75	75	95	0	-1	0	-5	-9	0	-5	-19
		55	0	1	0	5	11	0	6	30
55	55	95	0	-3	0	-12	-21	0	-13	-39
		75	0	-2	0	-6	-12	0	-7	-24
95	75	95	26	25	25	19	14	19	14	1
		75	26	27	25	25	25	19	20	23
		55	26	28	25	31	39	19	27	60
75	55	95	36	32	34	19	6	26	11	-18
		75	36	34	34	26	18	26	18	1
		55	36	36	34	34	34	26	27	32
95	55	95	72	67	67	48	32	48	32	1
		75	72	69	67	57	48	48	41	25
		55	72	72	67	67	68	48	50	61
75	95	95	-21	-21	-20	-20	-20	-17	-17	-19
		75	-21	-20	-20	-17	-13	-17	-13	-1
		55	-21	-19	-20	-14	-5	-17	-9	29
55	75	95	-27	-27	-26	-29	-33	-22	-27	-39
		75	-27	-27	-26	-26	-26	-22	-22	-25
		55	-27	-26	-26	-22	-17	-22	-17	-1
55	95	95	-42	-42	-41	-41	-41	-36	-37	-40
		75	-42	-41	-41	-39	-36	-36	-33	-25
		55	-42	-41	-41	-36	-30	-36	-30	-2

1) r_{-j} is the overall expected response rate among decrements from any cause other than j.

Table 14. Ratio between nonresponse bias and standard error[2] of $\hat{\mu}_j^*$ for selected values of μ_j and μ; z=1 n=100. Percent.

Expected response rates[1] %			Transition intensities							
			μ_j=0.01		μ_j=0.1			μ_j=0.01		
r_j	r_0	r_{-j}	μ=0.01	μ=0.1	μ=0.1	μ=0.5	μ=1	μ=0.5	μ=1	μ=5
95	95	75	0	1	0	11	19	0	24	56
		55	0	2	0	21	39	0	48	112
75	75	95	0	-1	0	-12	-22	0	-27	-63
		55	0	1	0	12	22	0	27	63
55	55	95	0	-2	0	-28	-51	0	-63	-148
		75	0	-1	0	-14	-26	0	-32	-74
95	75	95	20	19	60	44	30	99	67	2
		75	20	20	60	55	49	99	91	58
		55	20	21	60	65	69	99	115	115
75	55	95	23	20	68	38	12	112	48	-61
		75	23	21	68	50	34	112	76	2
		55	23	22	68	62	56	112	103	66
95	55	95	41	38	120	89	60	198	134	4
		75	41	39	120	99	79	198	158	61
		55	41	40	120	110	99	198	182	117
75	95	95	-23	-22	-68	-62	-56	-112	-103	-66
		75	-23	-21	-68	-50	-34	-112	-76	-2
		55	-23	-20	-68	-38	-12	-112	-48	61
55	75	95	-27	-27	-79	-86	-90	-130	-152	-151
		75	-27	-26	-79	-72	-65	-130	-120	-77
		55	-27	-25	-79	-58	-39	-130	-88	-3
55	95	95	-54	-52	-158	-144	-130	-261	-240	-154
		75	-54	-51	-158	-130	-104	-261	-208	-80
		55	-54	-50	-158	-117	-79	-261	-176	-6

1) r_{-j} is the overall expected response rate among decrements from any cause other than j.

2) The figures should be multiplied with 0.1 $\sqrt{n}$, where n is the sample size.

Table 15. Relative nonresponse bias of estimators of transition intensities, of transition probabilities and of survival probabilities. Percent.

Estim- ator[1)		Parameter values						
	$z\mu=$	0.01	0.10	0.69	1.00	2.00	3.00	5.00
	$p=$	0.99	0.90	0.50	0.37	0.14	0.05	0.01
	$q=$	0.01	0.10	0.50	0.63	0.86	0.95	0.99
				r_1 = 95%	r_0 = 75%			
$\hat{\mu}^*$		26.50	25.03	17.09	13.96	7.06	3.42	0.72
p'		-0.26	-2.47	-11.17	-13.03	-13.16	-9.76	-3.53
p"		-0.26	-2.47	-11.76	-14.42	-18.74	-20.22	-20.94
q'		26.33	23.50	11.17	7.58	2.06	0.51	0.02
q"		26.33	23.53	11.77	8.0	2.93	1.06	0.14
				r_1 = 95%	r_0 = 55%			
$\hat{\mu}^*$		72.10	66.76	41.22	32.46	15.18	7.09	1.45
p'		-0.72	-6.46	-24.85	-27.72	-26.19	-19.15	-6.99
p"		-0.72	-6.47	-26.66	-31.49	-38.61	-40.87	-41.94
q'		71.48	61.41	24.85	16.13	4.10	1.00	0.05
q"		71.49	61.55	26.67	18.33	6.04	2.14	0.28
				r_1 = 55%	r_0 = 75%			
$\hat{\mu}^*$		-26.57	-25.69	-20.13	-17.47	-10.22	-5.41	-1.22
p'		0.27	2.60	14.97	19.08	22.68	17.61	6.28
p"		0.27	2.60	15.38	20.27	29.97	33.94	36.03
q'		-26.47	-24.75	-14.98	-11.11	-3.55	-0.92	-0.04
q"		-26.47	-24.76	-15.39	-11.80	-4.69	-1.78	-0.24
				r_1 = 55%	r_0 = 95%			
$\hat{\mu}^*$		-41.98	-40.88	-33.52	-29.74	-18.54	-10.26	-2.41
p'		0.42	4.17	26.15	34.63	44.90	36.04	12.79
p"		0.42	4.17	26.66	36.27	57.25	66.69	71.88
q'		-41.86	-39.68	-26.15	-20.16	-7.03	-1.89	-0.09
q"		-41.86	-39.69	-26.67	-21.11	-8.96	-3.49	-0.49

1) $\hat{\mu}^*$ is the occurrence/exposure rate based on data for the respondents,

p', q' are the estimators of survival and transition probabilities, respectively, based on the estimated transition intensity: $p'=1-q'=\exp(-z\hat{\mu}^*)$,

p", q" are the corresponding estimators based on the observed proportion of survivors and decrements, respectively.

6. Two Ways of Adjusting for the Nonresponse Bias

Since the "naive" central rates in (2.1) are biased in the presence of nonresponse, two methods for adjusting for such bias are investigated. Both of them are based on the assumption that it is possible to get accurate estimates of the ratios $w_j = r_j/r_0$ ($j=1,2,\ldots, K$) between the response probabilities for the decrements and the response probability for the survivors. If such good estimates are available, we may decrease the nonresponse bias almost completely by adjustment methods. We take a risk, however: the nonresponse bias may in fact be increased by the "adjustment" methods. This happens if our estimates of the $\{w_j\}$ are inaccurate. This is a finding analogous to those made by Frankel (1969) and Thomsen (1973) for design-based adjustment methods (groupwise weighting and post-stratification).

I begin by presenting the two adjustment methods which provide consistent estimators if exact values of the $\{w_j\}$ are available. After that I illustrate how the estimators are affected by erroneous estimates of $w=r_1/r_0$ when there is only one cause of decrement. I also try to provide some rules of thumb for deciding when one adjustment method is better than the other. Finally, some other common adjustment methods are discussed and it is explained why they seldom can be used in life-history analysis.

6.1 Description of the methods

Suppose that we are willing to guess or use previous surveys to estimate the ratios between the response probabilities by $w_j' = r_j'/r_0'$ ($j=1,2,\ldots, K$). In the following we do not distinguish between pure nonrandom guesses and random estimates. Both are denoted w_j', and so is the expected value of a random estimate w_j'. The distinctions are unimportant here.

The first adjustment method is inspired by standard sampling methods, where an observation is weighted by the reciprocal of the probability of its inclusion in the sample. In our nonresponse situation this probability depends on the outcome of the life history, according to the model in Section 2. The weighted central rate becomes:

$$\hat{\mu}_j = \left[\sum_{v=1}^{n} \frac{Q^*_{jv}}{r_j'} \right] \Big/ \left[\sum_{v=1}^{n} \sum_{i=0}^{K} \frac{Q^*_{iv} T^*_v}{r_i'} \right] \qquad \text{for } j=1, 2, \ldots, K, \tag{6.1}$$

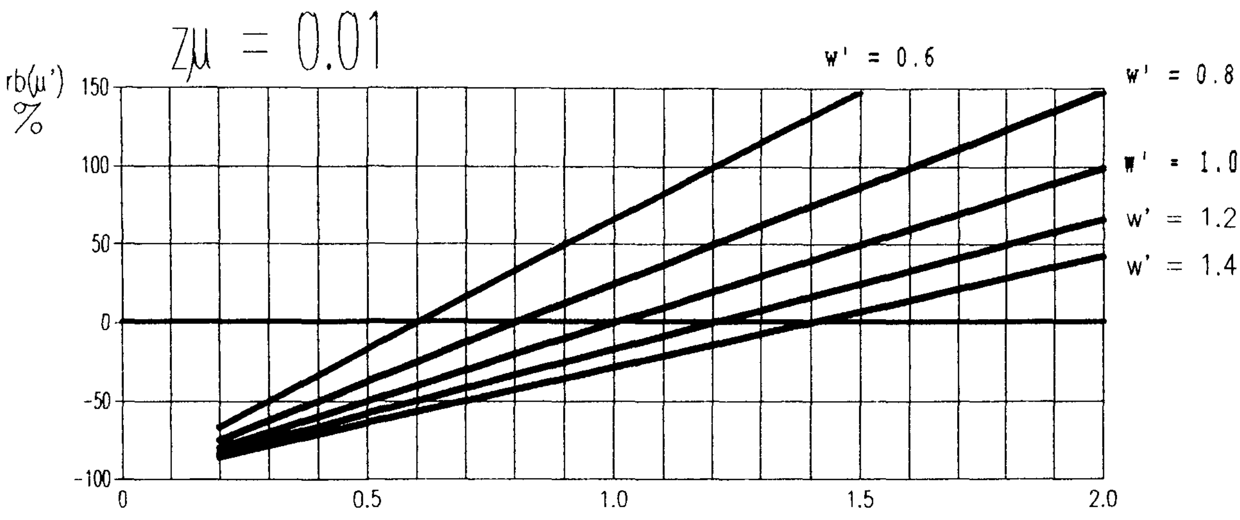
where $Q_0^* = R(1-Q) = 1$ if the individual is a survivor **and** a respondent, and $Q_0^* = 0$ otherwise. As before, the variables observed (Q_j^* and T^*) are equal to 0 for the nonrespondents.

Multiplying the numerator and denominator in (6.1) by r_0', we see that the estimator can be expressed as a function of the ratios w_j'. If the technical bias is insignificant, we can easily derive the expected value of the estimator in (6.1) by means of the results in the Appendix (Theorem 1). When there is only one cause of decrement, the nonresponse bias of $\hat{\mu}_j' = \hat{\mu}'$ can be expressed as

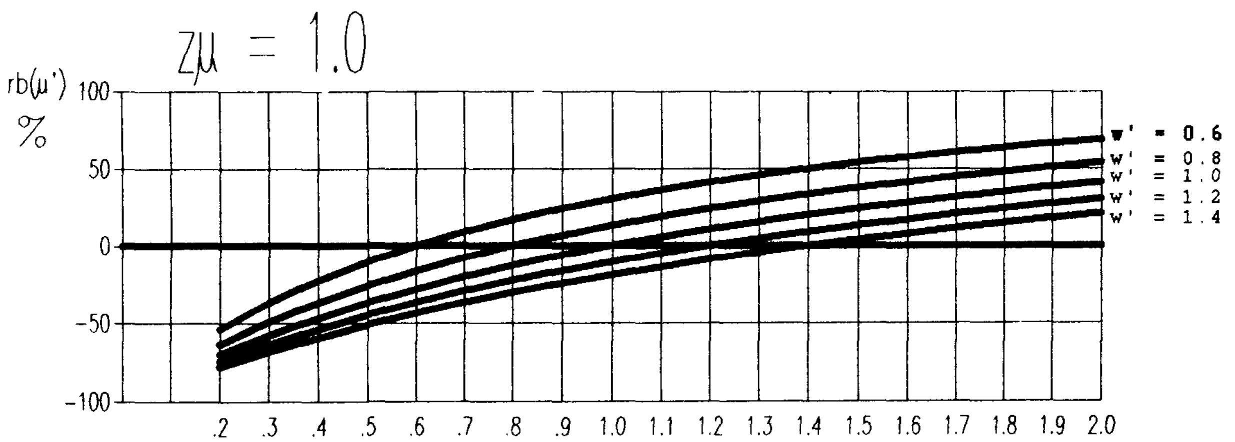
$$rb(\hat{\mu}') = [E(\hat{\mu}')-\mu]/\mu \approx \frac{pz\mu(w-w')}{qw - pz\mu(w-w')},$$

$$= (w-w')/[w' + wg(\mu)], \tag{6.2}$$

where $w=r_j/r_0$ and $w'=r_j'/r_0'$, and $g(\mu)$ was given in (5.5).



True ratio between response probabilities, w



True ratio between response probabilities, w

Figure 7. Nonresponse bias of the adjusted estimator $\hat{\mu}'$ for $z\mu=0.01$ and $z\mu=1$ and selected values of the guessed ratios, $w'=r_1/r_0$, between the response probabilities. Percent.

The adjusted estimator in (6.1) is approximately unbiased if the ratios between the response probabilities can be estimated (guessed) correctly. Figure 7 shows, for $z\mu = 0.01$ and $z\mu = 1$, and for various values of w and w', how the nonresponse bias may decrease or increase by using the adjusted estimator $\hat{\mu}'$. For instance, suppose $z\mu = 0.01$ and w=1.3, which are rather realistic values. If we do not adjust, i.e., if we let w'=1, then the nonresponse bias is about 30 percent ($z\mu^*=0.013$). Adjusting with $w'=1.2$ or $w'=1.4$ would decrease the nonresponse bias to about 10 percent and - 7 percent, respectively. Adjusting with $w'=0.8$, however, would increase the nonresponse bias to about 60 percent!

The second adjustment method was suggested by Hoem (1981). This method is based on the fact that the expected values of the unadjusted estimators $\hat{\mu}_j^*$ ($j=1,2,\ldots, K$) can be expressed as functions of the parameters $\underline{\mu} = (\mu_1,\ldots, \mu_K)$ and $\underline{w} = (w_1, w_2,\ldots, w_K)$. By (5.1),

$$E(\hat{\mu}_j^*) = \mu_j^* = f(\underline{\mu},\underline{w}) , \qquad \text{for } j=1,2,\ldots, K, \tag{6.3}$$

for a suitable function f. By replacing μ_j^* and $\underline{w}$ in (6.3) by $\hat{\mu}_j^*$ and $\underline{w}' = (w_1', w_2',\ldots, w_K')$, respectively, for $j=1,2,\ldots, K$, we get a system of K equations in which the unknown terms $\underline{\mu}$ can be found by an iteration method. With only one cause of decrement, the system reduces to the following single equation:

$$\hat{\mu}^* = \mu/[1 - \frac{pz\mu}{q}(1-\frac{1}{w'})], \tag{6.4}$$

were $w'=r_1'/r_0'$ is the guessed ratio between the response probabilities for decrements and survivors.

The equation in (6.4) can be solved, for instance, by the Newton-Raphson method. This gives the following iteration formula (see Appendix, Theorem 2):

$$\mu_{n+1} = \frac{[\hat{\mu}^* - \mu_n/s_n]s_n^2}{1 + \frac{z\mu_n}{q_n}(s_n-1)} , \tag{6.5}$$

where

$$s_n = 1 - \frac{p_n}{q_n} z\mu_n\left(1 - \frac{1}{w'}\right)$$

and

$$p_n = 1 - q_n = \exp(-z\mu_n).$$

A number of trial calculations with different starting values have shown that the iteration process converges very fast. Only a few steps are necessary to obtain accurate approximations. We have used as starting values the expected values of the unadjusted estimate $\hat{\mu}^*$ or the adjusted estimate $\hat{\mu}'$. The adjusted estimate found by this iterative method is approximately unbiased, provided that $w' = w$.

6.2 Comparison of the methods

Let us denote by $\hat{\mu}''$ the estimator (estimate) found by the iterative method. Figure 8 shows how the relations between the expected values of the three estimators $\hat{\mu}^*$, $\hat{\mu}'$ and $\hat{\mu}''$ on the one hand and the parameter value μ on the other, depend on the relation between w, w' and 1. (The results are derived in the Appendix, Theorem 3.) If the true ratio w is less than 1, the unadjusted estimators have a negative nonresponse bias. This is true also for the adjusted estimators $\hat{\mu}'$ and $\hat{\mu}''$ if $w'<w$, i.e., if we overestimate the ratio $w=r_1/r_0$ between the response probabilities.

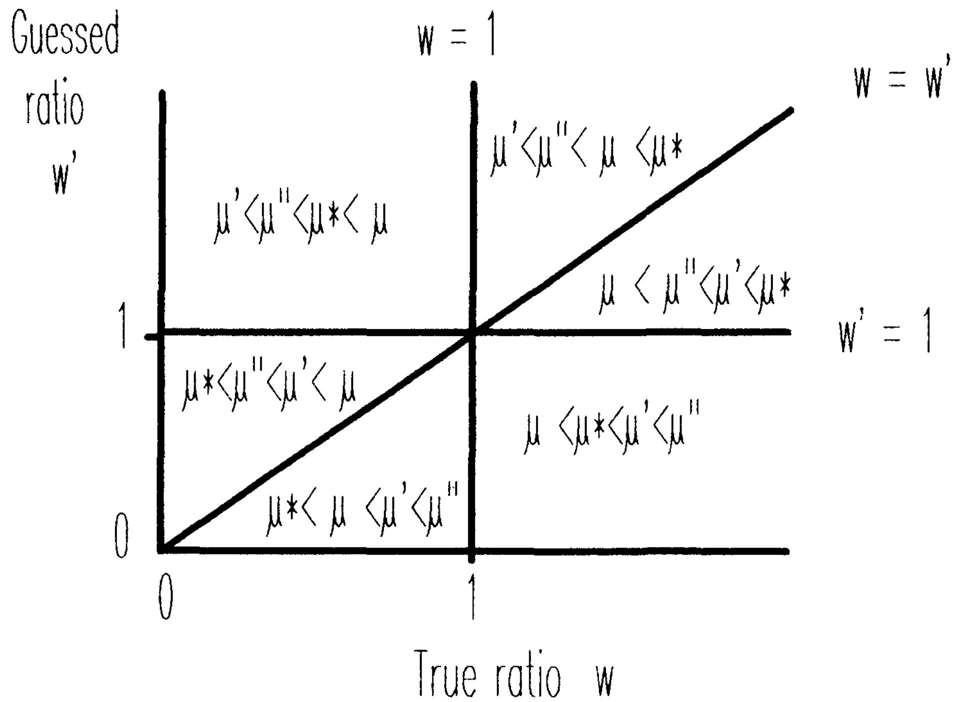


Figure 8. Relations between the expected values of the three estimators: $\hat{\mu}^*$ (unadjusted), $\hat{\mu}'$ (adjusted by weighting), $\hat{\mu}''$ (adjusted by the iterative method), and the true parameter value μ, for various relations between w, w' and 1.

Both adjustment methods reduce the size of the bias without changing its direction if $1<w'<w$ or $w<w'<1$, i.e., if w' is on the "right" side of 1. If we overdo our adjustment, i.e., $w'<w<1$ or $1<w<w'$., we change the direction of the bias and may even increase its size. Both adjustment methods always increase the bias (without changing its direction) if w' is on the "wrong" size of 1.

Figure 8 shows that the adjusted estimators work in the same way. In fact, via a number of calculations, we have found that they also change the nonresponse bias to approximately the same size if $z\mu$ is not too large. This is illustrated in Table 16. We had to choose $z\mu$ as large as 1 and 5 to obtain visible differences between μ' and μ''.

Table 16. Expected values of the three estimators: $\hat{\mu}^*$ (unadjusted), $\hat{\mu}'$ (adjusted by weighting), and $\hat{\mu}''$ (adjusted by the iterative method).

		True and estimated ratios between response probabilities								
		w = 0.8			w=1			w=1.2		
zμ		w'=0.8	w'=1.0	w'=1.2	w'=0.8	w'=1.0	w'=1.2	w'=0.8	w'=1.0	w'=1.2
.001	$z\hat{\mu}^*$	.00080	.00080	.00080	.00100	.00100	.00100	.00120	.00120	.00120
	$z\hat{\mu}'$	.00100	.00080	.00067	.00125	.00100	.00083	.00150	.00120	.00100
	$z\hat{\mu}''$	.00100	.00080	.00067	.00125	.00100	.00083	.00150	.00120	.00100
.010	$z\hat{\mu}^*$	.00801	.00801	.00801	.01000	.01000	.01000	.01199	.01199	.01199
	$z\hat{\mu}'$	.01000	.00801	.00668	.01248	.01000	.00834	.01496	.01199	.01000
	$z\hat{\mu}''$	.01000	.00801	.00668	.01249	.01000	.00834	.01496	.01199	.01000
.100	$z\hat{\mu}^*$	.08079	.08079	.08079	.10000	.10000	.10000	.11883	.11883	.11883
	$z\hat{\mu}'$	.10000	.08079	.06778	.12348	.10000	.08402	.14640	.11883	.10000
	$z\hat{\mu}''$	.10004	.08079	.06776	.12354	.10000	.08400	.14649	.11883	.09997
.500	$z\hat{\mu}^*$	.4192	.4192	.4192	.5000	.5000	.5000	.5737	.5737	.5737
	$z\hat{\mu}'$	.5000	.4192	.3609	.5911	.5000	.4332	.6729	.5737	.5000
	$z\hat{\mu}''$	.5005	.4192	.3610	.5923	.5000	.4331	.6749	.5737	.4996
1.00	$z\hat{\mu}^*$	.873	.873	.873	1.000	1.000	1.000	1.107	1.107	1.107
	$z\hat{\mu}'$	1.000	.873	.775	1.132	1.000	.896	1.241	1.107	1.000
	$z\hat{\mu}''$	1.001	.873	.776	1.135	1.000	.897	1.248	1.107	.999
5.00	$z\hat{\mu}^*$	4.958	4.958	4.958	5.000	5.000	5.000	5.028	5.028	5.028
	$z\hat{\mu}'$	5.000	4.958	4.917	5.034	5.000	4.966	5.057	5.028	5.000
	$z\hat{\mu}''$	5.000	4.958	4.928	5.041	5.000	4.971	5.069	5.028	5.000

It is better to adjust by weighting than by the iterative method when $w<w'<1$, and the opposite is true when $1<w'<w$. If, however, μ and $z\mu$ are small, the two methods give approximately the same results. Then there is no need to use the more complicated iterative method. If μ or $z\mu$ are large enough to give visible differences between the two methods, we have found that the weighting method (μ') seems to be less sensitive to erroneous estimates of w than the iterative method (Figure 9).

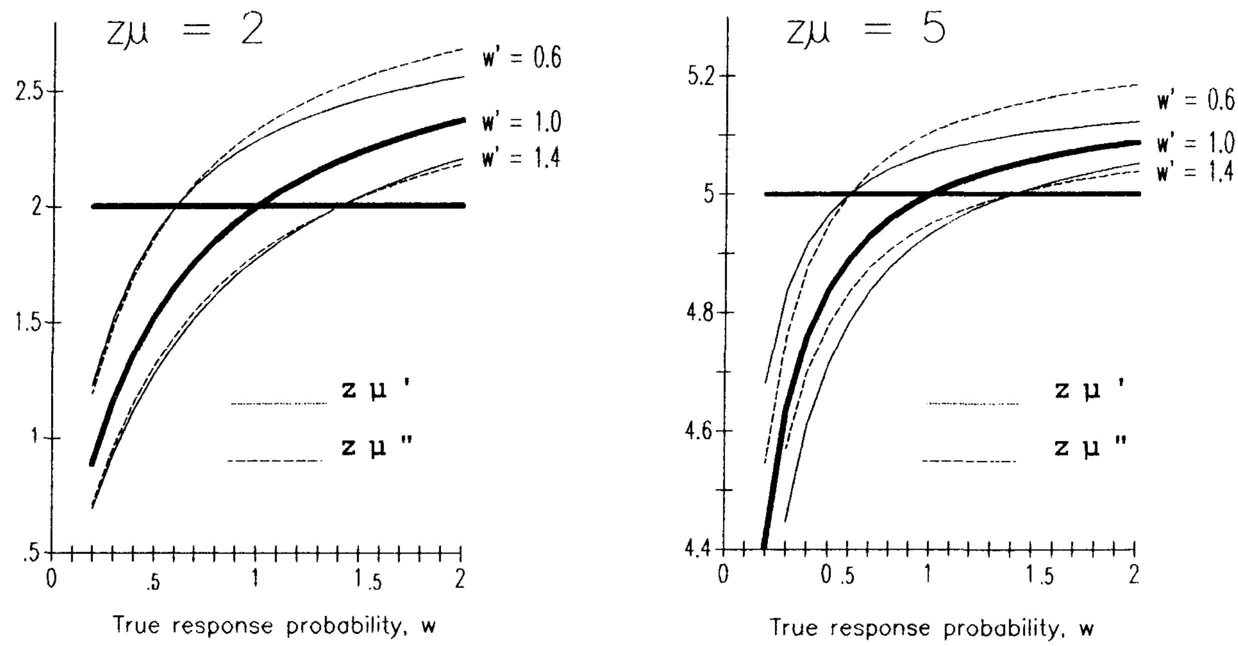


Figure 9. Expected values of the adjusted estimators $\hat{\mu}'$ and $\hat{\mu}''$ for $z\mu=2$ and $z\mu=5$ and selected values of the guessed ratios $w'=r1/r0$.

Unfortunately, both adjustment methods are sensitive to erroneous estimates of w when w is not close to 1, i.e., when the nonresponse bias is significant and there may be reasons to adjust for it. Thus, we have verified the old truth that the only safe way to reduce nonresponse bias is to reduce the nonresponse rate.

6.3 Other adjustment methods

For many reasons no global adjustment method was used in the 1981 Swedish Fertility Survey. The main reason was that the empirical study (Lyberg, 1983) suggested that the nonresponse bias was small compared with the random errors for the estimates of the main survey variables (fertility and nuptiality). Thus, it would not be wise to use an adjustment method that could increase the bias and decrease the precision. The two methods investigated in the previous section were found to be very sensitive to wrong guesses of the relation between the response probabilities. These methods could therefore not be recommended. No other adjustment method useful in life-history analysis was found in the literature.

Most of the nonresponse models discussed in Section 2.1 aim at adjusting for nonresponse. Survey statisticians often use weighting to adjust for nonresponse bias (see, e.g., Thomsen (1973, 1978), Lindström et al. (1979), Platek et al. (1978), Jagers (1986)). A review is given by Kalton and Kasprzyk (1986). The concept of response probabilities seems to have been commonly accepted. At least if such probabilities are formulated as conditional on the sample, as proposed by Särndal and Swensson (1987). A straightforward adjustment technique within the randomization theory is then to modify the Horvitz-Thompson estimator by weigthing with the "total inclusion probability" defined as the product of the sample inclusion probability and the response probability.

With many auxiliary variables the response probabilities can be estimated by logistic or probit regression and used directly for weighting individual values (see, for instance, Ekholm and Laaksonen (1990)). This method, however, can increase the variance substantially if the predicted response probabilities vary greatly. The two-phase sampling approach with conditional generalized regression estimation, developed by Swensson and Särndal (1987), is probably less sensitive.

The weighting adjustment methods found in the literature concern population totals (or means). For central rates (and other ratio estimators) the adjustment methods proposed can be applied to the numerator and denominator separately. If the groups are defined by the outcome of the life histories (i.e., the value of Q_j^*), the standard groupwise weighting method becomes equivalent to the weighting method described in the previous section. (If the values of Q_j were known for the nonrespondents it would also be possible to use an imputation technique for the missing exposures. Imputation for nonresponse is, however, not permitted by the data protection legislation in Sweden (Dalenius, 1979).) Auxiliary variables, other than the outcomes, could also be used to estimate response probabilities and weigh the occurrences and exposures separately.

Adjusting by weighting requires auxiliary variables that are correlated with the response mechanism (or rather reflect the correlation between the response mechanism and the outcome variables). For the 1981 Swedish Fertility Survey two different population registers could provide such auxiliary information: the Swedish Fertility Register (SFR) and the Total Population Register (TPR).

The SFR contained demographic data on life histories but no additional information that was expected to be associated with response behavior, for instance education, income, place of residence, etc.. The SFR information was therefore not expected to yield accurate estimates of response probabilities. Furthermore, it would not be easy to decide what life history data to account for when estimating response probabilities or defining homogeneous response groups (total fertility, fertility and timing of births, the fertility and marital history, or what?).

The TPR, which constituted the sampling frame, contained information about present address, marital status, children under 18 living with their mother, and income. That information could perhaps be useful to adjust estimates of the current situation, for instance, attitudes toward children. However, the central rates used in life history analyses concern subgroups defined by previous status, for instance background variables and duration in a state. The TPR-information could not be used for estimating response probabilities for such subgroups.

Weighting with response probabilities is a natural extension of the randomization theory for estimating population quantities. Life history analyses based on central rates rely on parametric outcome models and likelihood inference, however. The treatment of nonresponse in these analyses should therefore be derived within the framework of likelihood inference. One way is to model the joint distribution of the outcome variables (Q_j and T) and some auxiliary variable (x) so that the response mechanism can be ignored. This is the case if the loglikelihood can be decomposed into loglikelihoods (with distinct parameters) that correspond to likelihoods for complete data problems. (Little and Rubin, 1987, chapter 6).

Another (likelihood-based) way is the "stochastic censoring approach": The outcome variable (y) is observed if and only if the value of an unknown variable (u) exceeds a threshold value and both y and u are assumed to have a linear regression of covariates. The parameters can be estimated by maximum likelihood or the two-step method proposed by Heckman (1976, 1979) and used by Brehm (1990). (See, Little and Rubin, 1987, chapter 6). A third way is a Bayesian approach.

Of course, the best solution is to model the outcome variables so that the response mechanism can be ignored. Then all methods derived for the complete response situation can be applied, for instance, statistical tests, intensity regression (used in many analyses of the fertility survey), and so on. Likelihood methods for treating nonignorable response mechanisms were considered too complicated for the fertility survey as the survey was to be used for many different analyses.

7 Concluding remarks

The simple nonresponse model proposed in this essay has been useful for studying the effects of nonresponse on estimates of transition intensities in a competing risks model. It has been shown that central rates (occurrence exposure rates) based on the respondents only behave in the same way as in the complete response situation, provided that the response probabilities are equal for decrements and survivors. Then the response mechanism is ignorable: the central rates are asymptotically unbiased estimators of the transition intensities, the standard variance estimator is asymptotically unbiased, and the central rates for different causes are asymptotically uncorrelated.

When the response probabilities differ between decrements from different causes and survivors, the estimators are not unbiased and uncorrelated. In most cases the technical bias (due to ratio estimation), the bias of the variance estimator, and the correlation between central rates for different causes can be ignored. The nonresponse bias, however, may be very large. This bias can be expressed as a function of the differences between the response probabilities for decrements from different causes and the survivors and the underlying transition intensities.

Two adjustment methods were investigated, but no one was judged useful. Both methods require accurate estimates of the ratios between the response probabilities for decrements from different causes and the response probability for survivors. If erroneous estimates of these ratios are used the nonresponse bias may increase. As long as no robust adjustment method for competing risks models has been found the researcher is advised to use relevant covariates in the outcome model so that the response mechanism becomes ignorable.

ACKNOWLEDGEMENTS

I am very grateful to Jan M. Hoem for his great interest and very useful suggestions when I wrote the first version of this essay. I also want to thank Sven Berg and Carl-Erik Särndal for useful comments on earlier drafts. Lars Nordin and Hans Lundström made the calculations on empirical data possible: Lars by matching register and survey data and providing aggregated data on occurrences and exposures; Hans Lundström by transferring data between noncompatible media. I am very thankful to Daniel Thorburn who has encouraged me, checked the formulas and provided useful suggestions during the final work on this manuscript. Lars Lyberg and Martin Ribe have been helpful in the final preparation of the manuscript.

References

Andersson, H. (1979): On nonresponse and response probabilities. *Scandinavian Journal of Statistics*, Vol. 6, pp. 107-112.

Beyer, J.E., Keiding, M., and Simonsen, W., (1976): The exact behaviour of the maximum likelihood estimator in the pure birth process and the pure death process. *Scandinavian Journal of Statistics*, Vol. 3, pp. 61-72.

Bethlehem, J.G. (1988): Reduction of nonresponse bias through regression estimation. *Journal of Official Statistics*, Vol. 4. No. 3, pp.251-260.

Brehm, J. (1990): Towards understanding the effects of survey nonresponse on political research: two-stage correction for nonresponse. Paper presented at the Workshop on Household Survey Nonresponse, Stockholm, Sweden, 15-17 October, 1990. Unpublished report, Duke University, U.S.A.

Cassel, C.-M., Särndal, C.E,, and Wretman, J.H. (1983): Some uses of statistical models in connection with the nonresponse problem. In *Incomplete Data in Sample Surveys*, Vol III: Proceedings of the Symposium 1979. New York, Academic Press.

Cochran, W.G. (1977): *Sampling Techniques*. 3rd edition. New York, , Wiley.

Cialdini, R.B. (1990): Deriving psychological concepts relevant to survey participation from the literature on compliance, helping, and persuasion. Paper presented at the Workshop on Household Survey Nonresponse, Stockholm, Sweden, 15-17 October, 1990. Unpublished report, Arizona State University, Department of Psychology, Tempe, U.S.A.

Dalenius, T. (1983): Some reflections on the problem of missing data. In *Incomplete Data in Sample Surveys*, 3, Ed. W.G. Madow and I. Olkin, pp. 411-413. New York: Academic Press.

Dalenius, T. (1979): Data protection legislation in Sweden: A statistician's perspective. *Journal of Royal Statistical Society. A*, 142, Part 3, pp. 285-298.

Des Raj (1968): *Sampling Theory*. New York, McGraw-Hill.

Ekholm, A. and Laaksonen, S. (1990): Reweighting by nonresponse modeling in the Finnish household survey. Department of Statistics, University of Helsinki: Research Report No. 68.

Fay, R.E. (1986): Causal models for patterns of nonresponse. *Journal of American Statistical Assiciation*, Vol. 81, No. 394, pp. 354-365.

Frankel, L.R. (1969): Are survey data being over-adjusted? In *Current Controversies in Marketing Research.* Markham.

Groves, R. (1989): *Survey Errors and Survey Costs,* New York, Wiley.

Hansen, M.H. and Hurwitz, W.N. (1946): The Problem of Non-Response in Sample Surveys. *Journal of American Statistical Association,* Vol.41, pp. 517-529.

Heckman, J.J. (1976): The common structure of statistical models of truncation, sample selection and limited dependent variables and a simple estimator for such models. *Annals of Economic and Social Measurement, 5/4:475-492.*

Heckman, J.J. (1979): Sample selection bias as a specification error. *Econometrica, 47: 153-161*

Hoem, B. (1990): Alla goda ting är tre? Tredjebarnsfödslar bland svenska kvinnor födda 1936-60. (All good things are three? Third birth fertility among Swedish women born in 1936-60. In Swedish). Section of Demography, Universtity of Stockholm. Research Reports in Demography, No. 59.

Hoem, J.M. (1981): Skjevhet ved estimering av en avgangsintensitet ved intervjuundersøkelser der bortfallet avhenger av om den studerte avgang intreffet hos den enkelte respondent. Personal note, 1981-05-14.

Hoem, J.M. (1983): Weighting, misclassification, and other issues in the analysis of survey samples of life histories. University of Stockholm, Department of Statistics, Stockholm. Research Reports in Demography, No. 11.

Hoem, J.M. (1989): The issue of weights in panel surveys of individual behavior. University of Stockholm, Department of Statistics, Stockholm. Research Reports in Demography, No. 39. (May, 1987). IN Kasprzyk, D., Duncan, G., and Kalton, G. (eds.): *Panel Surveys,* pp. 539-565. New York, Wiley.

Hoem, J.M. and Funck Jensen, U. (1982): Multistate life table methodology: A probabilistic critique. University of Stockholm, Department of Statistics, Stockholm. Research Reports in Demography, No. 2. (March, 1982). In Land, K.C. and Rogers, A. (eds.): *Multidimensional Mathematical Demography,* pp. 155-264. New York, Academic Press.

Information i prognosfrågor (1982): Kvinnor och barn - Intervjuer med kvinnor om familj och arbete (Women and children - Interwiews with women about family and work. In Swedish) Sveriges officiella statistik, Statistiska centralbyrån (Statistics Sweden): Information i prognosfrågor, No. 1982:4

Information i prognosfrågor (1983): Arbete och barn - Kvinnors sysselsättning i de barnafödande åldrarna (Work and children - Employment patterns among women in the childbearing ages. In Swedish with English summary.) Sveriges officiella statistik, Statistiska centralbyrån (Statistics Sweden): Information i prognosfrågor, No. 1983:4.

Information i prognosfrågor (1984): Ha barn - men hur många? (Children - but how many? Interviews with women about children, family and work. In Swedish with English summary.) Sveriges officiella statistik, Statistiska centralbyrån (Statistics Sweden): Information i prognosfrågor, No. 1984:4.

Johansson, L. and Finnäs, F. (1983): *Fertility of Swedish women born in 1927–1960.* Statistics Sweden, Urval No. 14.

Kalton, G. and Kasprzyk, D. (1986): The treatment of missing survey data. *Survey Methodology*, Vol. 12, No. 1, pp. 1–16.

Lindström, H. and Lundström, S. (1974): A method to discuss the magnitude of the non-response error. *Statistisk tidskrift (Statistical Review), 1974:6, pp. 505–520.*

Lindström, H. et al. (1979): Standard methods for non-response treatment in statistical estimation. Stockholm: National Central Bureau of Statistics (Statistics Sweden), Survey Research Institute for Statistics on Living Conditions.

Little, R.J.A. (1982): Models for nonresponse in sample surveys. *Journal of Statistical Association*, 77, pp. 237–250.

Little, R.J.A. (1983): Superpopulation models for nonresponse. In *Incomplete Data in Sample Surveys, Vol. 2: Theory and bibliographies*, part VI, pp. 337–413. New York, Academic Press.

Little, R.J.A., and Rubin, D.B. (1987): *Statistical analysis with missing data.* New York, Wiley.

Lyberg, I. (1983): The effects of sampling and nonresponse on estimates of transition intensities: Some empirical results from the 1981 Swedish Fertility Survey. (Dec., 1983). University of Stockholm, Department of Statistics, Stockholm. Research Reports in Demography, No. 14.

Lyberg, I. (1984): Att fråga om barn. (To ask about children, In Swedish). Statistics Sweden, Bakgrundsmaterial från prognosinstitutet, 1984:4.

Oh, H. L. and Sheuren, F. J. (1983): Weighting adjustment for unit nonresponse. In *Incomplete Data in Sample Surveys. Vol. 2: Theory and Bibliografies*, Chapter 13, pp. 143–184. New York, Academic Press.

Panel on Incomplete Data (1983): *Incomplete data in sample surveys.* Vol I: Report and case studies; Vol II: Theory and bibliographies; Vol III: Proceedings of the Symposium. New York, Academic Press.

Platek, R., Singh, and Tremblay, V. (1978): Adjustment for nonresponse in surveys. In N.K. Namboodirei (ed.) *Survey Sampling and Measurement.* New York, Academic Press. Also in *Survey Methodology*(1977): 3(2), pp. 1-24.

Politz, A. and Simmons, W. (1949): I. An attempt to get the 'not at homes' into the sample without callbacks. II. Further theoretical considerations regarding the plan for eliminating callbacks. *Journal of the American Association*, 44, pp. 9-31.

Politz, A. and Simmons, W. (1950): Note on an attempt to get the 'not at homes' into the sample without callbacks. II. Further theoretical considerations regarding the plan for eliminating callbacks. *Journal of the American Association*, 45, pp. 136-137.

Quist, J. and Rennermalm, B. (1985): *Att bilda familj.* (Family formation. In Swedish with English summary). Statistics Sweden. Urval No. 17.

Quist, J. (1990): Kvalitets- och metodfrågor vid användning av registerdata. Tre fallstudier inom befolkningsstatistiken. (Quality and methodology issues in register data. Three case studies of population registers. In Swedish). Statistiska centralbyrån (Statistics Sweden), Bakgrundsmaterial från demografiska funktionen, No. 1990:2.

Rubin, D.B. (1983): Conceptual issues in the presence of nonresponse. In *Incomplete Data in Sample Surveys. Vol. 2: Theory and Bibliographies*, Chapter 12. New York, Academic Press.

Schou, G. and Væth, M. (1980): A small sample study of occurrence/exposure rates for rare events. *Scandinavian Actuarial Journal*, 1980(4), pp. 209-225.

Stasny, E.A. (1986): Some Markov-chain models for nonresponse in categorical data from panel surveys. *American Statistical Association 1986: Proceedings of the Section on Survey Research Methods.*, Chicago, Illinois, August 18-21, 1986, pp. 424-429.

Stasny, E.A. (1987): Some Markov-chain models for nonresponse in estimating gross labot force flows. *Journal of Official Statistics*, VOl. 3, No. 4, pp. 359-373.

Särndal, C.E. and Hui, T. (1981): Estimation for nonresponse situations: To what extent must we rely on models?. In Krewski, D., Platek, R. and Rao, J.N.K. (eds.): *Current Topics in Survey Sampling*, pp 227-246. New York, Academic Press.

Särnal, C.E. and Swensson, B. (1987): A general view of estimation for two phases of selection with applications to two-phase sampling and nonresponse. *International Statistical Review*, Vol. 55, No. 3, pp. 279-294.

Thomsen, I. (1973): A note on the efficiency of weighting subclass means to reduce the effects on non-response when analysing survey data. *Statistisk tidskrift* (Statistical Review), 1973(4), pp. 278-283.

Thomsen, I. (1973): A second note on the efficiency of weighting subclass means to reduce the effects on non-response when analysing survey data. *Statistisk tidskrift* (Statistical Review), 1978(3), pp. 191-196.

Vath, M., (1977): A note on the sampling distribution of the maximum likelihood estimators in a competing exponential risks model. *Scandinavian Actuarial Journal*, 1977(2), pp. 81-87.

APPENDIX

Theorem 1. The moments of the observed variables and observed central rates defined in Section 2 are as follows (for j=1,2,..., K):

$$E(Q_j^*) = E((Q_j^*)^2) = \mu_j q r_j/\mu \equiv q_j^* \,, \tag{A.1}$$

$$E(Q^*) = E((Q^*)^2) = q r_Q \equiv q^* \,,$$

$$E(T^*) = [q r_Q - pz\mu(r_Q - r_0)]/\mu \equiv t^* \,,$$

$$E((T^*)^2) = [2r_Q(q - pz\mu) - p(z\mu)^2(r_Q - r_0)]/\mu^2 \,,$$

$$E(Q_j^* T^*) = \mu_j r_j (q - pz\mu)/\mu^2 \,,$$

$$E(\hat{\mu}_j^*) \approx \mu_j^*\{1 + (q - pz\mu)\, r_Q[1 - (r_Q - r_0)\, \frac{pz\mu(z\mu - q)}{q^*(q - pz\mu)}/n(\mu t^*)^2\} \,,$$

and

$$\mathrm{Cov}(\hat{\mu}_i^* \hat{\mu}_j^*) \approx \mu_i^* \mu_j^* [d_{ij}/q_j^* - (r_Q - r_0)\, \frac{pz\mu(z\mu - 2q + pz\mu)}{q(\mu t^*)^2}]/n \,,$$

where $d_{ij} = 1$ if $i=j$ and $d_{ij} = 0$ if $i \neq j$, and

$$\mu = \Sigma \mu_j, \qquad r_Q = \Sigma r_j \mu_j/\mu \,, \tag{A.2}$$

and

$$p = 1 - q = \exp(-z\mu).$$

Furthermore, the covariance is estimated approximately unbiasedly by

$$\mathrm{cov}(\hat{\mu}_i^* \hat{\mu}_j^*) = \frac{n}{n-1}\, \Sigma_v\, (Q_{iv}^* - \hat{\mu}_i^* T_v^*)(Q_{jv}^* - \hat{\mu}_j^* T_v^*)/(\Sigma_v T^*)^2. \tag{A.3}$$

Proof. We use the following results concerning the model variables (for $j=1,2,\ldots, K$):

$$P(Q=1) = 1-\exp(-z\mu) = q \ ,$$

$$P(Q_j=1) = q\mu_j/\mu = q_j \ ,$$

$$E(T^v \mid Q=0) = z^v \ ,$$

and

$$E(T^v \mid Q_j=1) = [qv! - p \sum_{u=0}^{v-1} \frac{v!}{(v-u)!} (z\mu)^{v-u}]/q\mu^v, \quad \text{for} \quad v \geq 1.$$

The moments of T, conditional on $Q_j = 1$, are derived by using

$$P(T \leq t \mid Q_j=1) = \int_0^t \mu_j \exp(-s\mu) \, ds \, /q_j = [1-\exp(-t\mu)]/q$$

which yields

$$E(T^v \mid Q_j=1) = q^{-1} \int_0^z t^v \mu \exp(-t\mu) \, dt$$

$$= - \ q^{-1}\mu^{-v} \Big[\ \exp(-t\mu)[v! + \sum_{u=0}^{v-1} \frac{v!}{(v-u)!} (t\mu)^{v-u}] \ \Big]_0^z \ .$$

The results in (A.1) which have to do with the observed variables are then obtained by inserting the results of (A.4) in the following expressions: if $Y^* = R \cdot Y$, then

$$E(Y^*) = \Sigma \ P(Q_j=1) \cdot P(R=1 \mid Q_j=1) \cdot E(Y \mid Q_j=1)$$
$$+ \ P(Q=0) \cdot P(R=1 \mid Q=0) \cdot E(Y \mid Q=0)$$
$$= \Sigma \ q_j r_j \ E(Y \mid Q_j=1) \ + \ pr_0 \ E(Y \mid Q=0) \ .$$

To derive the moment of the observed central rates, we notice that they can be expressed as ratios of means: $\hat{\mu}_j^* = \bar{Q}_j^*/\bar{T}_j^*$, where each of $\bar{Q}_j^* = \Sigma \ Q_{j\,v}^*/n$ and $\bar{T}^* = \Sigma \ T_v^*/n$ is a mean of n stochastically independent variables, identically distributed as Q_j^* and T^*, respectively.

Let $\bar{y}$, $\bar{u}$, $\bar{x}$ and $\bar{v}$ be the means of n stochastically independent variables, identically distributed as Y, U, X and V with expected values m_y, m_u, m_x and m_v, respectively. By means of a Taylor expansion (see e.g., Des Raj, 1968, Chapters 5.4-5.5), it can be shown that the expected value of the ratio $\bar{y}/\bar{x}$ and the covariance of the ratios $\bar{y}/\bar{x}$ and $\bar{u}/\bar{v}$ are approximately

$$E(\bar{y}/\bar{x}) \approx \frac{m_y}{m_x}\{1 + [\, E(X^2) - \frac{m_x}{m_y} E(XY)]/n(m_x)^2\} \tag{A.5}$$

and

$$\mathrm{Cov}(\bar{y}/\bar{x},\bar{u}/\bar{v}) \approx \frac{m_y m_u}{m_x m_v}[RC(Y,U) + RC(X,V) - RC(Y,V) - RC(U,X)]/n$$

$$= \frac{m_y m_u}{m_x m_v}\left[\frac{C(Y,U)}{m_y m_u} + \frac{C(X,V)}{m_x m_v} - \frac{C(Y,V)}{m_y m_v} - \frac{C(U,X)}{m_u m_x}\right]/n,$$

where RC(.,.) denotes the relative covariance. Furthermore, a consistent estimator of the covariance above is given by

$$\mathrm{cov}(\bar{y}/\bar{x},\bar{u}/\bar{v}) = [\Sigma\ (Y_v - X_v\bar{y}/\bar{x})(U_v - V_v\bar{u}/\bar{v})]/\bar{x}\bar{v}n(n-1).$$

If we let $Y = Q_j^*$, $U = Q_i^*$, and $X = V = T^*$ in the expressions above we obtain, after some algebra, the results in (A.1). □

Theorem 2. Suppose that $\tau = \tau''$ is the unique solution to the equation

$$\tau^* = \tau\ /\ [1 - \frac{p(\tau)}{q(\tau)}\ \tau(1-b)] = \tau/s(\tau),$$

where $p(\tau) = 1 - q(\tau) = \exp(-\tau)$ and $b > 0$. Furthermore, suppose that the Newton-Raphson iteration process converges to τ'' and is sufficiently close at the nth step. Then the following is true for the nth value in that process:

$$\tau_n < \tau'' \text{ if and only if } (\tau_n/\tau^*) < s(\tau_n) = 1 - \frac{p(\tau_n)}{q(\tau_n)}\tau_n(1-b). \tag{A.6}$$

Proof. Let $f = f(\tau) = \tau/s(\tau) - y^*$,

where

$$s(\tau) = 1 - (1-b)\tau p/q \quad >0 \quad ,$$

$$b = 1/w' \quad >0 \quad ,$$

$$p = 1-q = \exp(-\tau) \quad (0 \leq p,q \leq 1).$$

The Newton-Raphson gives the following iteration formula to solve the equation $f(y) = 0$:

$$\tau_{n+1} = \tau_n - f(\tau_n)/f'(\tau_n). \qquad \text{(A.7)}$$

From $q'(\tau) = -p'(\tau) = -p(\tau) = -p(\tau)$ and $p+q=1$ it follows that:

$$\begin{aligned} f'(\tau) &= s^{-2}[s - \tau s'(\tau)] \\ &= s^{-2}\{s + \tau(1-b)q-2[q(-\tau p+p)-\tau p^2]\} \\ &= s^{-2}\{s - \tau(1-b)q^{-2}p(\tau-q)\} \\ &= s^{-2}[1 - (1-b)p\tau q^{-2}(q+\tau-q)] \\ &= s^{-2}[1 - (1-b)p(\tau/q)^2]. \end{aligned}$$

The last expression is always positive, since $b>0$ and $p(\tau/q)^2<1$, as shown by the following Taylor expansions:

$$
\begin{aligned}
1 - p(\tau/q)^2 &= (p/q)^2[e^{2\tau}(1 - 2e^{-\tau} + e^{-2\tau}) - \tau^2 e^{\tau}] \\
&= (p/q)^2[e^{2\tau} - 2\,e^{\tau} - \tau^2 e\tau + 1] \\
&= (p/q)^2[\Sigma_{v=0} (\tau^{v}/v!)[2^{v}-2] - \Sigma_{v=0} (\tau^{v+2}/v!) + 1] \\
&= (p/q)^2[\Sigma_{v=0} (\tau^{v}/v!)[2^{v}-2] - \Sigma_{v=2}(\tau^{v+2}/v!) + \tau^2+\tau^3+1] \\
&= (p/q)^2[\Sigma_{v=4} (\tau^{v}/v!)[2^{v}-2] - \Sigma_{v=4}(\tau^{v}/v!)(v-1)(v-2) \\
&\qquad +1+2v+4v^2/2+8v^3/6 - 2-2v-2v^2/2-2v^3/6 + \tau^2+\tau^3+1] \\
&= (p/q)^2[\Sigma_{v=4} (\tau^{v}/v!)[2^{v}-2 - v(v-1)] \quad > 0,
\end{aligned}
$$

since

$$2^{v}-2-v(v-1) = \sum_{i=2}^{v-2} \binom{v}{i} + 2[1+v(v-1)] - 2-v(v-1) > 0 \text{ for } v\geq 4. \qquad \square$$

This means that the correction factor in (A.7), $c_n = - f(\tau_n)/f'(\tau_n)$, is negative if $f(\tau_n) > 0$ and positive if $f(\tau_n) < 0$. As the process converges then

$$
\begin{aligned}
&\tau_n < \tau_{n+1} < \tau'' \qquad && \text{if } f(\tau_n) < 0 \,, \\
&\tau_n > \tau_{n+1} > \tau'' \qquad && \text{if } f(\tau_n) > 0 \,,
\end{aligned}
$$

i.e.,

$$\tau_n < \tau_{n+1} < \tau'' \quad \text{if and only if (iff) } \tau^* > \tau_n/s(\tau_n),$$

i.e.,

$$\tau_n < \tau_{n+1} < \tau'' \quad \text{iff} \quad \tau_n/\tau^* < 1 - \frac{p_n \tau_n}{q_n}(1-b). \qquad \square \quad \text{(A.7)}$$

Theorem 3. Let $\tau=z\mu$, $\tau^*=z\mu$, $\tau'=z\mu$, $\tau''=z\mu''$, and

$$\tau^* = \tau/[1-\frac{p\tau}{qw}(w-1)]$$

$$\tau' = \tau/[1-\frac{p\tau}{qw}(w-w')]$$

and let τ'' be the unique solution to the equation:

$$\tau^* = \tau/[1-\frac{p\tau}{qw}(w'-1)].$$

Then the following relations are true:

$$\begin{array}{llllllll}
\mu & < \mu'' & < \mu' & < \mu^* & \text{if} & 1 & < w' & < w, \\
\mu' & < \mu'' & < \mu & < \mu^* & \text{if} & 1 & < w & < w', \\
\mu & < \mu^* & < \mu' & < \mu'' & \text{if} & w' & < 1 & < w, \\
\mu^* & < \mu'' & < \mu' & < \mu & \text{if} & w & < w' & < 1, \\
\mu^* & < \mu & < \mu' & < \mu'' & \text{if} & w' & < w & < 1, \\
\mu' & < \mu'' & < \mu^* & < \mu & \text{if} & w & < 1 & < w'.
\end{array}$$

Proof. The relations between μ, μ^* and μ' are easily found by some algebra:

$$\tau/\tau^* = 1-\frac{p\tau}{qw}(w-1) \quad \rightarrow \tau<\tau^* \quad \text{iff } 1<w, \qquad \text{(a)}$$

$$\tau/\tau' = 1-\frac{p\tau}{qw}(w-w') \quad \rightarrow \tau<\tau' \quad \text{iff } w'<w, \qquad \text{(b)}$$

$$\tau^*/\tau' = 1+\frac{\frac{p\tau}{qw}(w'-1)}{1-\frac{p\tau}{q}(1-\frac{1}{w})} \quad \rightarrow \tau^*<\tau' \quad \text{iff } w'<1. \qquad \text{(c)}$$

To compare μ'' with μ, μ^* and μ', respectively, we use the result in Theorem 2. Assume that $\tau_n=\tau$, $\tau_n=\tau^*$ and $\tau_n=\tau'$, respectively, are sufficiently close to τ''. Inserting these terms into:

$$\tau_n<\tau'' \text{ iff } \tau_n/\tau^* < 1-\frac{p_n\tau_n}{q_n w'}(w'-1)$$

yields:

$\tau<\tau''$ iff

$$\tau[1-\frac{p\tau}{qw}(w-1)] < \tau[1-\frac{p\tau}{qw'}(w'-1)] \quad \rightarrow \tau<\tau'' \quad \text{iff } w'<w, \qquad \text{(d)}$$

$$\tau^*<\tau'' \text{ iff } 1 < 1-\frac{p^*q^*}{q^*w'}(w'-1) \quad \rightarrow \tau^*<\tau'' \quad \text{iff } w'<1, \qquad \text{(e)}$$

$\tau' < \tau''$ iff

$$\frac{\tau'}{\tau^*} < 1 - \frac{p(\tau')\tau'}{q(\tau')w'}(w'-1) \text{ , i.e., iff}$$

$$\frac{p(\tau')}{q(\tau')w'}(w'-1) < \frac{1}{\tau'} - \frac{1}{\tau^*} \text{ , i.e., iff}$$

$$\frac{p(\tau')}{q(\tau')w'}(w'-1) < \frac{1}{\tau}\left[-\frac{p\tau}{qw}(w-w') + \frac{p\tau}{qw}(w-1)\right], \text{ i.e., iff}$$

$$\frac{p(\tau')}{q(\tau')w'}(w'-1) < \frac{1}{\tau}\left[-\frac{p\tau}{qw}(w-w') + \frac{p\tau}{qw}(w-1)\right], \text{ i.e., iff}$$

$$\frac{p(\tau')q}{q(\tau')p}(w'-1) < \frac{w'}{w}(w'-1). \tag{f.1}$$

Inserting the approximation:

$$\frac{p(\tau')q}{q(\tau')p} = \frac{\exp(-\tau)-1}{\exp(-\tau')-1} \approx \frac{\tau}{\tau'} = 1 - \frac{p\tau}{qw}(w-w') \quad \text{into (f.1) yields:}$$

$$\left(1 - \frac{p\tau}{q}\right)\frac{(w'-1)(w-w')}{w} < 0 \quad \rightarrow \tau' < \tau'' \text{ iff } \begin{matrix} w' < w,1 \\ w' > w,1 \end{matrix} \text{ or } . \tag{f2}$$

Combining the results (a)-(f) yields:

if	$1 < w' < w$ then	$1 < w < w'$ then	$w' < 1 < w$ then	$w < w' < 1$ then	$w' < w < 1$ then	$w < 1 < w'$ then
(a)	$\tau < \tau^*$	$\tau < \tau^*$	$\tau < \tau^*$	$\tau^* < \tau$	$\tau^* < \tau$	$\tau^* < \tau$
(b)	$\tau < \tau'$	$\tau' < \tau$	$\tau < \tau'$	$\tau' < \tau$	$\tau < \tau'$	$\tau' < \tau$
(c)	$\tau' < \tau^*$	$\tau' < \tau^*$	$\tau^* < \tau'$	$\tau^* < \tau'$	$\tau^* < \tau'$	$\tau' < \tau^*$
(d)	$\tau < \tau''$	$\tau'' < \tau$	$\tau < \tau''$	$\tau'' < \tau$	$\tau < \tau''$	$\tau'' < \tau$
(e)	$\tau'' < \tau^*$	$\tau'' < \tau^*$	$\tau^* < \tau''$	$\tau^* < \tau''$	$\tau^* < \tau''$	$\tau'' < \tau^*$
(f)	$\tau'' < \tau'$	$\tau' < \tau''$	$\tau' < \tau''$	$\tau'' < \tau'$	$\tau' < \tau''$	$\tau' < \tau''$
(a)-(f)	$\tau<\tau''<\tau'<\tau^*$	$\tau'<\tau''<\tau<\tau^*$	$\tau<\tau^*<\tau'<\tau''$	$\tau^*<\tau''<\tau'<\tau$	$\tau^*<\tau<\tau'<\tau''$	$\tau'<\tau''<\tau^*<\tau$

List of notations

Parameters:

$\mu_j(t)$ = transition intensity as defined in Section 1.

$p=1-q$ = $\exp(-z\mu)$ = the survival probability for constant transition intensity μ and censoring time z.

q_j = $q\mu_j/\mu$ = the transition probability for cause j.

r_j, r_0 = the response probabilities among decrements from cause j (j=1,2, ..., K) and survivors, respectively.

Variables:

Q_j = 1 if transition due to cause j, $Q_j=0$ otherwise.

$Q = \Sigma Q_j$ = 1 if transition due to any cause, Q=0 otherwise.

R = 1 if the individual responds, R=0 otherwise.

T = min(U,z), where U is the time of transition out of State 0, and z is the censoring time.

Q_j^*, Q^*, T^* = RQ_j, RQ, RT, respectively, observed variables that are equal to 0 for the nonrespondents.

$\hat{r}_j$, $\hat{r}_0$ = observed response rates.

n_r = number of respondents.

Estimators:

$\hat{\mu}$ = central rate (occurrence/exposure rate) based on the whole sample.

$\hat{\mu}_j^*$ = corresponding rate based on the respondents (2.1).

$\hat{\mu}_j'$ = estimator adjusted by weighting (6.1).

$\hat{\mu}_j''$ = estimator adjusted by the iterative method (6.5).

q_j', p' = estimators of transition and survival probabilities based on estimated transition intensities (5.6).

q_j'', p'' = estimators of transition and survival probabilities based on observed proportions (5.7).

n_r = number of respondents.

Expected values:

r_Q = $\Sigma r_j\mu_j/\mu$ = the overall expected response rate among all departures.

r = $qr_Q + pr_0$ = the overall expected response rate.

r_{-j} = the overall expected response rate among decrements from any cause other than j.

q_j^*, q^*, t^* = expected values of Q_j^*, Q^*, and T^*, repectively (A.1).

μ_j^* = q_j^*/t^*.

R & D Reports är en för U/ADB och U/STM gemensam publikationsserie, som fr o m 1988-01-01 ersätter de tidigare "gula" och "gröna" serierna. I serien ingår även **Abstracts** (sammanfattning av metodrapporter från SCB).

R & D Reports Statistics Sweden are published by the Department of Research & Development within Statistics Sweden. Reports dealing with statistical methods have green (grön) covers. Reports dealing with EDP methods have yellow (gul) covers. In addition, abstracts are published three times a year (light brown /beige/ covers).

Reports published during 1991:

1991:1 (grön)	Computing elementary aggregates in the Swedish consumer price index **(Jörgen Dalén)**
1991:2 (grön)	Översikt av estimatorer för stora och små redovisningsgrupper **(Sixten Lundström)**
1991:3 (grön)	Interacting nonresponse and response errors **(Håkan L Lindström)**

Kvarvarande **beige** och **gröna** exemplar av ovanstående promemorior kan rekvireras från Inga-Lill Pettersson, U/LEDN, SCB, 115 81 STOCKHOLM, eller per telefon 08-783 49 56.

Kvarvarande **gula** exemplar kan rekvireras från Ingvar Andersson, U/ADB, SCB, 115 81 STOCKHOLM, eller per telefon 08-783 41 47.